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Comparing the Black Hole and the Soccer League Competition Algorithms Solving the Set Covering Problem

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***Abstract*—The development of techniques to solve the Set Covering Problem (SCP) have given rise a wide range of metaheuristic alternatives, some of them designed from the beginning to operate in binary search spaces, and other considering continuos spaces that requires adaptation intended to work with binary spaces. Black Hole and Soccer League Competition they were designed to work with continuous spaces and they have been adapted to operate in binary spaces: Binary Black Hole and Binary Soccer League Competition, respectively, aimed to solve problems in a binary domain, particularly the SCP. The present paper compare both implementation in a statistical way, involving the use of non-parametric tests and supported by R statistical computing enviroment, considering regularity and consistency of their results when both algorithm implementations are tested on the same benchmark sets.**

***Index Terms*—Optimization, set covering problem, constraint satisfaction, binary black hole algorithm, soccer league competition algorithm, algorithm adaptation, algorithm comparison**

I. Introduction

THE need to find solutions to optimization problems either using complete or approximative techniques, has allowed the development of several alternatives with different approaches and models. Metaheuristic alternatives are suitable for high dimensionality problems where the main target is to find good solutions, but not the ideal optimal, in an acceptable time spend. The question that arises is how to establish if one algorithm implementation has better behavior than other one, or how to quantify the improvements achieved when some modifications or tuning have been introduced to a specific implementation.

This paper address the comparison of two population-based metaheuristic algorithms adapted to work on binary search spaces to solve the Set Covering Problem (SCP). The comparison is performed from an statistical point of view considering the regularity and consistency of their results when they are tested in the same set of benchmarks.

As is the usual in the domain of complex optimization problems helped by the metaheuristics field, each solution strategy arises from behaviors observed from the nature and then mapped to algorithms. The first one, named Soccer League Competition Algorithm (SLC), is based on soccer competitions where the best teams conformed by exceptional players improve their chances to win each match and each player attempts to become a soccer star or a super soccer star player [1]. The second one, named Black Hole Algorithm (BH) [2], [3] is based on previous work of the particle swarm optimization algorithm with newly convergence elements [4], [5]. It defines an universe of a constant number of stars moving around static locations called black holes and when a star is swallowed by a black hole then a new random star is born.

Both algorithms work with a set of individuals moving around a search space but with different strategies, all of them aimed to reach best regions that improve they fitness and escape from the local optima. Both SCP and BH were designed to work in continuous search spaces and its adaptation to a binary domain have been performed by different ways: the Binary Black Hole (BBH) algorithm lies on transfer and binarization functions [6], Binary Soccer League Competition (BSLC) lies on the Hamming distance reduction approach instead.

As mentioned above, the comparison is performed considering a statistical approach based on regularity and consistency of the results. A methodological mean-analysis applying non-parametric statistical tests is performed according preconditions required by each of them. Shapiro-Wilk, Kolmogorov-Smirnoff-Lilliefors, Wilcoxon-Mann-Whitney, Levene, ANOVA an unpaired t-test are used to define a best choice in each of the 55 different scenarios.

The section II formulates the SCP with its main elements. The BH and how it works in searching for optimal is discussed in section III. The section IV describes the original SLC designed for continuous search spaces, while in section IV-C a binary adaptation of SLC is introduced. The comparison for both algorithm implementations are addressed in section VII and in section IX the conclusions are drawn.

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II. The Set Covering Problem

The Set Covering Problem is one of 21 NP-Hard problems [7] presents in a wide variety of optimization scenarios.

Since its introduction in 1972 by Karp [8] it has been used in optimization problems of elements locations providing spatial coverage such as telecommunications antennas [9], community services [10], urban transportation crews planning [11], metallurgical industry [12], safety and robustness of data networks [13], construction structural calculations [14], focus of public policies [15] among others.

$$\min \quad C = \sum_{j=1}^{n} c_j x_j \tag{1}$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1 \quad \forall i \in \{1, 2, ..., m\} \tag{2}$$

$$x_j \in \{0, 1\} \quad \forall j \in \{1, 2, ..., n\} \tag{3}$$

In general words, let $\mathcal{S}$ be the union of n sets. An element is covered by a set if the element is in the set. A *cover* of $\mathcal{S}$ is a group of the n sets such that every element of $\mathcal{S}$ is covered by at least one set in the group. The SCP challenge is to find a *cover* of $\mathcal{S}$ with minimum size. That is, minimizing expression Eq. (1) and complying with Eq. (2) and Eq. (3).

III. The Black Hole Algorithm

Farahmandian and Hatamlou presents in [2] a strategy intended to find solutions for optimization problems, conceiving the idea of an universe conformed by stars orbiting a unique and fixed center, a *black hole* refered as $\mathbf{X}_{BH}$, in a population-based algorithm approach similar to those used in genetic techniques [16] or particles swarm [17].

The $\mathbf{X}_{BH}$ is a fixed star in the search space, having the best fitness value regarding other stars or, equivalently, the lowest value for a defined function called *objective function* intended to minimize.

The star's motion is performed by an operator of rotation that moves each of them iteratively around $\mathbf{X}_{BH}$, causing along the process the collapse of some stars into the black hole by gravitational effect, the creation of new stars randomly as an exploration strategy, or bringing the creation of a new black hole as an exploitation strategy. The universe's motion process ends when a detention criteria is reached, being the current $\mathbf{X}_{BH}$ the best known solution found for the problem.

A. The Big Bang stage

This stage consists in the creation of an initial universe conformed by a set of $nStar$ stars built randomly. Stars may be replaced during the iteration process but its amount remains fixed throughout the process. The algorithm 1 shows the mechanism for building a new universe, also applied in intermediate steps of star replacement. Let $\mathbf{X}_i$ be a star, then: where $StarBuilder(n)$ function creates a new feasible random binary star, i.e. a feasible solution vector with dimension n.

Algorithm 1 Initial random star builder

```
for i ← 1, nStar do
    X_i ← StarBuilder(n)
```

B. Fitness evaluation

Let $f_{BH}(\mathbf{X}_i)$ be a fitness evaluation function, $f_{BH} : \mathbb{R}^n \to \mathbb{R}$. The black hole $\mathbf{X}_{BH}$ will be those $\mathbf{X}_i$ with the lowest fitness value regarding the rest of stars in the universe.

C. The rotation operator

The operator of rotation sets a new position for each star $\mathbf{X}_i$ other than $\mathbf{X}_{BH}$ which remains in a fixed position. The new position of $\mathbf{X}_i$ at iteration $t+1$, considering its initial position at t iteration is defined by Eq. (4) below:

$$\mathbf{X}_i^d(t+1) = \mathbf{X}_i^d(t) + rand()(\mathbf{X}_{BH}^d - \mathbf{X}_i^d(t)), \tag{4}$$

where $i \in \{1, 2, ..., nStars\}$, $\mathbf{X}^d$ stands for any d-dimension of the solution, $\mathbf{X}_{BH}$ is black hole position, $rand()$ is a random number with uniform distribution in [0,1].

D. Collapsing into the black hole

A star closer to the black hole at a distance called *event horizon* is inevitably captured and permanently absorbed by it, being replaced by a new star generated randomly. In other terms, the collapse of a star occurs when it exceeds the radius of Schawarzchild (R).

