A Comparison between Two Metaheuristics Applied to the Cell Formation Problem with Alternative Routings

Orlando Durán A., Luis Pérez P., and Felipe Olmos de Aguilera

***Abstract*—This work proposes a genetic algorithm for optimization of the cell formation problem with alternative routings. A series of test problems were generated and used to evaluate the performance of the proposed Genetic Algorithm and a Simulated Annealing algorithm as well. The novelty of the proposed work lies in the representation technique and the transformations that allow treating the original multidimensional problem as a two-dimensional one. That simplified the programming tasks and the resolution method.**

***Index Terms*—Manufacturing systems design, manufacturing cells, computational intelligence, genetic algorithms, simulated annealing.**

I. Introduction

Cellular manufacturing is an organizational approach based on group technology (GT) that structures a plant as a set of manufacturing cells. Each cell consists in a set of production equipment that process similar components. Cellular based organizations provide considerable cost and productivity benefits to practical manufacturing environments. A list of advantages derived of grouping machines into cells can be found in [1]. The main challenge in the design of manufacturing cells is the identification of machines and components that will make part of a cell. This identification process requires an effective approach to form part families so that similarity within a part family can be maximized. Clustering analysis is the most frequently used method for manufacturing cell design. However, there is still the challenge of creating an efficient clustering method because the cellular formation problem (CFP) is a NP-complete problem. Additional difficulties arise if one considers that many components could have alternative processing methods or routings. This last aspect increases significantly the complexity of this type of problems.

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A manufacturing cell is an organizational structure that groups similar machines that have similar design features or processing capabilities to constitute more efficient production systems. Each cell is composed of a number of machines so that they can produce and maintain continuous production flows in order to reduce the time lost in transfer between workstations. In Figure 1, a basic outline of a manufacturing cell is shown, where the input of a part, its processing sequence, and its output are displayed.

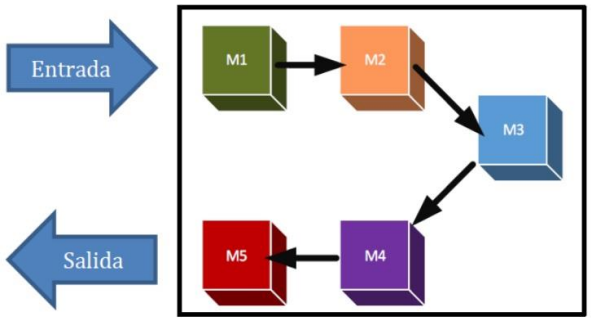

Fig. 1. Magnetization Schematic representation of a manufacturing cell

The most used structure to represent a set of machine-workpiece relationships is the incidence matrix. Figure 2 represents a binary incidence matrix; in which each one of the 1S correspond to the machines that a certain workpiece uses in its production process.

S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	1	2	1	2	3	3
S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	1	1	2	1	2	3
S_{30}	S_{31}	S_{32}	S_{33}	S_{34}	S_{35}	1	2	3	2	1	3
S_{40}	S_{41}	S_{42}	S_{43}	S_{44}	S_{45}	1	2	2	3	1	3

Fig. 2. The incidence matrix

From the incidence matrix in Figure 2 it can be said:

- Workpiece 1 pass through machines 3 and 4.
- Workpiece 2 will go through machines 1 and 2.
- Workpiece 3 will pass through machines 3 and 5.
- Workpiece 4 pass through machines 1 and 2.
- Workpiece 5 pass through machines 3 and 4.
- Workpiece 6 will pass through machines 5 and 6.

To form manufacturing cells it is necessary to apply some clustering method to detect which are the machines and parts to be grouped so as to obtain all the advantages mentioned above. Figure 3 shows how it is an incidence matrix made after the grouping.

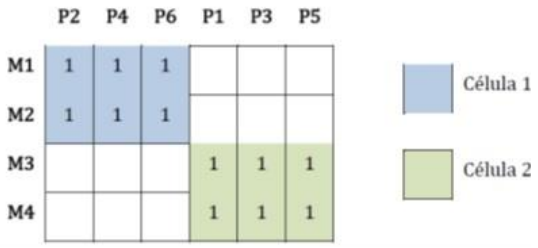

Fig. 3. An illustrative example after the clustering process.

