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Multi-day-ahead traffic accidents forecasting based on Singular Spectrum Analysis and Stationary Wavelet Transform combined with Linear Regression: A comparative study

Lida Barba and Nibaldo Rodríguez

***Abstract*—Numerous statistical and mathematical methods have been developed in order to explain the complexity of nonstationary time series. Singular Spectrum Analysis (SSA) and Wavelet Transform (WT) are two potent theories with different mathematical foundations that have been used in several applications with successful results; however in most studies SSA and WT have been presented separately, then there is a lack of systematic comparisons between SSA and WT in time series forecasting. Consequently the aim of this work is to evaluate the performance of two hybrid models, one is based on SSA combined with the Autoregressive model (SSA-AR), and the other is based on Stationary Wavelet Transform combined with AR (SWT-AR). The models are described in two stages, the first stage is the time series preprocessing and the second is the prediction. In the preprocessing the low frequency component is obtained, and by difference the high frequency component is computed. Whereas in the prediction stage the components are used as input of the Autoregressive model. The empirical data applied in this study corresponds to the traffic accidents domain, they were daily collected in the Chilean metropolitan region from 2000 to 2014 and are classified by relevant causes; the data analysis reveals important information for road management and a challenge for forecasters by the nonstationary characteristics. The direct strategy was implemented for 7-days-ahead prediction, high accuracy was observed in the application of both models, SWT-AR reaches the best mean accuracy, while SSA-AR reaches the highest accuracy for farthest horizons.**

I. Introduction

Singular Spectrum Analysis (SSA) and Wavelet Transform (WT) are two potent methods with different mathematical foundations that have been successfully applied in time series analysis; nevertheless, in the literature review there is a lack of systematic comparisons between SSA and WT even more in forecasting. Singular Spectrum Analysis is a nonparametric spectral estimation method which is used to decompose a time series into a sum of components such as trend, cyclical, seasonal, and noise. SSA is defined in four steps, embedding, Singular Value Decomposition (SVD), grouping and diagonal averaging, which are summarized in Decomposition and Reconstruction [1]. The beginning of the SSA method is attributable to Loève (1945)[2], Karhunen (1946), [3] and Broomhead-King (1986) [4]. Some researches have taken advantage of the SSA flexibility to apply it in diverse fields; favorable results were obtained in climatic series [5], [6], energy [7], industrial production [8], tourist arrivals [9], trade [10], among others. Although the SSA flexibility allows its usage in a wide range of relatively short time series, there is a lack of standard methods to select the window length, which is a principal parameter in the decomposition.

On the other hand the wavelet decomposition is a popular method of nonstationary time series analysis in the time-frequency domain. Successful results have been obtained in a wide number of applications such as hydrology [11, 12], biological signals [13], common energy consumption [14], financial market [15], [16], marketing [17], among others. The wavelet analysis provides spectral and temporal information in different spatial and temporal scales. The Continuous Wavelet transform (CWT) [18] and the Discrete Wavelet Transform (DWT) [19, 20] are used to obtain a representation form of a time series. CWT calculates wavelet coefficients at every possible scale, which requires a significant amount of computational resources and it generates redundant information. Whereas DWT calculates the wavelet coefficients based on discrete dyadic scales; DWT reduces the computational complexity and generally non-redundant information, however it is prone to shift sensitivity, which is an undesirable feature in forecasting [21]. Stationary Wavelet Transform (SWT) is based on a nonorthogonal multiresolution algorithm for which the DWT is exact [22], besides SWT is shift invariant. The SWT decomposition is dependent of two parameters, the wavelet function and decomposition levels, regrettably as in SSA, there is no standard methods to take decisions about these parameters configurations.

This paper contribution is a systematic comparison of hybrid models based on Singular Spectrum Analysis combined with Autoregressive model (SSA-AR) and Stationary Wavelet Transform combined with the same Autoregressive model

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(SWT-AR) for multi-step ahead forecasting of nonstationary time series. A daily time series of injured in traffic accidents is used to evaluate the hybrid models; the data were collected in the Chilean metropolitan region (Santiago) from year 2000 to 2014 [23]. This paper is organized as follows. Section II describes the methodology used to implement SSA and SWT. Section III presents the prediction based on components. Section IV shows the efficiency criteria. The Results and Discussion are described in Section V. Finally Section VI concludes the paper.

II. Methodology

The forecasting methodology is described in two stages, preprocessing and prediction (Fig. 1). Singular Spectrum Analysis (SSA) and Stationary Wavelet Transform (SWT) are implemented in the preprocessing stage. The aim of SSA and SWT is to decompose an observed signal x in components of equal size and different dynamic, in this case of low and high frequency, c_L and c_H respectively. In the prediction stage the Autoregressive (AR) model is implemented to predict the components.