Farahmandian and Hatamlouy intend in [16] to determinate the distance of a star $\mathbf{X}_i$ to the radius R as:

$$R = \frac{f_{BH}(\mathbf{X}_{BH})}{\sum_{i=1}^{nStars} f(\mathbf{X}_i)} \tag{5}$$

where $f_{BH}(\mathbf{X}_{BH})$ and $f_BH(\mathbf{X}_i)$ are the black hole and the $\mathbf{X}_i$ star fitness value, respectively.

A star $\mathbf{X}_i$ will collapse when its distance at the black hole is less than R as indicated in Eq. (5). Aimed to manage the tolerance threshold calculating the event horizon, we incorporate an additional parameter $s \in [0, 1]$ to the algorithm, to modify the minimum allowable proximity to the black hole, measured in function of its fitness. Thus, a star $\mathbf{X}_i$ will colapse into the black hole if:

$$|f_{BH}(\mathbf{X}_{BH}) - f_{BH}(\mathbf{X}_i)| < sR \tag{6}$$

IV. The Soccer League Competition Algorithm

SLC is introduced by Moosavian in [1] and defines a set of n_{teams} set of players or feasible solutions called $teams$. Each team $\mathcal{T}$ is conformed by n_{fp} fixed players $\mathbf{FP}$ and n_{sp} substitute players $\mathbf{SP}$.

A player $\mathbf{X} = (x^1, x^2, ..., x^d)$ will belong to the fixed or substitute class depending of its performance level rank. The performance level or *power player* is defined by a function $PP : \mathbb{R}^n \to \mathbb{R}$. If two solutions $\mathbf{X}_i$ and $\mathbf{X}_j$ verifies that

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$PP(\mathbf{X}_i) > PP(\mathbf{X}_j)$, then we will say that $\mathbf{X}_i$ has a better performance than $\mathbf{X}_j$.

For each team $\mathcal{T}$, the player having the higest player power value is called *super player*, $\mathbf{X}_{SP}$. Likewise, considering all teams we can find the *super star player*, $\mathbf{X}_{SSP}$, as the player with the best power player.

Given the player power function PP, we can generalize and define the *team power* TP as follow:

$$TP = \sum_{\mathbf{X}_k \in \mathcal{T}} \frac{PP(\mathbf{X}_k)}{n_{fp} + n_{sp}} \tag{7}$$

A. Stochastic criteria

Two teams faced in a match will result in one single winner always. If TP_A and TP_B are the team power for $\mathcal{T}_A$ and $\mathcal{T}_B$, respectively, the probability of victory for $\mathcal{T}_A$ facing $\mathcal{T}_B$ is given as follow:

$$PV_A = \frac{TP_A}{TP_A + TP_B} \tag{8}$$

In a similar way, we can calculate the probability of victory for $\mathcal{T}_B$, PV_B. It results that $PV_A + PV_B = 1$. Then, given a random number $r \in [0, 1]$ and PV_A defined as Eq. (8) we can define the winner team in a time t as shown in algorithm 2:

Algorithm 2 Definition of the winner team between $\mathcal{T}_A$ and $\mathcal{T}_B$

```
PV_A ← GetProbabilityOfVictory(T_A, T_B)
r ← rnd(0, 1)
if 0 ≤ r ≤ PV_A then
    T_A   is the winner
else
    T_B   is the winner
```

B. Movement operators

For the winner team defined above, the **imitation** and **provocation** operators are defined. In the other hand, the **mutation** and **substitution** operators are defined for the looser team. The **imitation** operator will attemp to move each fixed player of the winner team towards $\mathbf{X}_{SSP}$ or $\mathbf{X}_{SP}$ aimed to improve its player power, calculating two feasible candidate solutions, $\mathbf{X}_a$ and $\mathbf{X}_b$, using Eq. (9) and Eq. (10) as follow:

$$\mathbf{X}_a = \mu_1 \mathbf{FP}(t) + \tau_1(\mathbf{X}_{SSP} - \mathbf{FP}(t)) + \tau_2(\mathbf{X}_{SP} - \mathbf{FP}(t)) \tag{9}$$

$$\mathbf{X}_b = \mu_2 \mathbf{FP}(t) + \tau_1(\mathbf{X}_{SSP} - \mathbf{FP}(t)) + \tau_2(\mathbf{X}_{SP} - \mathbf{FP}(t)) \tag{10}$$

where $\mu_1 \sim U(\theta, \beta)$, $\quad \mu_2 \sim U(0, \theta)$, $\theta \in [0, 1]$, $\beta \in [1, 2]$ and $\tau_1, \tau_2 \sim (0, 2)$ are random numbers with uniform distribution as is indicated in [1]. The algorithm 3 shows how imitation operation does work, moving $\mathbf{FP}(t)$ to the new position $\mathbf{FP}(t+1)$ when its player power is improved.

The **provocation** operator will attempt to move each substitute player **SP** towards the centroid or gravitational center **G** defined by Eq. (11), aimed to improve its player power.

Algorithm 3 Imitation operator

```
X_a ← GetCandidate_a()
X_b ← GetCandidate_b()
if PP(X_a) > PP(FP(t)) then
    FP(t + 1) ← X_a
else if PP(X_b) > PP(FP(t)) then
    FP(t + 1) ← X_b
```

$$\mathbf{G}^d = \frac{\sum_{\mathbf{FP}_i \in \mathcal{T}} \mathbf{FP}_i^d}{n_{fp}} \tag{11}$$

Then, two new candidates X_r and X_s are calculated as follow:

$$\mathbf{X}_r = \mathbf{G} + \chi_1(\mathbf{G} - \mathbf{SP}) \tag{12}$$

$$\mathbf{X}_s = \mathbf{G} + \chi_2(\mathbf{SP} - \mathbf{G}) \tag{13}$$

where $\chi_1 \sim U(0.9, 1)$, $\chi_2 \sim U(0.4, 0.6)$ are random numbers with uniform distribution as is indicated in [1]. The algorithm 4 shows how the provocation operator does work, moving $\mathbf{SP}(t)$ to the new position $\mathbf{SP}(t+1)$ when its player power is improved. In other case, it is replaced by a new random generated feasible solution.

Algorithm 4 Provocation criteria

```
X_r ← GetCandidate_r()
X_s ← GetCandidate_s()
if PP(X_r) > PP(SP(t)) then
    SP(t + 1) ← X_r
else if PP(X_s) > PP(SP(t)) then
    SP(t + 1) ← X_s
else
    SP(t + 1) ← NewPlayer()
```

For the looser team, the fixed players will attempt to apply small changes to avoid repeating the match failure by using some **mutation** operator like Genetic Algorithm (GA). Also, some substitute players will be replaced by promising young talents by applying a crossover operator but not considered in the binary adaptation of SLC.

