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Application of Adaptive PD Control to a Biomechanics System Known as Exoskeleton

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***Abstract*—The control in biomechanical systems has become an active field of biomedical engineering. This paper describes a design of adaptive control to achieve desired in the system defined as exoskeleton moves, the adaptive method is located is selected taking into account energy savings because the adaptive controller is based on a scheme of variable gain in time Proportional and Derivative respect to PD control. The structure of adaptive gain is determined using a type of control Lyapunov function. The adaptation law uses velocity estimation based on a robust exact differentiator (RED) implemented as a variation of Super- Twisting algorithm. The derivative adaptive proportional controller is evaluated on simulated exoskeleton structure. The set of simulations considers the presence of an external disturbance. The controller proves efficient to counter the effects of external mechanical system. Proposed controller performance was superior to standard proportional derivative controller, and has been shown in this study.**

***Index Terms*—Biomechanical system; Adaptive Control PD; Exoskeleton.**

I. Introduction

THE development of exoskeletons has boomed in the last 20 years and because it has helped humans to solve a myriad of problems, especially in the biomedical area. In particular exoskeletons have been an interesting line of research and widely studied by many researchers in the military, industrial and medical area, the latter being the greatest social impact, because it provides a direct benefit to patients with mobility problems neuromuscular. Here, to potentiate the processes of physical rehabilitation of persons with motor disabilities are developing new robotic devices such as exoskeletons [8][9].

Exoskeletons are a kinematic chain which engages externally individuals whose joints and links correspond to the joints of the human body which tries to emulate. The main feature of these mechanisms is direct contact between the user and the exoskeleton, which transfers the mechanical power through information signals[1].

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In the area of biomedical, exoskeletons can help mainly in the field of rehabilitation, and this is achieved by controlling the movements of the patient through a process of stimulation between mechatronic systems and patient. The design and control of an exoskeleton is focused on achieving patient develop a uniform to finally get a steady gait rehabilitation[10]. This is classified as static and dynamic walk, wherein the first center of gravity is maintained in a second region and its center of gravity is not kept in the same region. Both are capable of balancing a person, obtaining stability. This paper presents the design of an exoskeleton and development of control algorithms based on a control known as adaptive control [3].

II. Notation

The following notation was used in this study: $\mathbb{R}^n$ represents the vector space with n-components, τ is used to define the transpose operation, $\|k\|$ is used to define the Euclidean norm of is $k\epsilon\mathbb{R}^n$. $\|k\|_H^2 := k^\intercal Hk$ is the weighted norm of the real valued vector $k \in \mathbb{R}^n$ with weight matrix $H > 0, H = H^\intercal$, $H \in \mathbb{R}^{n\times n}$. The matrix norm labeled as $\|D\|^2, D \in \mathbb{R}^{n\times n}$ is defined as the maximum eigenvalue of the matrix D. If two matrices $N \in \mathbb{R}^{n\times n}$ and $D \in \mathbb{R}^{n\times n}$, fulfills $M > N$, that means that M-N is a positive definite matrix. The symbol $\mathbb{R}^+$ represents the positive real scalars. The symbols $I_{n\times n}$ and $0_{n\times n}$ were used to represent the identity matrix $I\in \mathbb{R}^{n\times n}$ and the matrix formed with zeroes of dimension $n \times n$. This is just some harmless text under a subsection.

III. State space formulation of Exoskeleton

Whereas the mechanical structure of an exoskeleton, this can be formulated in an equation of state space,

$$M(q)\ddot{q} + C(q,\dot{q}) + g(q) + \Delta(t,q,\dot{q}) = u(t)$$

Where $q \in \mathbb{R}^n$ is the vector of generalized coordinates, M is the inertia matrix, C is the matrix of Coriolis and centrifugal forces, g is the gravitational force term Δ is the term of uncertainty y u denotes the vector of controllable forces provided by the torque required to move the actuators. The control input u is assumed that some functions is given by known feedback. Note M (q) is invertible, where $M(q) = M(q)^\intercal$ and is strictly positive definite.