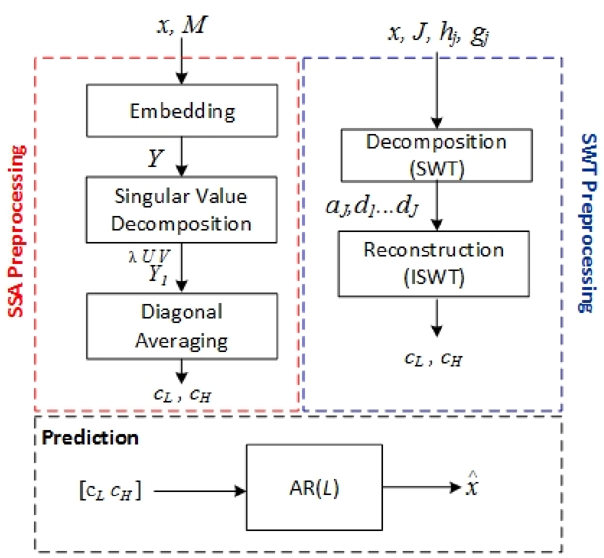

Fig. 1. Forecasting methodology

The one-step ahead forecasting model is defined with the next expression:

$$\hat{x}(n+1) = f\left[c_L(n), c_H(n)\right], \tag{1}$$

where $\hat{x}$ is the predicted value and n represents the time instant.

A. Preprocessing based on Singular Spectrum Analysis

The preprocessing through Singular Spectrum Analysis was illustrated in Fig. 1. The observed signal x through SSA is decomposed and the low frequency component c_L is extracted; while the component of high frequency c_H is computed by simple difference between the observed signal x and the component c_L. Conventionally the SSA implementation is defined in four steps: embedding, decomposition, grouping, and reconstruction by diagonal averaging [1]; but in this work the grouping step is not performed.

1) Embedding: The embedding step maps the time series x of length N in a sequence of M multidimensional lagged vectors of length K; the embedding process is shown below

$$Y = \begin{pmatrix} x_1 & x_2 & \dots & x_K \\ x_2 & x_3 & \dots & x_{K+1} \\ \vdots & \vdots & \vdots & \vdots \\ x_M & x_{M+1} & \dots & x_N \end{pmatrix}, \tag{2}$$

where Y is a real matrix of MxK dimension, with $M < K$, and $K = N - M + 1$. The matrix Y is a Hankel matrix which means that the elements x_{ij} on the anti-diagonals $i + j$ are equal.

2) Decomposition: The decomposition step implements the SVD of the trajectory matrix Y. The SVD of an arbitrary nonzero $M \times K$ matrix $Y = [Y_1 : ... : Y_K]$ is a decomposition of Y in the form

$$Y = \sum_{i=1}^{M} \sqrt{\lambda_i} U_i V_i^{\top}, \tag{3}$$

where λ_i is the ith eigenvalue of matrix $S = YY^{\top}$ arranged in decreasing order of magnitudes. $U_1, \dots, U_M$ is the corresponding orthonormal system of eigenvectors of the matrix S.
Standard SVD terminology calls $\sqrt{\lambda_i}$ the ith singular value of matrix Y; U_i is the ith left singular vector, and V_i is the ith right singular vector of Y. The collection $(\sqrt{\lambda_i}, U_i, V_i)$ is called ith eigentriple of the SVD.

3) Reconstruction: The first eigentriple is used to extract the low frequency component c_L, the remainder eigentriples are not used, in that reason the grouping is not performed. The elemental matrix which contains the c_L component is computed with:

$$Y_1 = \sqrt{\lambda_1} U_1 V_1^{\top}. \tag{4}$$

The reconstruction is performed by diagonal averaging over Y_1. The elements $c_L(i)$ are extracted as follows:

$$c_L = \begin{cases} \frac{1}{k-1} \sum_{m=1}^{k} Y_1(m, k-m), & 2 \le k \le M, \\ \frac{1}{M} \sum_{m=1}^{M} Y_1(m, k-m), & M < k \le K+1, \\ \frac{1}{K+M-k+1} \sum_{m=k-K}^{M} Y_1(m, k-m), & K+2 \le k \le K+M. \end{cases} \tag{5}$$

The complementary component is the component of high frequency c_H, therefore $c_H = x - c_L$. Although c_H was not directly extracted by SSA, it was calculated from the component c_L, therefore c_H is an indirect product of the SSA decomposition.