C. Binary versions of BH and SLC

Gómez introduces in [18] a strategy to allow BH to work in binary search spaces, using transfer and binarization functions, thus mapping non-binary values to the $\{0, 1\}^n$ domain. Jaramillo presents in [19] a binary adaptation approach using Hamming distance reduction instead vectorial algebra, dicretization and binarization functions. For the **imitation** operator it proposes two new candidates $\mathbf{X}_a$ and $\mathbf{X}_b$ as follow:

$$\mathbf{X}_a^d = \begin{cases} \mathbf{X}_{SP}^d & \text{if } rand() \leq p_{imitation} \\ \mathbf{FP}^d(t) & \text{other case} \end{cases} \tag{14}$$

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$$\mathbf{X}_b^d = \begin{cases} \mathbf{X}_{SSP}^d & \text{if } rand() \leq p_{imitation} \\ \mathbf{FP}^d(t) & \text{other case} \end{cases} \quad (15)$$

where $rand() \sim U(0,1)$ is a random generated value with uniform distribution and $p_{imitation}$ is a probability of imitation defined as an initial parameter of the model. The **provocation** operator uses a new centroid point definition, $\mathbf{BG}$ built from $\mathbf{G}$ in Eq. (11) but considering the probability to have 1 or 0 in the dimension d as follow:

$$\mathbf{BG}^d = \begin{cases} 1 & \text{if } \mathbf{G}^d \geq 0.5 \\ 0 & \text{other case} \end{cases} \quad (16)$$

A **mutation** operator for fixed players could be considered as follow:

$$\mathbf{FP}^d(t+1) = \begin{cases} \mathbf{FP}^d(t) & \text{if } rand() \leq p_{mutation} \\ \neg\mathbf{FP}^d(t) & \text{other case} \end{cases} \quad (17)$$

where $rand() \sim U(0,1)$ is a random generated value with uniform distribution and $p_{mutation}$ is a probability of mutation defined as initial parameter of the model.

V. Solving SCP using BBH and BSLC

BBH and BSLC require both a fitness function definition. For BBH the Eq. (1) of SCP can be used to define $f_{BH}(\mathbf{X}) = 1/\sum x_i c_i$ when SCP faces a minimum optimization problem, or its inverse in case of maximum. In the same way, BSLC can define its player power function as $PP(\mathbf{X}) = 1/\sum x_i c_i$ when SCP faces a minimum optimization problem, or its inverse in case of maximum. Feasibility based on constraints Eq. (2) and Eq. (3) are specific of each implementation and are not covered in this paper.

VI. Performing BBH and BSLC experiments

The implementation of both algorithms were tested using the same SCP benchmark problem sets. Problem sets 4 - 6 are taken from Balas and Ho [20]. Problem sets A - D are from Basley [21]. Problem sets NRE and NRF are taken from [22]. The data sets is provided by the OR-Library website [23] and available for free download in Internet. Problem sets 4 and 5 contain 10 instances each. Problem sets 6, A - D, NRE and NRF contain 5 instances each. Table I shows details for each problem set. The density value corresponds to the percentage of ones in the constraint matrix.

The goal of the comparison is focused in contrasting regularity in the BBH and BSLC outcomes and proximity to the best known solution for each instance tested.

BSLC did use 10 teams, conformed each one of them by 15 fixed and 10 substitute players. BBH did use an universe conformed by 250 stars. Each instance test was run 31 times for both BBH and BSLC in order to obtain a median value for each experiment.

The summary of results is shown in the Table II. The number of constraints, dimensions and the best known solution value Z_{BKS} are shown. The relative percentage difference rpd is defined as $rpd = \frac{min - Z_{BKS}}{Z_{BKS}}$. Each implementation shows min, max, mean and median values obtained per instance.

VII. Statistical comparison approach

In the metaheuristic field, performing a statistical analysis using parametric tests is not suitable as result of the stochastic nature of the evolutionary algorithms. This due to the fact that required conditions as normality, homoscedasticity and independence are not satisfied, as is demonstrated in [24] and [25]. However, when normality is not guaranteed, **Wilcoxon** and **Wilcoxon-Mann-Whitney** might be an appropriate option in order to compare populations, considering medians instead means.

Lanza and Gómez use in [26] a methodological approach to compare different algorithms implementation, considering the regularity and consistency of their results, as is shown in Fig. (1).

Using the Mann-Whitney-Wilcoxon test is possible to check if the population distributions of two evolutionary algorithm outcomes are identical without assuming them to follow the normal distribution. For this purpose, the non-normality and independence conditions must be checked first in order to choose a suitable contrast test.

A. Non-Normality condition

In order to check that a normal distribution is not present in the outcomes, Kolmogorov-Smirnov-Lilliefor (KSL) and Shapiro-Wilk (SW) tests are performed for each instance, helped by R statistical computing environment.

KSL-test [27] is used to compare the accumulative distribution observed with a reference probability distribution, in this case a *normal* distribution; a $p_{value} > 0.05$ implies that there is no evidence to reject the null-hypothesis H_0, that accumulative distribution observed and the reference distribution are the same. In the other hand, SW-test [28] defines a null-hypothesis H_0 as the population is normally distributed; a $p_{value} > 0.05$ implies that there is no evidence to reject H_0.

As the result sets were obtained by evolutionary algorithms, the main idea of this stage is to prove that there is evidence to reject a normal distribution, i.e. the null-hypothesis H_0 in KSL, or SW or both, for each instance.

B. Independence of the samples

Two data samples are independent if they come from distinct populations and the samples do not affect each other. Each run executed corresponds to independiente process running in a virtualized computing environment; the values in one sample (run result) does not affect the values in the other sample, and values in one sample reveal no information about those of the other samples, thus we can establish the sample independence.

C. Statistical results

1) Non-normality.: The results obtained using the R's parametric tests $ks.test$ and $shapiro.test$ are summarized in Table III. It shows the p_{value} obtained per test, algorithm

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TABLE I
DETAILS OF SCP PROBLEM SETS.

Problem set	Constraints	Dimensions	Density	Instances
4	200	1000	2%	10
5	200	2000	2%	10
6	200	1000	5%	5
A	300	3000	2%	5
B	300	3000	5%	5
C	400	4000	2%	5
D	400	4000	5%	5
NRE	500	5000	10%	5
NRF	500	5000	20%	5

TABLE II
RESULTS PER BENCHMARK INSTANCE AND ALGORITHM IMPLEMENTATION.