Many works related to manufacturing cell formation assume that each piece has a unique sequence. This is far from reality because any operation on a given workpiece can be performed on alternative machines. This introduces new variables to the cell formation problem. In Figure 4, alternative manufacturing sequences are shown for the same parts and machines of Figure 2.

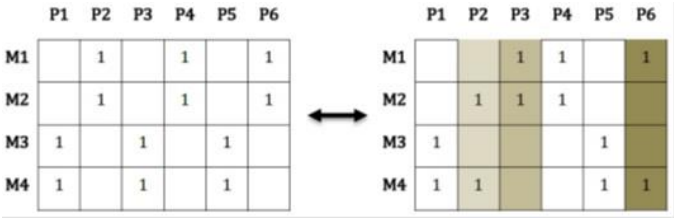

Fig.4. Alternative manufacturing sequences.

From the incidence matrix in Figure 4, it can be said:

- Workpieces 1, 4 and 5 have no alternative routes.
- Workpiece 2 can be processed by machines 1 and 2, or 2 and 4.
- Workpiece 3 can be processed by machines 3 and 4, or 1 and 2.
- Workpiece 6 can be processed by machines 1 and 2, or 1 and 4.

II. Metaheuristics and Machine Grouping

In recent years, different metaheuristic methods have been used to solve the cell-formation problem. An extensive review of the use of metaheuristics in cellular manufacturing is presented by [2]. From the point of view of alternative routings fewer works have been reported in literature. Rodrigues and Weller [3] considered alternative routing to minimize extra-cellular processing of task applying a Tabu search combined with a branch and bound based strategy.

Caux et al. [4] proposed a method that considers alternative routings and machine capacity constraints. The proposed algorithm simultaneously deals with the cell formation problem and the part-routing assignment problem where one of problems was then solved from the solutions of the other. Jayaswal and Adil [5] proposed a SA-based heuristic methodology considering operational sequence, machine replication, alternative process routings to minimize the inter-cell movements and machine investments and operating costs. Wu et al. [6] proposed a hybrid SA method with genetic operation considering alternative process routing and insertion move was utilized in solution improvement stage in order to speed up solution search and to escape from local optima.

Chan et al. [7] presented a multi-objective optimization model using a GA approach to solve the proposed model. Hu and Yasuda [8] addressed the cell formation problem with alternative process routes developing a GA methodology with new chromosome representation, separating crossover heuristic and special mutation technique which produced efficient and optimal solution. Kao and Lin [9] proposed a PSO based algorithm for cell definition. The proposed approach considers a twofold procedure: machine partition and part-routing assignment. Experimental results demonstrated that the algorithm found equal or fewer exceptional elements than existing algorithms for most of the test problems selected from the literature.

III. Chromosome Representation

To model the problem we used a set of vector and matrices that we describe in the following:

- An incidence matrix,(a_{ij}).
- A multilayer matrix where the third dimension is associated to alternative routings for each one of the component (o_{ijn}).
- Machine matrix (y_{ik}).
- Component matrix (z_{jk}).

The multilayer matrix (o_{ijn}) represents the union of a series of alternative routings for every component that make part of the problem, where each layer (n: number of layers) represents an alternative way of manufacture each one of the components. Therefore, the number of layers will be equal to the maximum number of alternative routings that any component has. Figure 5 shows a multilayer matrix with three alternative routings for each component.

Fig. 5. A three-layer incidence matrix.

The algorithm works with a 2-dimension matrix (MxP), thus the initialization process consists in selecting for each one of the component one of the alternative routings from o_{ijn} to construct a 2-dimensional matrix (a_{ij}) that takes part of the

optimization process. Considering, for instance, Figure 5, the matrix shown in Figure 6 is obtained according to the following process: the first row indicates that the component 1 is using the second alternative routing; the component 2 uses the first alternative routing and the components 3 and 4 uses the third alternative routing. The Machine matrix (y_{ik}) represents, with a 1, where each one of the machines (indicated by the row i) is assigned to a specific cell (indicated by the column k). Component matrix (z_{jk}) represents, with a 1, where each one of the components (indicated by the row j) is assigned to a specific cell (indicated by the column k).