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B. Stationary Wavelet Transform

The preprocessing through Stationary Wavelet Transform was illustrated in Figure 1. SWT is based on the Discrete Wavelet Transform, its implementation is defined in the algorithm of Shensa [22]. SWT implements up-sampled filtering [24, 25].
In SWT the length of the observed signal must be an integer multiple of 2^j, with $j = 1, 2, \ldots, J$; where J is the scale number. The signal is separated in approximation coefficients and detail coefficients at different scales, this hierarchical process is called multiresolution decomposition [26].

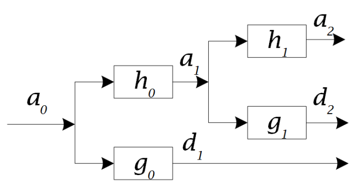

Fig. 2. Decomposition scheme of SWT (with 2 levels)

The observed signal a_0 (which was named x in previous section) is decomposed in approximation and detail coefficients through decomposition low pass filters $(h_0, h_1, \ldots, h_{J-1})$, and decomposition high pass filters $(g_0, g_1, \ldots, g_{J-1})$, one to each level as the scheme of Fig. 2. Each level filters are up-sampled versions of the previous ones.
At the first decomposition level, the observed signal a_0 is convoluted with the first low pass filter h_0 to obtain the first approximation coefficients a_1 and with the first high pass filter g_0 to obtain the first detail coefficients d_1. The process is defined as follows

$$a_1(n) = \sum_i h_0(i) a_0(n-i), \tag{6a}$$

$$d_1(n) = \sum_i g_0(i) a_0(n-i), \tag{6b}$$

This process follows iteratively, for $j = 1, \ldots, J$-1 and it is defined bellow:

$$a_{j+1}(n) = \sum_i h_j(i) a_j(n-i), \tag{7a}$$

$$d_{j+1}(n) = \sum_i g_j(i) a_j(n-i), \tag{7b}$$

Inverse Stationary Wavelet Transform (ISWT) performs the reconstruction. The implementation of ISWT consists in applying the operations that were done in SWT in inverse order and based on low-pass and high-pass reconstruction filters. The last coefficient approximation a_J reconstructs the component of low frequency c_L, whereas all detail coefficients reconstruct the component of high frequency c_H, both components are never decimated, therefore they have the same length as the observed signal.

III. Prediction Stage: Autoregressive model based on Components

The prediction is the second stage in the forecasting methodology (Fig. 1), and it depends of the first stage of preprocessing. The multistep-ahead forecasting is based on the direct method, which implements τ AR models of equal structure to predict the variable at time $n+h$ based on the linear relationship between L previous values of the components and the future value of the component at time $n+h$. The model uses the time series of length N which is split in two groups, training and testing, with length N_r and N_t respectively.
The following equation defines the general structure of the AR model in matrix notation:

$$\hat{y} = \beta Z, \tag{8}$$

where $\hat{y}$ is the predicted value of y with length N_r, Z is the regressor matrix and β is the linear coefficients matrix. The regressor matrix Z has order N_rx$2L$ due to L lagged values of c_L and the L lagged values of c_H. The matrix of coefficients β has order 2x$2L$ (one row for each component), and they are computed with the Least Square Method (LSM) [27], as follows:

$$\beta = Z^{\dagger} y, \tag{9}$$

where $Z^{\dagger}$ is the Moore-Penrose pseudoinverse matrix [27].
The direct strategy estimates τ models between regressors to compute h-step ahead prediction [28]. Each model returns a direct forecast of $\hat{x}(n+h)$. This strategy can be expressed as

$$\hat{x}(n+h) = f\left[Z(n), Z(n-1), \ldots, Z(n-L+1)\right], \tag{10}$$

where $h = 1, \ldots, \tau$.

IV. Efficiency Criteria

Two efficiency criteria are computed to evaluate the prediction accuracy of multi-step ahead prediction. One is the modified version of Nash-Suctlife Efficiency (*MNSE*); *MNSE* is computed to overcome the oversensitivity to extreme values induced by the mean square error of original Nash-Sutcliffe efficiency and to increase the sensitivity for lower values [29]:

$$MNSE = 1 - \frac{\sum_{i=1}^{N_t} |x_i - \hat{x}_i|}{\sum_{i=1}^{N_t} |x_i - \bar{x}|}, \tag{11}$$

where x_i is the ith observed value $\hat{x}_i$ is the ith predicted value, $\bar{x}$ is the mean of x, and N_t is the testing sample size.
The prediction accuracy was also evaluated through Mean Absolute Percentage Error (*MAPE*), Coefficient of Determination (R^2), and Relative Error (*RE*).