Instance	Constr.	Dimension	Z_{BKS}	BBH					BSLC				
				min	$median$	$mean$	max	rpd %	min	$median$	$mean$	max	rpd %
4.1	200	1000	429	447	735.5	685	1268	4.2	429	723.3	722	1395	0
4.2	200	1000	512	517	741.5	696	1154	0.98	512	862.9	831	1375	0
4.3	200	1000	516	527	860.8	769	1780	2.13	517	898.7	824	1450	0.19
4.4	200	1000	494	499	900.8	843	1945	1.01	504	835.2	791	1290	2.02
4.5	200	1000	512	518	863.5	758	1452	1.17	518	851.2	777	1367	1.17
4.6	200	1000	560	612	983.8	861	1821	9.29	562	902.5	815	1496	0.36
4.7	200	1000	430	460	728.7	668	1187	6.98	435	707.8	650	1352	1.16
4.8	200	1000	492	496	843.1	773	1506	0.81	492	772.7	691	1406	0
4.9	200	1000	641	649	1052.8	1079	1818	1.25	646	1002.3	1029	1494	0.78
4.10	200	1000	514	575	837.1	813	1423	11.87	548	832.6	758	1419	6.61
5.1	200	2000	253	255	440.5	414	756	0.79	254	448.3	425	738	0.4
5.2	200	2000	302	316	535.6	489	1109	4.64	305	506.5	483	902	0.99
5.3	200	2000	226	227	372	334	804	0.44	232	387.2	360	684	2.65
5.4	200	2000	242	251	382.6	316	702	3.72	264	369.8	357	614	9.09
5.5	200	2000	211	217	338.2	321	613	2.84	213	329.6	312	519	0.95
5.6	200	2000	213	216	332.8	293	658	1.41	218	325.5	308	456	2.35
5.7	200	2000	293	304	468.5	430	1066	3.75	299	468.8	478	911	2.05
5.8	200	2000	288	302	499.8	487	804	4.86	309	471.2	424	911	7.29
5.9	200	2000	279	311	480.2	459	854	11.47	286	455.9	413	840	2.51
5.10	200	2000	265	287	435.1	399	890	8.3	290	451.4	421	653	9.43
6.1	200	1000	138	139	227	221	381	0.72	144	234.5	212	430	4.35
6.2	200	1000	146	149	251.3	247	410	2.05	154	241	233	453	5.48
6.3	200	1000	145	149	234.5	217	450	2.76	154	230.2	194	379	6.21
6.4	200	1000	131	131	208	181	396	0	136	224.9	223	382	3.82
6.5	200	1000	161	167	239.5	238	386	3.73	161	254.8	237	483	0
A.1	300	3000	253	258	397.1	351	654	1.98	258	397.7	383	722	1.98
A.2	300	3000	252	253	411.9	361	739	0.4	254	424.9	413	689	0.79
A.3	300	3000	232	234	355.4	336	687	0.86	237	375.2	337	709	2.16
A.4	300	3000	234	238	360.8	352	582	1.71	257	392.8	360	703	9.83
A.5	300	3000	236	240	343.9	308	700	1.69	238	352	349	569	0.85
B.1	300	3000	69	70	108.1	100	188	1.45	70	103.6	100	159	1.45
B.2	300	3000	76	78	124.5	116	195	2.63	77	122	112	201	1.32
B.3	300	3000	80	81	133.1	116	228	1.25	83	121	113	212	3.75
B.4	300	3000	79	80	134.8	133	250	1.27	80	127.8	118	283	1.27
B.5	300	3000	72	73	114.3	103	204	1.39	74	114.3	107	225	2.78
C.1	400	4000	227	232	360	322	695	2.2	229	365.1	362	578	0.88
C.2	400	4000	219	220	344.1	321	550	0.46	223	336.5	337	534	1.83
C.3	400	4000	243	246	419.2	381	815	1.23	248	384.9	383	626	2.06
C.4	400	4000	219	219	367.9	344	621	0	219	361.7	334	576	0
C.5	400	4000	215	222	385.3	379	644	3.26	220	333.5	291	569	2.33
D.1	400	4000	60	65	100	91	179	8.33	73	111.6	109	164	21.67
D.2	400	4000	66	72	109.9	100	176	9.09	67	99.6	90	172	1.52
D.3	400	4000	72	74	108.3	95	194	2.78	73	122.3	117	214	1.39
D.4	400	4000	62	66	99.6	96	155	6.45	69	101	93	178	11.29
D.5	400	4000	61	62	92.7	94	147	1.64	63	97.9	86	162	3.28
NRE.1	500	5000	29	30	47.5	44	92	3.45	30	49.5	50	80	3.45
NRE.2	500	5000	30	30	49.2	45	88	0	32	52.1	48	88	6.67
NRE.3	500	5000	27	27	40.8	37	83	0	29	45.2	44	68	7.41
NRE.4	500	5000	28	29	46.3	42	77	3.57	29	46.2	43	76	3.57
NRE.5	500	5000	28	28	44.9	44	81	0	28	41.9	42	68	0
NRF.1	500	5000	14	14	22.7	23	45	0	16	22.2	21	36	14.29
NRF.2	500	5000	15	15	26.5	24	45	0	15	23.9	24	36	0
NRF.3	500	5000	14	15	24	24	44	7.14	14	22.7	22	37	0
NRF.4	500	5000	14	14	21.9	21	35	0	14	23.1	23	43	0
NRF.5	500	5000	13	14	20.9	18	43	7.69	14	21.6	20	37	7.69

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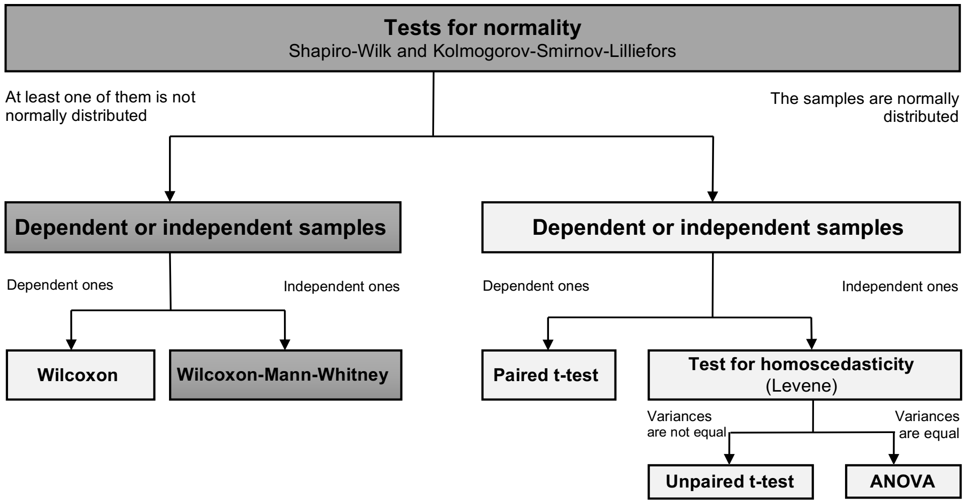

Fig. 1. Statistical methodology chart for 2 samples

implementation and instance. Values less than 0.05 are marked with * symbol. The ~~H_0~~ in the labeled column $result$ indicates that there is no evidence for normality assumption, i.e. rejecting H_0. When BBH and BSLC normality test give simultaneously evidence to accept H_0, then we can be facing unexpected outcomes, needing a second revision.

Figure (1) addresses the test to apply in each instance and the Table III shows the results. Note that the Wilcoxon-Mann-Whitney test is the predominant. This fact is expected due the no-normality and independence conditions is met by almost all the instance outcomes.

2) Wilcoxon-Mann-Whitney test execution: A cross test between the BBH and BSLC outcomes is performed using R. In both cases, a redefinition of the alternative-hypothesis H_1 is set and the main idea is to reject the null-hypothesis H_0 in order to accept H_1.

The first test defines a H_1 as $\overline{X}_{BBH} > \overline{X}_{BSLC}$. If a $p_{value} < 0.05$ is obtained, then there is evidence to reject H_0, accepting H_1, i.e. BSLC's outcomes are statistically better than BBH. In a similar way, the second test defines a H_1 as $\overline{X}_{BSLC} > \overline{X}_{BBH}$. If a $p_{value} < 0.05$ is obtained then there is evidence to reject H_0, accepting H_1, i.e. BBH's outcomes are statistically better than BSLC.

The summary of results for each instance is shown in Table IV. Values less than 0.05 are marked with * symbol. As we can see, there is not significant evidence to choose one implementation or other for most instances. Figures (2-4) shows boxplot for each instance.

In respect of instances 4.1, 4.2, 6.5, the unpaired test was defined as the suitable test to perform according the methodology in Fig. (1). In a similar way, the ANOVA test has been selected in order to perform comparison in instances 4.9, 5.5, A.3, NRE.5, NRF.2 and NRF.4. However, a normal distribution is an unexpected result in evolutionary algorithms, as previously mentioned in Section VII due to its stochastic nature.

As suggested by Demšar in [29], the Kolmogorov-Smirnov and similar normality test have a little power in detecting abnormalities on small samples. With the purpose of having a third evidence, D'Agostini-Pearson (DA) normality test is applied. The values obtained are shown in Table V, keeping the same evidence of KSL and SW tests. Figures (5-6) show a boxplot for these instances. Regardless the results evidenced in Table V, Wilcoxon-Mann-Whitney test is performed on these datasets to address location comparison under a non-normality assumption.