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Using the state variable representation of the mechanical structure (1), the second order nominal model presented above can be represented as follows:

$$\frac{d}{dt}x_a(t) = x_b(t)$$

$$\frac{d}{dt}x_b(t) = f\big(x(t)\big) + g\big(x(t)\big)u(t) + \Delta(x(t),t) \tag{1}$$

The vector x_a represents the position in each degree of freedom of the exoskeleton; the associate vector x_b is the corresponding velocity. Finally, the function Δ represents the uncertainties and perturbations. In this paper, it is assumed that

$$\|\Delta(x(t),t)\|^2 \leq \eta_0 + \eta_1\|x\|^2, \eta_0, \eta_1 \in \mathbb{R}^+$$

The control structure was proposed following the adaptive PD scheme. This class of control model obeyed

$$u(t) = \left(g\big(x(t)\big)^{-1}\right)\left(k_p(t)\right)e(t) + k_d(t)\frac{d}{dt}e(t)$$

Where

$$x = \left[e^{\intercal} \quad \frac{de}{dt}\right]^{\intercal}, e = [x]^{\intercal}$$

$$\frac{de}{dt} = [x_b]^{\intercal}$$

The mechanical nature of exoskeleton is used here to consider that a nonlinear system described by a feasible distributed second order nonlinear differential equation can be used for representing it mathematically.

He drif term $f: \mathbb{R}^{2n} \to \mathbb{R}^n$ is a Lipschitz function. The following assumption is considered valid in this study.

Assumption 1. The nonlinear system (1) is controllable.
Based o the previous fact, the input associated term $g: \mathbb{R}^n \to \mathbb{R}^{n\times n}$satisfies.

$$0 < g^- \leq \|g(k)\|_F \leq g^+ < \infty, k \in \mathbb{R}^n \tag{2}$$

It is evident that matrix g(z(t)) is invertible $t \geq 0$.
Assumption 2. The nonlinear function f (·) is unknown but satisfies the Lipschitz condition

$$\|f(x) - f(x')\| \leq L_1\|x - x'\| \tag{3}$$

I the previous inequality, $\leq x, x' \in \mathbb{R}^{2n}$and $L_1 \in \mathbb{R}^+$.

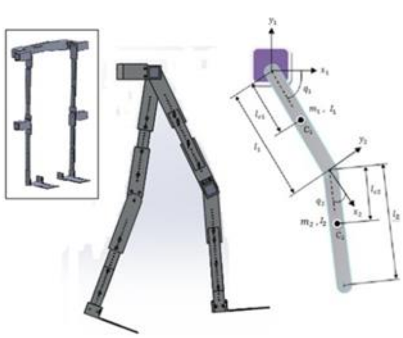

Fig.1 Represents the exoskeleton and its free body diagram to obtain its mathematical model of the system by the Euler-Lagrange method.

The Euler-Lagrange generate a mathematical model of the system is represented by the following two equations:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + \Delta = u$$

$$\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix}\ddot{q} + \begin{bmatrix} C_{11}(q,\dot{q}) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q,\dot{q}) \end{bmatrix}\dot{q} + \begin{bmatrix} g_1(q) \\ g_2(q) \end{bmatrix} + \Delta(t,q,\dot{q}) = u$$

IV. Controller Structure

A PD controller is designed using the assumption regarding $e(t)$ and $\frac{dy}{dx}(t)$ are measured simultaneously where e is the tracking or the regulation error. This is not the regular case in real building mechanical structure represented in Figure 1. Otherwise, an important resources investment. Therefore, in classical in classical literature, one can find two important solutions: to construct an observer or using a …first order filter to approximate the error derivative. The first one requires the system structure (that is in this paper is assumed to be unknown because the presence of external perturbations and internal uncertainties) and in the second case, the derivative approximation is usual poor, especially if the output information is contaminated with noises. One additional option is considering a class of RED that can provide a suitable and accurate approximation of the error derivative. Super Twisting Algorithm (STA) has demonstrated to be one of the best RED in several times[4][5].