$$MAPE = \frac{1}{N_t} \sum_{i=1}^{N_t} \left| \frac{x_i - \hat{x}_i}{x_i} \right|. \tag{12}$$

$$R^2 = 1 - \frac{\sigma^2(x - \hat{x})}{\sigma^2(x)}, \tag{13}$$

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where σ^2 is the variance.

$$RE = \frac{(x-\hat{x})}{x}. \quad (14)$$

V. Experimental Results and Discussion

A. Data

The forecasting performance of hybrid models SSA-AR and SWT-AR are evaluated through a time series of injured in traffic accidents. The data were daily collected by the Chilean police and the National Traffic Safety Commission (CONASET) [23] from year 2000 to 2014 in Santiago, Chile with N =5479 records; the data reveals 260073 injured persons due to 58 causes defined by CONASET. In this work the problem is focused on principal causes via ranking; it was found that 15 causes are present in 80% of injured people in traffic accidents, which are categorized in *imprudent driving*, *pedestrian recklessness*, *signal disobedience*, *alcohol in driver*, *vehicle control loss*, and *mechanical causes*. The complex dynamic of the series is observed in Fig. 3. Nonstationary characteristics were probed with the KPSS test [30].

B. Prediction based on Singular Spectrum Analysis and the Autoregressive model

This forecasting is based on the component of low frequency extracted with SSA, and its complementary component of high frequency computed by subtraction. Therefore the performance model depends of the components that were provided. The order of the AR model was set by information of the Autocorrelation Function (ACF). Fig. 4a shows 7, 14, 21, 28, 31, and 35 day peaks; the multiple harmonic $L = 14$ was chosen to implement a parsimonious model.

One-step ahead forecasting was implemented with $L = 14$; the window length was evaluated through the metric *MNSE*, as it is shown in Fig. 4b. The effective window length was set in $M = 7$, which reaches a *MNSE* =98.4%; consequently the Hankel matrix has 7×5473 dimension. The component of low frequency c_L was extracted with SSA, and the component of high frequency is its complement. The SSA components are shown in Fig. 5. From Fig. 5, the c_L component presents slow fluctuation, whereas the c_H component presents quick fluctuation, the components quality is now evaluated in the prediction stage.

One step-ahead forecasting via SSA-AR is extended to multi-step ahead forecasting keeping the same settings ($L = 14$, $M = 7$), via direct strategy; Figures 6a, 6b and Table I present the results for multi-step ahead prediction of Injured in traffic accidents. From Figures 6a, 6b, and Table I, high accuracy was reached through the application of SSA-AR hybrid model for multi-step ahead prediction of injured in traffic accidents. SSA-AR presents a mean *MNSE* of 93.5% and a mean *MAPE* of 2.6%.

TABLE I
Multi-step ahead prediction, metrics *MNSE* and *MAPE*

	MNSE(%)		*MAPE*(%)	
h	SSA-AR	SWT-AR	SSA-AR	SWT-AR
1	98.4	99.8	0.6	0.07
2	96.7	99.4	1.3	0.2
3	94.8	98.8	2.1	0.5
4	93.1	97.6	2.7	1.0
5	91.5	95.1	3.4	2.0
6	90.2	87.4	3.9	5.2
7	89.4	82.5	4.3	7.1
Min	89.4	82.5	0.6	0.07
Max	98.4	99.8	4.2	7.1
Mean	93.5	94.4	2.6	2.3
Mean Gain		0.96%		13.0%

The observed signal vs the predicted signal for 7-days ahead prediction via SSA-AR is shown in Fig. 7a, good fitting is reached. The performance evaluation through *MNSE*, *MAPE*, R^2, and *RE* are presented in Table II. The SSA-AR model reaches high accuracy with a *MNSE* of 89.4%, a *MAPE* of 4.3%, a R^2 of 98.8%, and 97.4% of the predicted points show a relative error lower than ±15%.

C. Prediction based on Stationary Wavelet Transform and the Autoregressive model

The wavelet decomposition was implemented with the mother wavelet function Daubechies of order 2 (Db2), in reason that Db2 presents better performance than other wavelets the time series decomposition with long term non-linear trend and periodic component [21]. The number of decomposition levels was evaluated in the prediction stage via AR(14) model. The decomposition level was set by evaluation of different values in the range $j = 1, \ldots, 4$ for one-step ahead prediction. As it is shown in Fig. 4c, the effective number of decomposition levels was set in $J = 2$, which reaches the highest efficiency *MNSE* of 98.8%. The last coefficient approximation reconstructs the component of low frequency c_L, whereas the addition of all detail coefficients reconstruct the component of high frequency c_H. The components extracted through SWT from the time series were illustrated in Fig. 5c and 5d. The c_L component present long-term fluctuations, whereas c_H present short-term fluctuations.