VIII. Analysis of Results

Based on the statistical results obtained by Wilcoxon-Mann-Whitney test, there is a slight tendency to define the BBH algorithm as better than BSLC, regarding all instances covered in the experiments performed in this work.

For instances 5.3, D.1, D.3, NRE.3, NRF.5 there is evidence towards BBH and it can be confirmed by their respectives boxplot in Fig. (2) and Fig. (4) where BBH have better location of the median than BSLC. In the other hand, instances C.5 and D.2 show evidence towards BSLC, and its respective boxplots in Fig. (4) confirms the above, where BLSC shows better location of the median than BBH.

The non-normality condition appears to be not required in order to perform a Wilcoxon-Mann-Whitney test when data sets have been generated by an evolutive algorithm. This is evidenced comparing numerical results in Table VI against the boxplot representation for this instances in Fig. (5 - 6), where both conclusions that can emerge do fit. In particular, the instance 4.2 shows a better location of the BBH's median value than BSLC and it can be evidenced numerically by the result in Table VI . The other instances referred in the same group with unexpected normality distribution evidence, as result of

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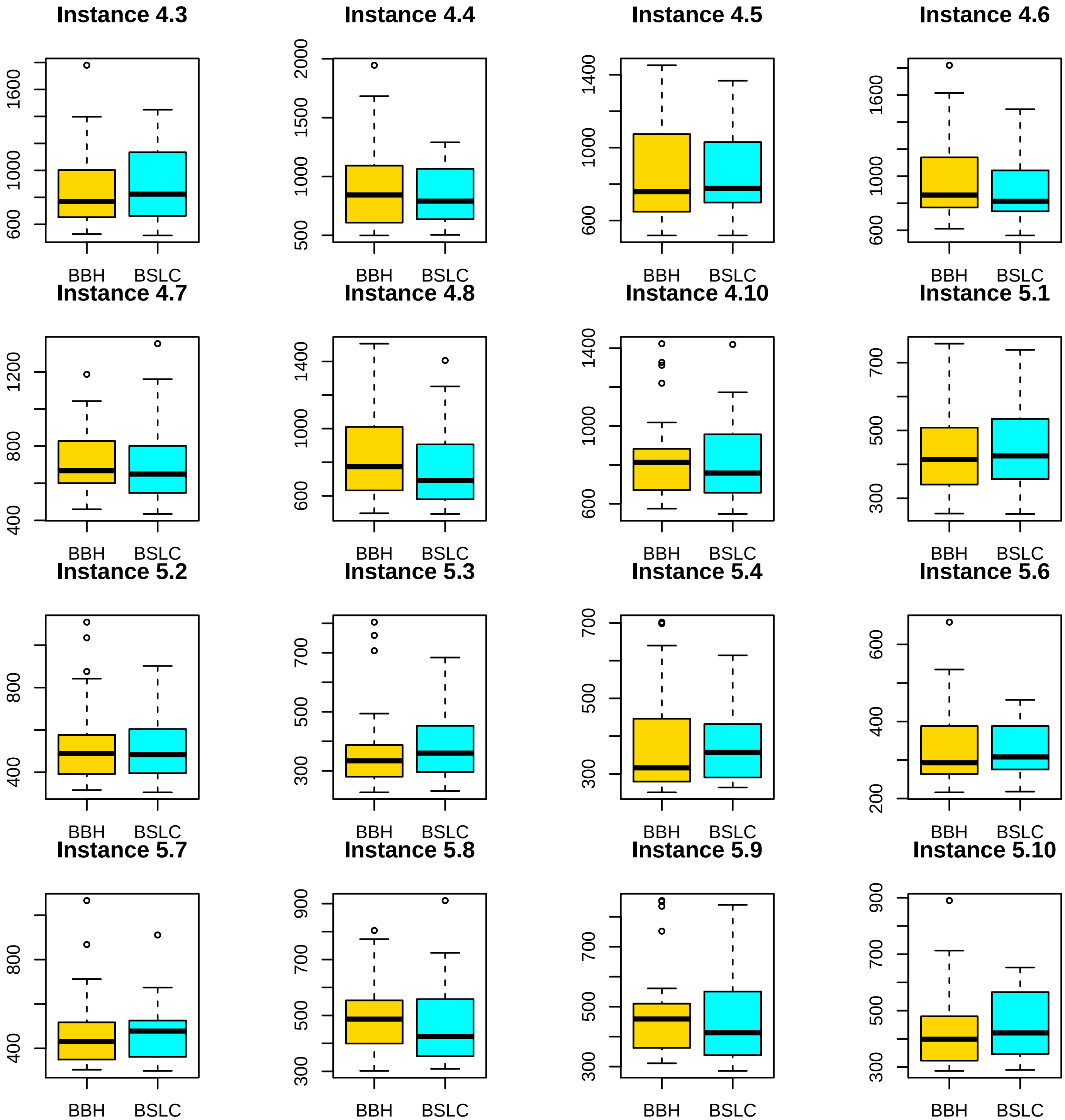

Fig. 2. Boxplot for outcomes addressed by Wilcoxon-Mann-Whitney test.

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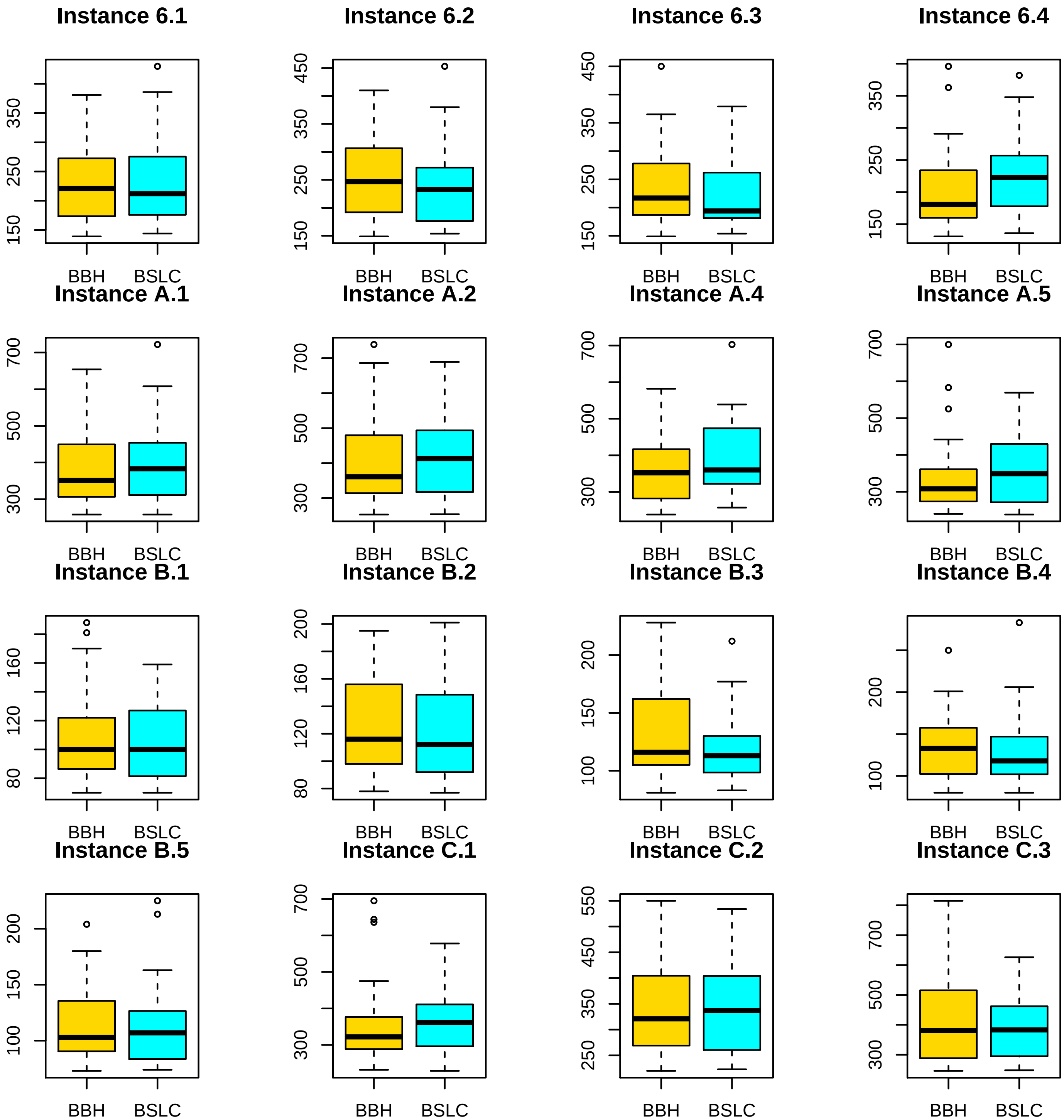