IV.1 Super-Twisting Algorithm

In counterpart of some others second order sliding modes algorithms, the STA can be used with systems having relative degree one with respect to the chosen output Levant (1993). The STA application as a RED is described as follows. If $w_1 = r(t)$ where $r(t) \in \mathbb{R}$ is the signal to be differentiated, $w_2 = \frac{dr}{dt}(t)$ represents its derivative and under the assumption of $\frac{dr}{dt}(t) \leq r^+$, the following auxiliary equation is gotten $\frac{dw_1}{dt}(t) = w_2(t)$ and $\frac{dw_2}{dt}(t) = \frac{d^2r}{dt}(t)$. The previous set of differential equation is a state representation of the signal $r(t)$.

The STA algorithm to obtain the derivative of $r(t)$ looks like

$$\begin{gathered} \frac{d}{dt}\overline{w_1}(t) = \overline{w_2}(t) - \lambda_1|\widehat{w}_1(t)|^{\frac{1}{2}}\text{sign}\big(\widehat{w}_1(t)\big) \\ \frac{d}{dt}\overline{w_2}(t) = -\lambda_2\text{sign}\big(\widehat{w}_1(t)\big) \\ \widehat{w}_1 = \overline{w_1} - w_1 \\ d(t) = \frac{d}{d(t)}w_1(t) \end{gathered} \tag{4}$$

where $\lambda_1, \lambda_2 > 0$ are the STA gains. Here $d(t)$is the output of the differentiator Levant[2].

$$\text{sign}(z) := \begin{Bmatrix} 1 & \text{if} & z > 0 \\ [-1,1] & \text{if} & z = 0 \\ -1 & \text{if} & z < 0 \end{Bmatrix} \tag{5}$$

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IV.2 PD controller with the Super-Twisting Algorithm

A single adaptive PD controller is applied over each section of the building-like mechanical structure. This is a class of ATMD. Each adaptive PD controller proposed in this study obeys the following structure

$$u_i(t) = -k_{1,i}(t)e_i(t) - k_{2,i}(t)d_i(t) \qquad (6)$$

where e_i is x^a. The gains in the PD controller are determined by

$$k_1(t) = g_i^{-1}(x_a(t))(k_1(t) + k_1^*)$$

(7)

$$k_2(t) = g_i^{-1}(x_a(t))(k_2(t) + k_2^*)$$

With $k_1(t)$ and $k_2(t)$ are time varying scalars adjusted by a special tracking error dependent adaptive law described by the following ordinary differential equations:

$$\frac{d}{dt}\bar{k}_{1,i}(t) = -\pi_1^{-1}e_i(t)M_a^{\top}P_2E_i(t)$$

(8)

$$\frac{d}{dt}\bar{k}_{2,i}(t) = -\pi_2^{-1}e_{i+n}(t)M_b^{\top}P_2E_i(t)$$

Where $\pi_{1,i}$ and $\pi_{2,i}$ are free parameters to adjust the velocity of convergence for the adjustable gains. In (7), the parameters $k_{1,i}^*$ and $k_{2,i}^*$ are positive constants. The matrices Ma and Mb are given by $M_a = [1\ 0]^{\top}$ and $M_b = [0\ 1]^{\top}$: Additionally, the term $E_i = [e_i \ \ e_{i+n}]$. The matrix $P_{2,i}$ is positive definite and it is presented in the main statement of the main theorem of this article.

The variable $d_i(t)$ is obtained from the following particular application of the STA as RED:

$$\frac{d}{dt}\bar{x}_a^i(t) = \bar{x}_a^i(t) - \lambda_1|\hat{x}_1(t)|^{\frac{1}{2}}\mathrm{sign}(\hat{x}_1(t))$$
$$\frac{d}{dt}\bar{x}_b^i(t) = -\lambda_2\mathrm{sign}(\hat{x}_a(t)) \qquad (9)$$
$$\hat{x}_a^i(t) = x_a^i - \bar{x}_a^i$$

Considering that displacements on building-like structures are small and considering the assumption 1 and 2, it is easy to get that $\left\|\frac{d}{dt}x_b^i(t)\right\| \le h^*$ where h^* is a finite positive scalar.