One step-ahead forecasting via SWT-AR is extended to multi-step ahead forecasting keeping the same settings ($L = 14$, $J = 2$), via direct strategy; Figures 6a, 6b and Table I present the results for multi-step ahead prediction of Injured in traffic accidents. From Figures 6a, 6b, and Table I, high accuracy was reached through the application of SWT-AR hybrid model for multi-step ahead prediction of injured in traffic accidents. SWT-AR presents a mean *MNSE* of 94.4% and a mean *MAPE* of 2.3%.

The observed signal vs the predicted signal for 7-days ahead prediction via SWT-AR is shown in Fig. 8a, good fitting is reached. The performance evaluation through *MNSE*, *MAPE*, R^2, and *RE* are presented in Table II. The SWT-AR model

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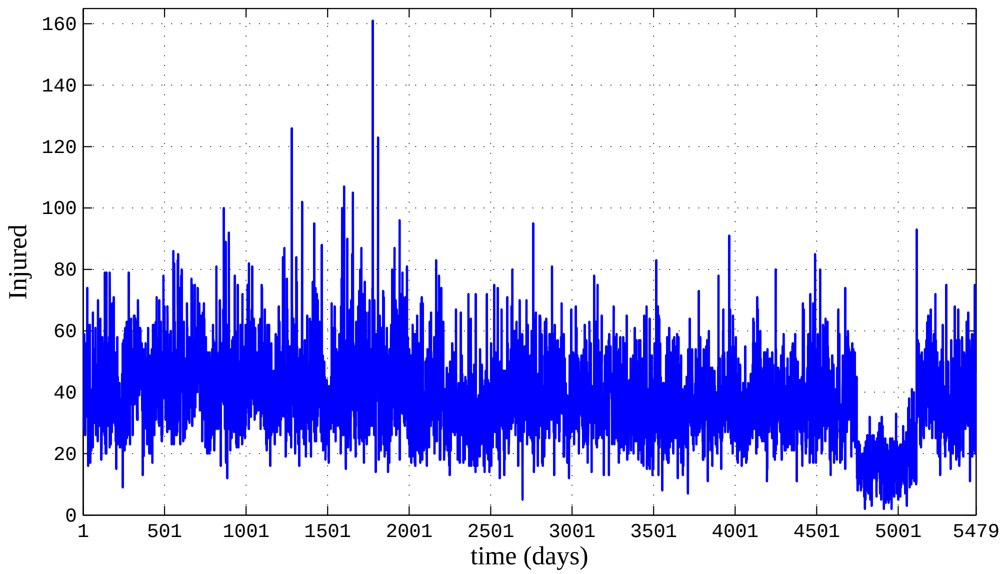