Fig. 3. Boxplot for outcomes addressed by Wilcoxon-Mann-Whitney test.

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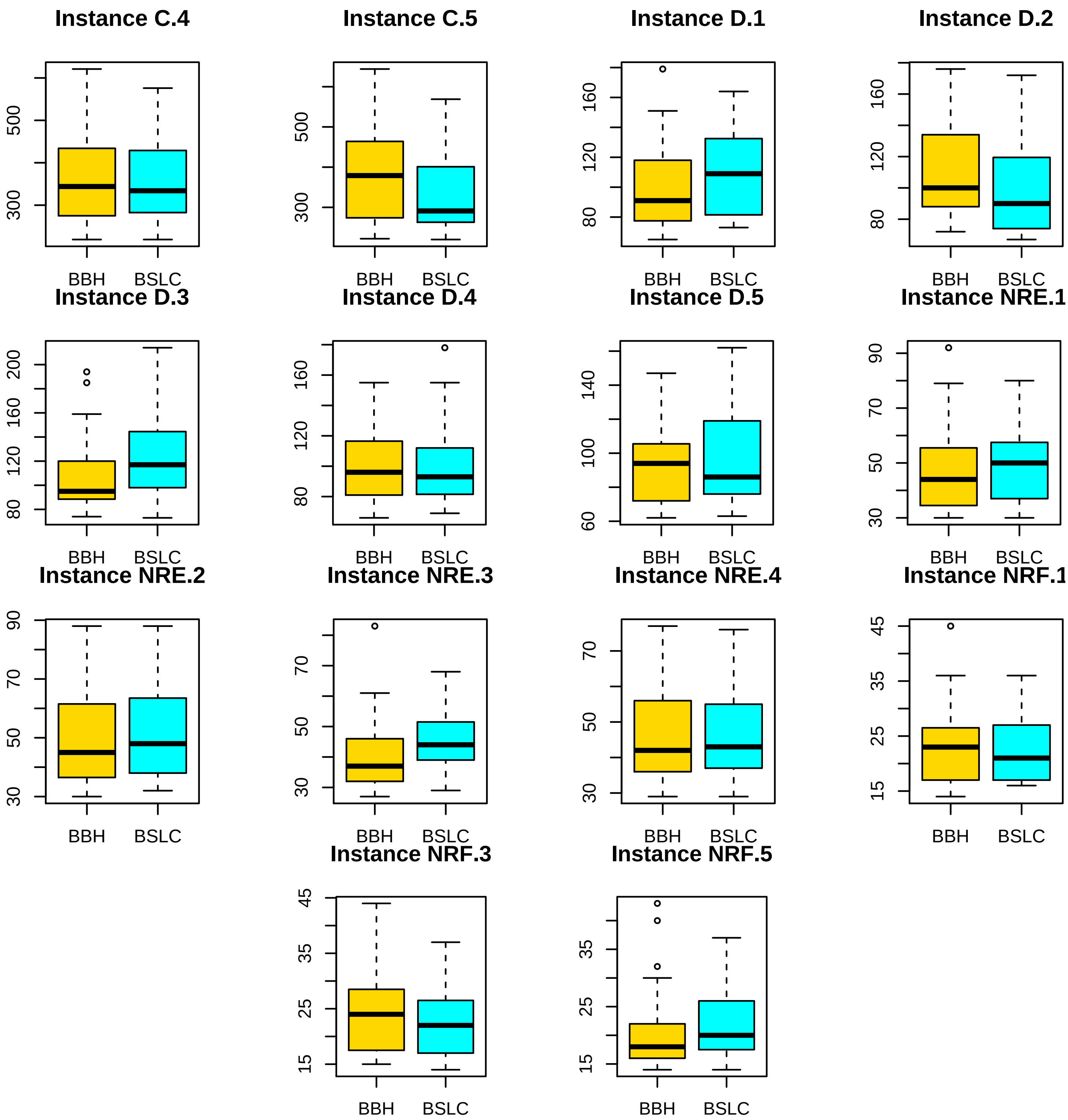

Fig. 4. Boxplot for outcomes addressed by Wilcoxon-Mann-Whitney test.

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TABLE III
SUMMARY OF RESULTS FOR THE NORMALITY PARAMETRIC TEST AND THE TEST TO APPLY FOR ALGORITHM OUTCOMES.