The following extended system describes the complete dynamics of the error signal in close-loop with an adequate implementation of (4) and the controller proposed in (6):

$$\frac{d}{dt}x_a^i(t) = x_b^i(t)$$
$$\frac{d}{dt}x_b^i(t) = f(x^i(t)) - \bar{k}_{1,i}(t)e_i(t) - \bar{k}_{2,i}(t)d_i(t) + \Delta^i(x^i(t), x^{i+1}(t), x^{i-1}(t), t)$$

$$\frac{d}{dt}\hat{x}_a^i(t) = \hat{x}_a^i(t) - \lambda_{1,i}|\hat{x}_a(t)|^{\frac{1}{2}}\mathrm{sign}(\hat{x}_a(t))$$
$$\frac{d}{dt}\hat{x}_b^i(t) = -\lambda_{2,i}(\hat{x}_a(t)) - \frac{d}{dt}x_b^i(t)$$

(10)

$$\frac{d}{dt}\bar{k}_{1,i}(t) = -\pi_{1,i}^{-1}e_i(t)M_a^{\top}P_{2,i}E_i(t)$$
$$\frac{d}{dt}\bar{k}_{2,i}(t) = -\pi_{2,i}^{-1}e_{i+n}(t)M_b^{\top}P_{2,i}E_i(t)$$

The following section shows the main result of this paper. The theorem introduced in that section gives a constructive way to adjust the gains of the STA and it provides the applicability of using the adaptive gains for the PD controller.

IV.3 Convergence of the adaptive PD controller

The stability of the $e = 0$ is justified by the result presented in the following theorem:

Theorem 1. Consider the nonlinear system given in (1), supplied with the control law (6) adjusted with the gains given in (7) and the derivative of the error signal obtained by means of equation (9), if there exist a positive scalar i and if the gains are selected as $\lambda_{1,i} > 0, \lambda_{2,i} > 0$the next Lyapunov inequalities always have a positive definite solution $P_{1,i}$.

$$A_{1,i}^{\top}P_{1,i} + P_{1,i}A_{1,i} \le -Q_{1,i}$$

$$A_{1,i} = \begin{bmatrix} -\lambda_{1,i} & 1 \\ -\lambda_{2,i} & 0 \end{bmatrix}; Q_{1,i} = Q_{1,i}^{\top} > 0; Q_{1,i} \in \mathbb{R}^{2\times 2} \qquad (11)$$

then for every positive value of L1 satisfying equation (3) and positive value of h^+, there exist positive gains $\bar{k}_{1,i}, \bar{k}_{2,i}$ such that if the Riccati equations given by

$$P_{2,i}(A_{2,i} + \alpha_i I) + (A_{2,i} + \alpha_i I)P_{2,i}^{\top} + P_{2,i}R_{2,i}P_{2,i} + Q_{1,i} \le 0$$
(12)

have positive definite solution $P_{2,i}$ with

$$A_{1,i} = \begin{bmatrix} 0 & 1 \\ -k_{1,i}^* & -k_{2,i}^* \end{bmatrix}; R_{2,i} = \Lambda_{a,i} + \Lambda_{b,i}$$

(13)

$$Q_{2,i} = 4\lambda_{\max}\{\lambda_{b,i}^{-1}\}I_{2\times 2} + \bar{\Lambda}_{a,i}; \bar{\Lambda}_{a,i} = L_1\,\Lambda_{a,i}$$

$$\Lambda_{a,i}, \Lambda_{b,i} > 0, \text{and symmetric}, \Lambda_{a,i}, \Lambda_{b,i} \in \mathbb{R}^{n\times n}, \alpha_i \in \mathbb{R}^+$$

and if the adaptive gains of the PD controller are adjusted by (8), thus the trajectories of

$$E^{\top} = [x_1^a, \dots, x_n^a, x_1^b, \dots, x_n^b]$$

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$$\lim_{t\to\infty} E^{\top}(t)P_2E(t) \le \sum_{i=1}^{n}\frac{\gamma_i}{\alpha_i} \qquad (14)$$

where

$$P_2 = \begin{bmatrix} P_{2,1} & 0_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & P_{2,2} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} & P_{2,n} \end{bmatrix} \qquad (15)$$

and $\gamma_i = 2\lambda_{max}\{\Lambda_{b,i}^{-1}\}(h_i^{+} + 2\Delta_{+}^{*} + \eta_{0,i})$

V. Numerical Results

Fig.2 Represents the Exoskeleton that it was built and where the control tests were performed.