Fig. 3. Days vs Number of Injured by 15 principal causes

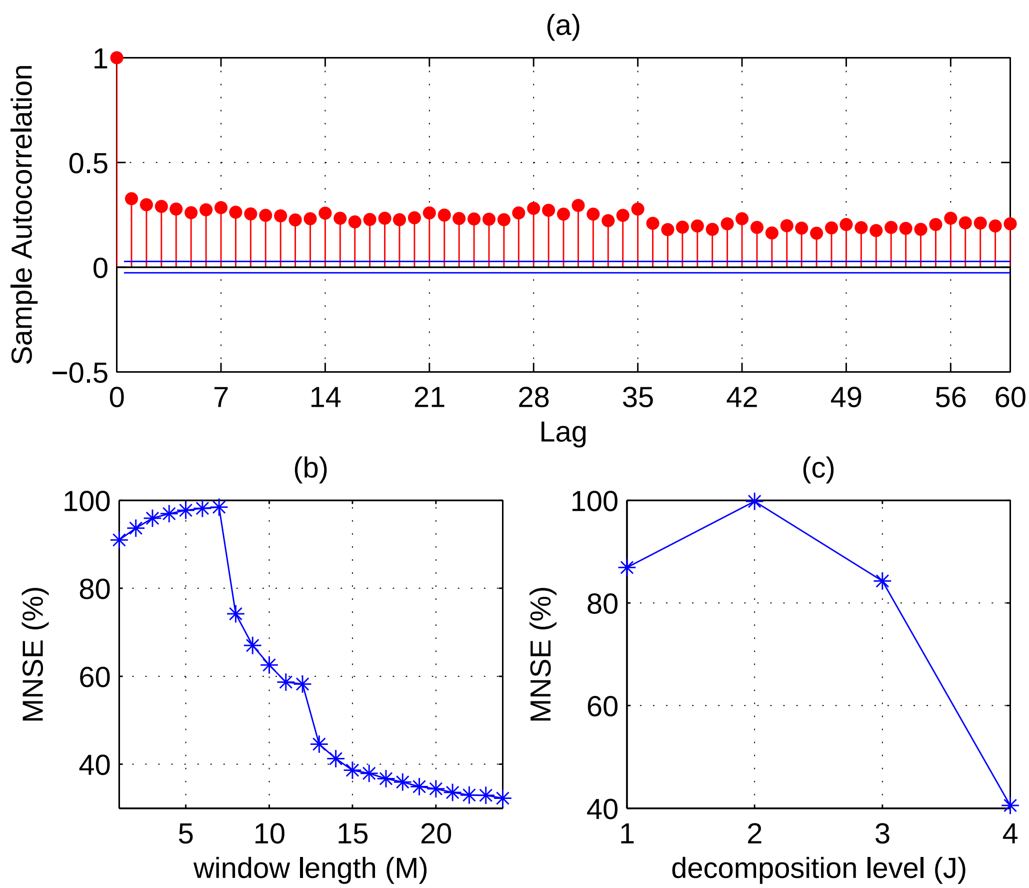

Fig. 4. (a) Injured in Traffic Accidents - ACF, (b) Evaluation of variable window length M for SSA, (c) Evaluation of variable decomposition level J for SWT

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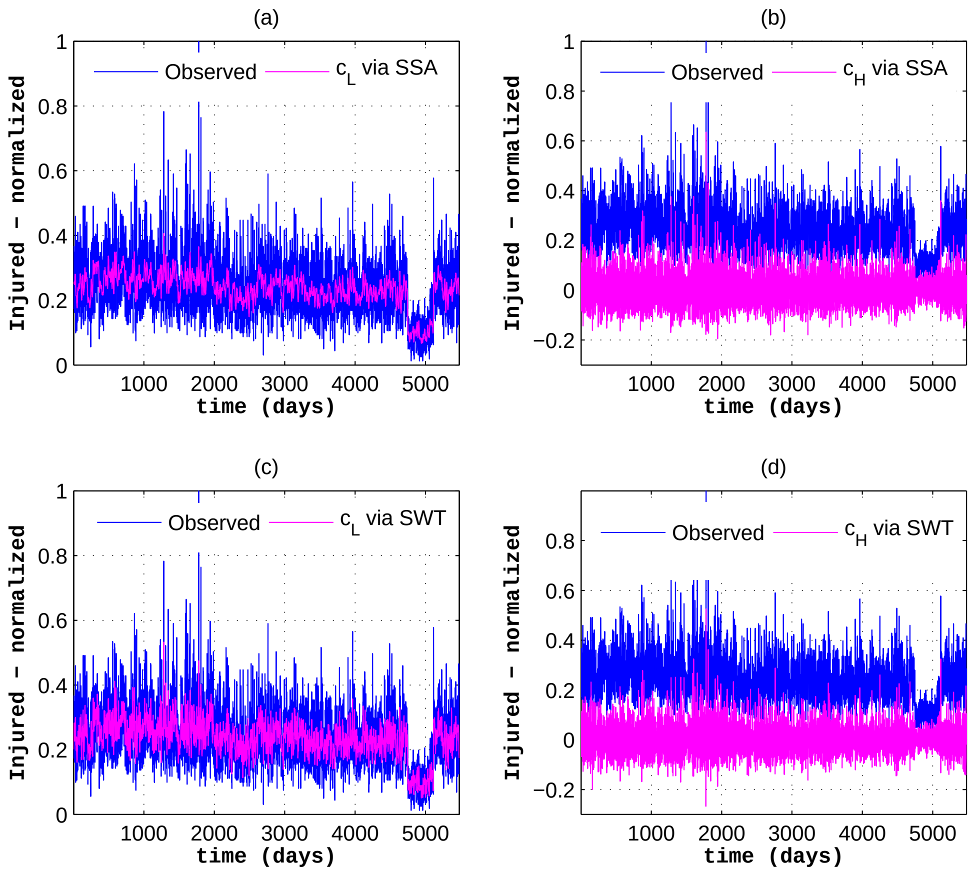

Fig. 5. Injured in traffic accidents Decomposition (a) c_L via SSA and SWT, (b) c_H via SSA and SWT