Instance	**BBH**			**BSLC**			**Levene**	**Test to apply**
	KSL p_{value}	**SW** p_{value}	result	**KSL** p_{value}	**SW** p_{value}	result	p_{value}	
4.1	0.078	0.933	H_0	0.857	0.220	H_0	0.012	Unpaired t-test
4.2	0.052	0.065	H_0	0.713	0.080	H_0	0.019	Unpaired t-test
4.3	0.589	*0.013	~~H_0~~	0.453	0.053	H_0		Wilcoxon-Mann-Whitney
4.4	0.711	*0.029	~~H_0~~	0.711	*0.046	~~H_0~~		Wilcoxon-Mann-Whitney
4.5	0.327	*0.025	~~H_0~~	0.481	0.259	H_0		Wilcoxon-Mann-Whitney
4.6	0.411	*0.006	~~H_0~~	0.327	*0.024	~~H_0~~		Wilcoxon-Mann-Whitney
4.7	0.364	0.059	H_0	0.483	*0.024	~~H_0~~		Wilcoxon-Mann-Whitney
4.8	0.251	*0.008	~~H_0~~	0.421	*0.018	~~H_0~~		Wilcoxon-Mann-Whitney
4.9	0.051	0.370	H_0	0.634	0.059	H_0	0.319	ANOVA
4.10	0.965	0.378	H_0	0.513	*0.036	~~H_0~~		Wilcoxon-Mann-Whitney
5.1	0.623	*0.039	~~H_0~~	0.272	*0.021	~~H_0~~		Wilcoxon-Mann-Whitney
5.2	0.860	*0.007	~~H_0~~	0.441	*0.049	~~H_0~~		Wilcoxon-Mann-Whitney
5.3	0.891	0.071	H_0	0.495	*0.007	~~H_0~~		Wilcoxon-Mann-Whitney
5.4	0.079	*0.000	~~H_0~~	0.655	*0.008	~~H_0~~		Wilcoxon-Mann-Whitney
5.5	0.690	0.060	H_0	0.860	0.084	H_0	0.969	ANOVA
5.6	0.271	*0.028	~~H_0~~	0.639	0.093	H_0		Wilcoxon-Mann-Whitney
5.7	0.495	*0.017	~~H_0~~	0.633	0.074	H_0		Wilcoxon-Mann-Whitney
5.8	0.944	0.351	H_0	0.632	*0.011	~~H_0~~		Wilcoxon-Mann-Whitney
5.9	0.309	*0.022	~~H_0~~	0.539	*0.025	~~H_0~~		Wilcoxon-Mann-Whitney
5.10	0.398	*0.003	~~H_0~~	0.350	*0.007	~~H_0~~		Wilcoxon-Mann-Whitney
6.1	0.543	0.069	H_0	0.456	*0.013	~~H_0~~		Wilcoxon-Mann-Whitney
6.2	0.919	0.214	H_0	0.864	*0.030	~~H_0~~		Wilcoxon-Mann-Whitney
6.3	0.645	*0.043	~~H_0~~	0.104	*0.000	~~H_0~~		Wilcoxon-Mann-Whitney
6.4	0.220	*0.032	~~H_0~~	0.916	0.127	H_0		Wilcoxon-Mann-Whitney
6.5	0.052	0.168	H_0	0.674	0.056	H_0	0.009	Unpaired t-test
A.1	0.294	*0.005	~~H_0~~	0.847	0.070	H_0		Wilcoxon-Mann-Whitney
A.2	0.085	*0.001	~~H_0~~	0.357	0.072	H_0		Wilcoxon-Mann-Whitney
A.3	0.938	0.095	H_0	0.508	0.061	H_0	0.256	ANOVA
A.4	0.937	0.068	H_0	0.505	*0.043	~~H_0~~		Wilcoxon-Mann-Whitney
A.5	0.695	0.072	H_0	0.593	*0.050	~~H_0~~		Wilcoxon-Mann-Whitney
B.1	0.536	*0.029	~~H_0~~	0.783	0.093	H_0		Wilcoxon-Mann-Whitney
B.2	0.630	*0.018	~~H_0~~	0.475	0.053	H_0		Wilcoxon-Mann-Whitney
B.3	0.118	*0.012	~~H_0~~	0.683	*0.012	~~H_0~~		Wilcoxon-Mann-Whitney
B.4	0.387	0.137	H_0	0.585	*0.049	~~H_0~~		Wilcoxon-Mann-Whitney
B.5	0.409	*0.036	~~H_0~~	0.715	0.086	H_0		Wilcoxon-Mann-Whitney
C.1	0.515	*0.039	~~H_0~~	0.908	0.077	H_0		Wilcoxon-Mann-Whitney
C.2	0.377	*0.026	~~H_0~~	0.718	0.089	H_0		Wilcoxon-Mann-Whitney
C.3	0.722	*0.010	~~H_0~~	0.615	0.120	H_0		Wilcoxon-Mann-Whitney
C.4	0.709	*0.020	~~H_0~~	0.608	*0.035	~~H_0~~		Wilcoxon-Mann-Whitney
C.5	0.602	0.134	H_0	0.110	*0.009	~~H_0~~		Wilcoxon-Mann-Whitney
D.1	0.372	*0.014	~~H_0~~	0.483	*0.008	~~H_0~~		Wilcoxon-Mann-Whitney
D.2	0.267	0.063	H_0	0.216	*0.001	~~H_0~~		Wilcoxon-Mann-Whitney
D.3	0.313	*0.014	~~H_0~~	0.918	0.292	H_0		Wilcoxon-Mann-Whitney
D.4	0.713	0.183	H_0	0.430	*0.035	~~H_0~~		Wilcoxon-Mann-Whitney
D.5	0.420	*0.021	~~H_0~~	0.307	*0.005	~~H_0~~		Wilcoxon-Mann-Whitney
NRE.1	0.687	*0.015	~~H_0~~	0.665	0.080	H_0		Wilcoxon-Mann-Whitney
NRE.2	0.642	*0.041	~~H_0~~	0.665	*0.028	~~H_0~~		Wilcoxon-Mann-Whitney
NRE.3	0.108	*0.001	~~H_0~~	0.994	0.794	H_0		Wilcoxon-Mann-Whitney
NRE.4	0.237	*0.002	~~H_0~~	0.510	0.051	H_0		Wilcoxon-Mann-Whitney
NRE.5	0.763	0.057	H_0	0.926	0.222	H_0	0.119	ANOVA
NRF.1	0.154	0.051	H_0	0.366	*0.005	~~H_0~~		Wilcoxon-Mann-Whitney
NRF.2	0.070	0.019	H_0	0.062	0.098	H_0	0.281	ANOVA
NRF.3	0.698	*0.046	~~H_0~~	0.812	0.082	H_0		Wilcoxon-Mann-Whitney
NRF.4	0.919	0.058	H_0	0.900	0.055	H_0	0.134	ANOVA
NRF.5	0.188	*0.007	~~H_0~~	0.093	*0.011	~~H_0~~		Wilcoxon-Mann-Whitney

Fig. 5. Boxplot for outcomes with normal distribution evidence for unpaired-test.

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TABLE IV
WILCOXON-MANN-WHITNEY TEST RESULTS.

Instance	BKS	Median		RPD		Wilcoxon-Mann-Whitney test		Algorithm Selection
		BBH	BSLC	BBH	BSLC	$H_1 : \overline{X}_{BBH} > \overline{X}_{BSLC}$	$H_1 : \overline{X}_{BSLC} > \overline{X}_{BBH}$	
4.3	516	769	824	0.02	0	0.8	0.2	indistinct
4.4	494	843	791	0.01	0.02	0.36	0.64	indistinct
4.5	512	758	777	0.01	0.01	0.54	0.46	indistinct
4.6	560	861	815	0.09	0	0.27	0.74	indistinct
4.7	430	668	650	0.07	0.01	0.2	0.81	indistinct
4.8	492	773	691	0.01	0	0.11	0.89	indistinct
4.10	514	813	758	0.12	0.07	0.75	0.26	indistinct
5.1	253	414	425	0.01	0	0.57	0.43	indistinct
5.2	302	489	483	0.05	0.01	0.76	0.24	indistinct
5.3	226	334	360	0	0.03	0.95	*0.05	BBH
5.4	242	316	357	0.04	0.09	0.7	0.31	indistinct
5.6	213	293	308	0.01	0.02	0.68	0.33	indistinct
5.7	293	430	478	0.04	0.02	0.79	0.22	indistinct
5.8	288	487	424	0.05	0.07	0.12	0.88	indistinct
5.9	279	459	413	0.11	0.03	0.58	0.42	indistinct
5.10	265	399	421	0.08	0.09	0.88	0.12	indistinct
6.1	138	221	212	0.01	0.04	0.51	0.5	indistinct
6.2	146	247	233	0.02	0.05	0.18	0.83	indistinct
6.3	145	217	194	0.03	0.06	0.34	0.66	indistinct
6.4	131	181	223	0	0.04	0.94	0.06	indistinct
A.1	253	351	383	0.02	0.02	0.44	0.57	indistinct
A.2	252	361	413	0	0.01	0.78	0.23	indistinct
A.4	234	352	360	0.02	0.1	0.86	0.15	indistinct
A.5	236	308	349	0.02	0.01	0.91	0.09	indistinct
B.1	69	100	100	0.01	0.01	0.55	0.45	indistinct
B.2	76	116	112	0.03	0.01	0.45	0.56	indistinct
B.3	80	116	113	0.01	0.04	0.09	0.91	indistinct
B.4	79	133	118	0.01	0.01	0.21	0.79	indistinct
B.5	72	103	107	0.01	0.03	0.34	0.67	indistinct
C.1	227	322	362	0.02	0.01	0.92	0.08	indistinct
C.2	219	321	337	0	0.02	0.44	0.56	indistinct
C.3	243	381	383	0.01	0.02	0.35	0.66	indistinct
C.4	219	344	334	0	0	0.43	0.58	indistinct
C.5	215	379	291	0.03	0.02	*0.04	0.96	BSLC
D.1	60	91	109	0.08	0.22	0.97	*0.04	BBH
D.2	66	100	90	0.09	0.02	*0.05	0.95	BSLC
D.3	72	95	117	0.03	0.01	0.99	*0.01	BBH
D.4	62	96	93	0.06	0.11	0.37	0.64	indistinct
D.5	61	94	86	0.02	0.03	0.68	0.33	indistinct
NRE.1	29	44	50	0.03	0.03	0.86	0.14	indistinct
NRE.2	30	45	48	0	0.07	0.77	0.23	indistinct
NRE.3	27	37	44	0	0.07	0.99	*0.01	BBH
NRE.4	28	42	43	0.04	0.04	0.62	0.39	indistinct
NRF.1	14	23	21	0	0.14	0.63	0.38	indistinct
NRF.3	14	24	22	0.07	0	0.22	0.78	indistinct
NRF.5	13	18	20	0.08	0.08	0.97	*0.03	BBH