When the PD is calculated adaptive controller for the system, the derivative obtained by the STA provides some advantages. The robustness of STA is applied as a wrapper performs best for any driver applied to second order systems when the only information available is the output signal.

Then the first part of the numerical simulations is employed to evaluate the performance of the STA as RED.

The derivative of exoskeleton like positions is compared with the information provided by the measurements obtained directly from the simulation of the system presented in Figure 1 and the derivative of reference signal.

Calculations were done in Matlab software with the following parameters of Table 1.

Table 1 exoskeleton parameters were obtained with the Solidworks software

i	$m_i[Kg]$	$l_i[m]$	$l_{ci}[m]$	$I_{xxi}[Kgm^2]$	$I_{yyi}[Kgm^2]$	$I_{zzi}[Kgm^2]$
1	0.734	0.48	0.24	257.681	136.610	121.386
2	0.564	0.48	0.24	429.689	86.953	345.264

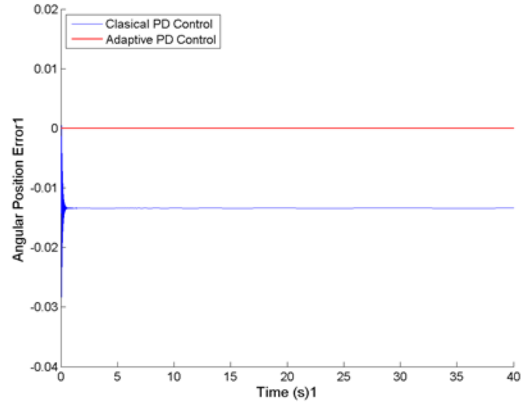

Fig.3 Evaluate the adaptive PD control and classical PD for the board of the exoskeleton located in the thigh.

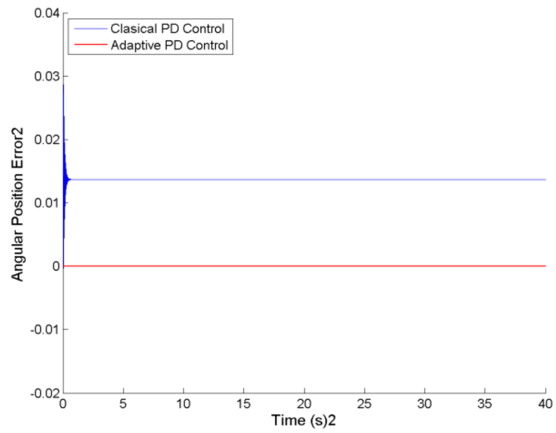

Fig.4 Evaluate the adaptive PD control and classical PD for the board of the exoskeleton located in the calf.

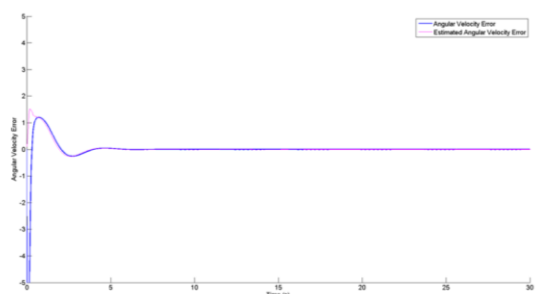

Fig.5 Evaluate the error adaptive PD control and classical PD for the board of the exoskeleton located in the thigh.

VI. Conclusion

An adaptive output based controller based on the proportional derivative controller was implemented to the exoskeleton. The controller was fed with the information of the velocity estimated by a RED based on the application of the super twisting algorithm. The closed loop controller forced the ultimate boundedness of the tracking errors to a region around the origin. A special class of Lyapunov function was the main tool for obtaining the adaptive gains of the PD controller as well as the convergence of the STA used as RED.

Simulation observed in Fig. 3 shows that the algorithm STA

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is zero, while the adaptive PD gets that stability in a close to zero but is never zero, so does the board located in the calf fig. 4, therefore the algorithm STA obtains better control for this system.

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