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Fig.6. An incidence matrix resulting from the initial transformation.

Therefore, the machine and the component matrices will have many columns as manufacturing cells will be defined. For instance, the representation of a random solution with 2 cells (columns), 4 machines, and 6 components is shown in Figure 7.

1	0
0	1
1	0
0	0
0	0

0	1
1	0
1	0
0	0
0	1
0	1

Fig 7. a) Machine matrix; b) Component matrix.

Figure 7a shows the machine cell and Figure 7b shows the component matrix with 6 components and the same 2 cells. To obtain the incidence matrix (a_{ij}) the union of matrices y_{ik} and z_{jk} is generated. Following, to perform the genetic operators we transformed the matrix into a vector shown in Figure 8.

0	1	0	2	2	1	0	0	2	1	1

Fig.8. Partial chromosome representation

In addition, to the end of the vector a sequence is added with the information corresponding to the columns that were taken of the multilayer matrix to form the incidence matrix. Thus, the starting point will be a vector of length (M + P + P). It will contain the machines and parts locations along with the information of the layer (alternative routing) that it was considered for each one of the workpieces (Figure 9). Concerning the incidence matrix, columns 1, 2 and 5 were taken from the first layer, and the 3, 4 from the second.

0	1	0	2	2	1	0	0	2	1	1	1	1	2	2	1	2

Fig.9. Final chromosome representation

IV. Optimization Model

The fitness function is designed to deliver the number of items that were outside the cells. For this, the incoming vector virtually divided into three parts (see Figure 13) is taken, and from the first M components (M: Number of machines) matrix y is formed (M × C). With the following P components (P: Number of workpieces) the z matrix is formed (P × C), and the last M components determine which columns are taken from the multilayer matrix to form the incidence matrix A.

The cell formation problem with alternative routings can be formulated as follows:

- M be the number of machines,
- P, the number of parts,
- C, the number of cells,
- i, the index of machines ($i = 1, \ldots, M$),
- j, the index of parts ($j = 1, \ldots, P$),
- k, the index of cells ($k = 1, \ldots, C$),
- $A = [a_{ij}]$, M × P binary machine-part incidence matrix,
- M_{max}, the maximum number of machines per cell.

$$y_{ik} = \begin{cases} 1 & \text{if machine } i \in \text{cell } k; \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{jk} = \begin{cases} 1 & \text{if part } j \in \text{family } k; \\ 0 & \text{otherwise.} \end{cases}$$

We selected as the objective function to be minimized the number of times that a given part must be processed by a machine that does not belong to the cell that the part has been assigned to. The problem is represented by the following mathematical model:

Minimize N:

$$N = \sum_{k=1}^{C} \sum_{i=1}^{M} \sum_{j=1}^{P} a_{ij} z_{jk} (1 - y_{ik})$$

Subject to:

$$\sum_{k=1}^{C} y_{ik} = 1 \qquad \forall i,$$

$$\sum_{k=1}^{C} z_{jik} = 1 \qquad \forall j,$$

$$\sum_{i=1}^{M} y_{ik} \leq M_{max} \qquad \forall k .$$

V. Numerical Illustration

The purpose of this section is to show, through numerical examples, how the proposed formulation can be used to design a cellular manufacturing system with alternative routings. To test the performance of the proposed model, we randomly

generated a set of problems with 16 machines, 30 components, and 2 alternative routings.

The genetic algorithm was applied to four randomly generated problems. A set of multilayer matrices filled with 20%, 40%, 60% and 80% of 1s respectively was generated; where 0% represents an empty matrix without operations, and 100% represents a saturated matrix, where each workpiece should go through every machine in the manufacturing system.

A set of tests were performed using the Genetic Algorithm and a Simulated Annealing with the same solution representation. The following control parameters were set to the Simulated Annealing algorithm: Initial Temperature: 8; The logarithmic function was used as the temperature reduction function. In addition, we used the following annealing intervals: 50, 100, 150, y 200. For the Genetic Algorithm, a traditional crossover operator was chosen by which the two parents produce two children and are replaced by them. For the mutation operator, it is applied randomly to 10% of the individuals. Finally, the GA stops when one of two conditions is met: (i) the fitness value of the best individual does not improve after P iterations, or (ii) the total number of iterations exceeds a maximum number ($P \times P$). The use of the number of parts (P) in the stopping criteria draws from the fact that the size of the search space is directly dependent of that number.