TABLE V
NORMALITY TEST RESULTS FOR INSTANCES WITH NORMAL DISTRIBUTION EVIDENCE ACCORDING KS AND SW TESTS.

Instance	BBH			BSLC		
	KSL p_{value}	**SW** p_{value}	**DA** p_{value}	**KSL** p_{value}	**SW** p_{value}	**DA** p_{value}
4.1	0.078	0.933	**0.89**	0.857	0.220	**0.36**
4.2	0.052	0.065	**0.07**	0.713	0.080	**0.12**
4.9	0.051	0.370	**0.31**	0.634	0.059	**0.12**
5.5	0.690	0.060	**0.13**	0.860	0.084	**0.13**
6.5	0.052	0.168	**0.26**	0.674	0.056	**0.11**
A.3	0.938	0.095	**0.23**	0.508	0.061	**0.08**
NRE.5	0.763	0.057	**0.18**	0.926	0.222	**0.35**
NRF.2	0.070	0.019	**0.18**	0.062	0.098	**0.33**
NRF.4	0.919	0.058	**0.36**	0.900	0.055	**0.29**

KSL and SW tests, show statistically no preference towards BBH or BSLC. That is a preliminary conclusion that must be confirmed or rejected by performing more comprehensive experiments that are beyond the scope of this paper.

IX. CONCLUSIONS

In this paper an approach to compare two algorithms implementation has been performed using non-parametric test. The outcomes for each algorithm implementation have been analyzed by a statistical approach which concludes that BBH's outcomes shows in general a better regularity and consistency than BSLC when they are tested over 55 benchmark for the SCP. It is possible to observe a correlation between the statical and the empirical implementation selection, indicating that 85.5% of the instances there is no preferences towards BBH or BSLC.

The Shapiro-Wilk and Kolmogorov-Smirnov tests could be not enough in order to confirm o reject evidence for a normal distribution when a sample is obtained from an evolutionary algorithm, as has been shown in the experiments for certain instances in this paper. This can be due irregularity in the sample or a result related with the sample size, where small values can give distorted results, as

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TABLE VI
WILCOXON-MANN-WHITNEY TEST RESULTS.

Instance	BKS	Median		RPD		Wilcoxon-Mann-Whitney test		Algorithm Selection
		BBH	BSLC	BBH	BSLC	$H_1 : \overline{X}_{BBH} > \overline{X}_{BSLC}$	$H_1 : \overline{X}_{BSLC} > \overline{X}_{BBH}$	
4.1	429	685	722	0.04	0	0.68	0.33	indistinct
4.2	512	696	831	0.01	0	0.96	*0.04	BBH
4.9	641	1079	1029	0.01	0.01	0.31	0.7	indistinct
5.5	211	321	312	0.03	0.01	0.51	0.49	indistinct
6.5	161	238	237	0.04	0	0.76	0.25	indistinct
A.3	232	336	337	0.01	0.02	0.73	0.28	indistinct
NRE.5	28	44	42	0	0	0.17	0.83	indistinct
NRF.2	15	24	24	0	0	0.28	0.73	indistinct
NRF.4	14	21	23	0	0	0.75	0.25	indistinct

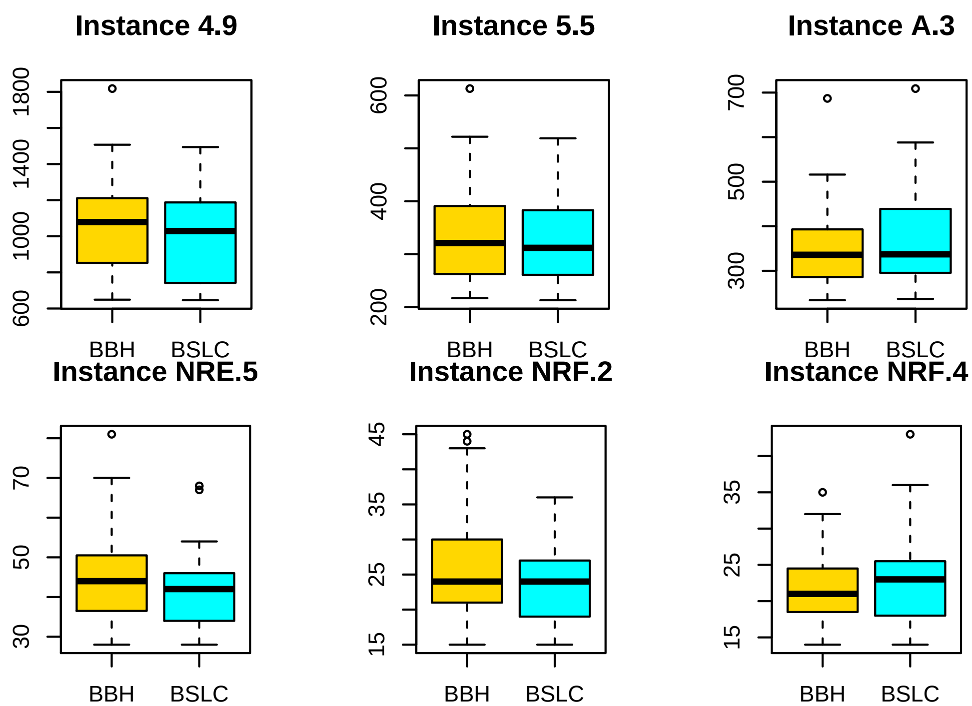

Fig. 6. Boxplot for outcomes with normal distribution evidence for ANOVA test.

is suggested by Demšar in previous papers, requiring more experiments to be addressed in order to set a conclusion properly. How ever, regardless Kolmogorov-Smirnov and Shapirho-Wilk, non-normality condition appears to be not required in order to perform a Wilcoxon-Mann-Whitney test when data sets have been generated by an evolutive algorithm.

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