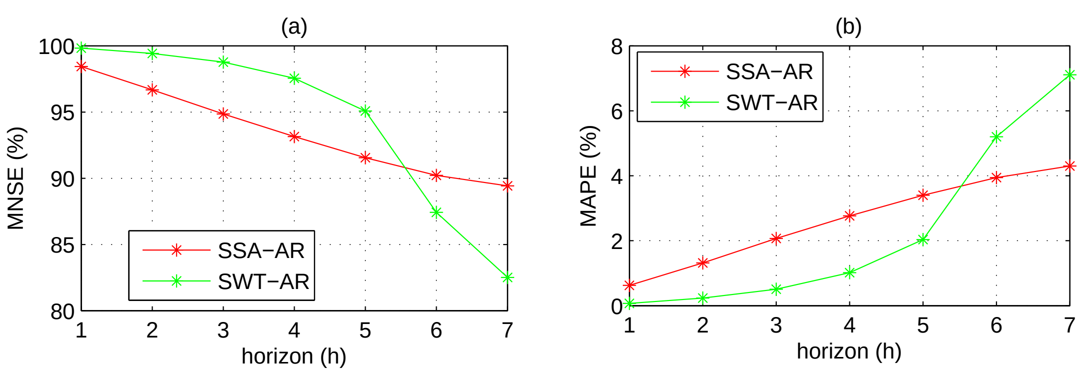

Fig. 6. Efficiency criteria for multi-step ahead prediction of Injured in traffic accidents, via SSA-AR and via SWT-AR (a) MNSE (b) MAPE

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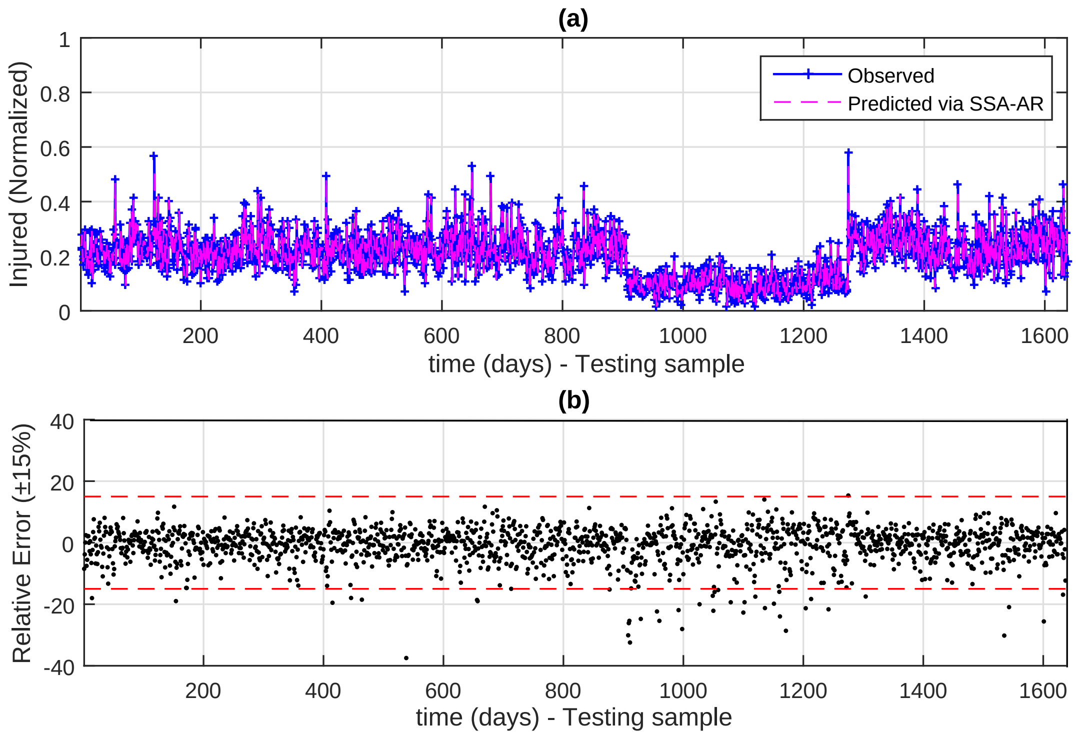

Fig. 7. Results for 7-day ahead prediction of Injured via SSA-AR (a) Observed signal vs Predicted signal, (b) Relative Error

reaches high accuracy with a *MNSE* of 82.5%, a *MAPE* of 7.1%, a R^2 of 96.5%, and 90.4% of the predicted points show a relative error lower than ±15%.

TABLE II
RESULTS FOR 7-DAYS AHEAD PREDICTION OF INJURED IN TRAFFIC ACCIDENTS

	MNSE(%)	*MAPE*(%)	R^2(%)	*RE*(%)
SSA-AR	89.4	4.3	98.8	97.4±15%
SWT-AR	82.5	7.1	96.5	90.4±15%
SSA-AR Gain	8.4%	65.1%	2.4%	7.7%±15%

From Table I, SWT-AR reaches a *MNSE* mean gain over SSA-AR of 0.96%, and a *MAPE* mean gain of 13.0%. However SSA-AR shown superiority with respect to SWT-AR for the farthest horizons ($h = 6$, $h = 7$) as it is presented in Fig. 6 and Table I. By instance from Table II, SSA-AR achieves a *MNSE* gain of 8.4%, a *MAPE* gain of 65.1%, a R^2 gain of 2.4% and 7.7% of *RE* gain (±15%) with respect to SWT-AR for 7-days ahead prediction.