TABLE I
COMPARISON OF THE AVERAGE VALUES OBTAINED BY THE GA AND SA.

	$M_{max} = 6$		$M_{max} = 8$		$M_{max} = 10$		$M_{max} = 12$	
	SA	GA	SA	GA	SA	GA	SA	GA
20% of 1s	31,65	25,62	29,65	17,88	26,28	14,3	21,03	8,66
40% of 1s	90,05	82,76	71,4	60,4	58,08	52,86	35,63	33,12
60% of 1s	151,18	144,8	116	108	92,83	89,26	59,65	58,3
80% of 1s	214,4	204,4	163,5	156,7	123,3	125,1	82,4	82

TABLE II
COMPARISON OF THE BEST VALUES OBTAINED BY THE GA AND SA.

	$M_{max} = 6$		$M_{max} = 8$		$M_{max} = 10$		$M_{max} = 12$	
	SA	GA	SA	GA	SA	GA	SA	GA
20% of 1s	29	22	24	13	19	10	14	0
40% of 1s	85	77	63	56	52	49	32	33
60% of 1s	146	137	110	104	90	86	58	58
80% of 1s	210	202	159	154	125	125	82	82

The first test consisted in defining 3 cells with varying the maximum number of machines per cell (Mmax= 6, 8, 10, and 12). In addition, the number of 1s in each initial incidence matrix varied according to the following: 20%, 40%, 60% y 80%; the population size was also tested according to the following: 1,000, 3,000, 5,000, 7,000 and 9,000 individuals. Each test was performed 100 times. In Table I, the average results obtained by both techniques are shown using 3,000 as the population size of the GA. Table II shows the best results obtained by the experiments detailed previously. It can be observed that in every test (except one) GA outperforms the results obtained by the use of SA.

The second test consisted in obtain the arrangements varying the number of cells (2, 3 and 4 cells). In addition, the number of 1s in each initial incidence matrix varied according the same strategy of the first test; the population size was also tested according to the following: 1,000, 3,000, 5,000, 7,000 and 9,000 individuals. Each test was performed 100 times. In Tables 3 and 4 the results obtained by both techniques are shown using 3,000 as the population size of the GA. It can be observed that in every test GA outperforms in average the results obtained by the use of SA (Tables III and IV).

TABLE III
COMPARISON OF THE AVERAGE VALUES OBTAINED BY THE GA AND SA IN THE 2ND TEST.

		2 cells		3 cells		4 cells	
		SA	GA	SA	GA	SA	GA
20% of 1s	Average	17,15	14,14	26,28	14,3	34,18	14,36
40% of 1s	Average	53,88	51,12	57,3	52,34	67,28	52,96
60% of 1s	Average	92,63	87,66	93,93	89,64	100,15	89,96
80% of 1s	Average	129,13	87,66	127,6	125,1	130,93	125,3

TABLE IV
COMPARISON OF THE BEST VALUES OBTAINED BY THE GA AND SA IN THE 2ND TEST ($M_{MAX} = 10$).

2 cells		3 cells		4 cells	
SA	GA	SA	GA	SA	GA
13	11	19	10	25	11
50	48	51	48	61	48
88	86	89	87	94	87
125	125	125	125	125	125

VI. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

A novel representation scheme for solving the cell formation problem with alternative routes is proposed and tested. The proposed technique is feasible and simple. In addition, comparisons were performed. The SA proved that performs well, however the genetic algorithm outperforms the S.A. In addition, we can conclude that this behavior does not depend on the number of 1s in the incidence matrix, i.e. the density of operations or how intensive are the process routings, because in every tested scenario (number of 1s) GA surpasses the SA algorithm. We recommend using genetic algorithm, due to better explore the space on the basis of population, and along with that simulated annealing converges to local minima easily could be better.

The novelty of the proposed work lies in the representation technique and the transformations that allow treating the original multidimensional problem as a two-dimensional one. This simplified the programming tasks and the resolution method.

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