VI. CONCLUSIONS

In this study have been presented and compared two hybrid prediction models based on Singular Spectrum Analysis and Stationary Wavelet Transform combined with the Autoregressive model. The models have been evaluated with a nonstationary time series daily collected from the traffic accidents domain in the period 2000 to 2014; the data characterize the fifteen most relevant causes of injured people in traffic accidents in Santiago, Chile.

The component obtained with the first eigentriple in SSA corresponds to the approximation signal obtained in the last decomposition level in SWT. On the other hand, the regarding eigentriples of SSA reconstruct the component of high frequency, which corresponds to the detail coefficients computed in SWT to reconstruct the component of high frequency. Both SSA and SWT, obtain components of low frequency with long-term fluctuations, and components of high frequency of short-term fluctuations.

The prediction results of SSA-AR and SWT-AR are similar in curve fitting and accuracy. SWT-AR shows the highest mean accuracy for multi-step ahead prediction with a mean *MNSE* gain of 0.96% and a mean *MAPE* gain of 13%. However SSA-AR achieves the best accuracy for farthest horizons; 7-days ahead prediction present a *MNSE* gain of 8.4%, a *MAPE* gain of 65.1%, a R^2 gain of 2.4%, and 7.7% (±15%) of *RE* gain.

Finally both models are suitable for traffic accidents forecasting, however further research can be undertaken to evaluate this potential techniques and the proposed strategies in the solution of other nonstationary problems.

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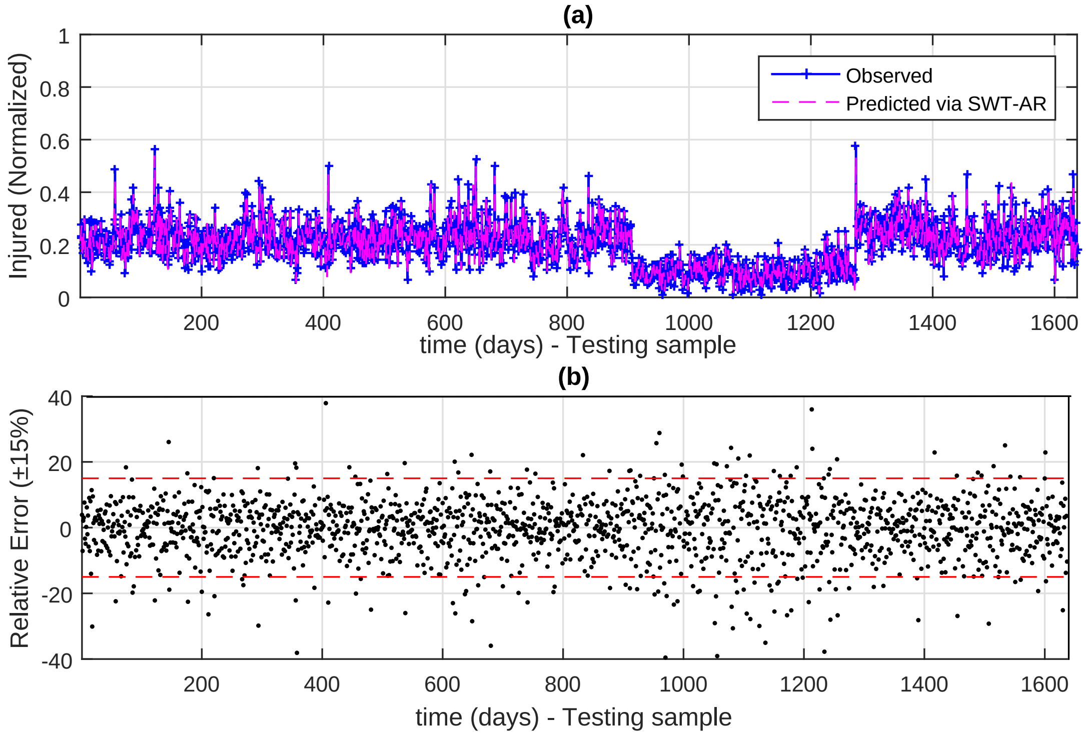

Fig. 8. Results for 7-day ahead prediction of Injured via SWT-AR (a) Observed signal vs Predicted signal, (b) Relative Error

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