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An Application of the Lie Product to the Higher Singular Configurations Problem of Serial Manipulators

Una Aplicación del Producto Lie al Problema de Configuraciones Singulares Superiores en Manipuladores Seriales

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Abstract

It is strongly advisable that manipulators must be able to operate into singular regions. A singularity occurs when the end-effector of a manipulator loses, at least, one freedom and consequently cannot perform an arbitrary planned task into the workspace. In this contribution, given a serial manipulator at a singular configuration, a computer procedure to transform a singular configuration into a non-singular configuration one is introduced. The Lie product —a fundamental operation of the Lie algebra, e(3)— plays a central role in the proposed method. Finally, a numerical example is provided.
Keywords: Lie algebra, Lie product, Singular configuration, Velocity state, Kinematics.

Resumen

Es altamente recomendable el que los manipuladores sean capaces de operar dentro de regiones singulares. Una singularidad ocurre cuando el órgano terminal de un manipulador pierde, cuando menos, un grado de libertad y consecuentemente no puede ejecutar una tarea arbitraria planificada dentro del espacio de trabajo. En esta contribución, dado un manipulador serial en una configuración singular, se describe un procedimiento de computadora para transformar una configuración singular en una no singular. El producto de Lie — una operación fundamental del álgebra de Lie, e(3) — desempeña un papel central en el método propuesto. Finalmente, se proporciona un ejemplo numérico.
Palabras Clave: Algebra de Lie, Producto de Lie, Configuración singular, Estado de velocidad, Cinemática.

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1 Introduction

Assume that a serial manipulator, hereafter manipulator for brevity, is at a singular configuration; then, there is at least one loss of freedom and, in order to satisfy a planned task, the end-effector cannot have an arbitrary velocity state in the singular configuration. Moreover, the reverse velocity analysis solution has either extremely or infinite values of actuator velocities, and the manipulator has poor dexterity. These drawbacks affect the performance of the manipulator, however it might be still controllable.

Several procedures to maneuver manipulators in singular regions have been reported in the literature. At a singular configuration it is possible to overcome it by switching, without meeting the singularity, one related configuration to another (Innocenti and Parenti-Castelli, 1998). If the singular configuration is inevitable, the lost of motion can be recovered by isolating and actuating the screw or screws responsible for the singularity (Hunt, 1986), or by selecting a feasible trajectory that pass through the singularity (Chevallereau, 1996).

It is straightforward to demonstrate that isolating and actuating the screw or screws responsible of a singular configuration can quickly eliminate or escape the singularity. As far as the authors are aware, the first comprehensive study dealing with this option, namely the escapement from singularities, is due to Hunt (1986) who used the matrix of cofactors of the Jacobian matrix and the theory of screw systems to determine both the singularity and the screw or screws responsible for such singularity. However, the method proposed by Hunt is applicable only to non-redundant manipulators and, in addition, has serious limitations when the end-effector loses more than one freedom.

Recently, the application of the Lie algebra — a distributive, anticommutative and non-associative algebra that satisfies the Jacobi identity — which is isomorphic to screw theory also called motor algebra (Sugimoto, 1990) to the study of higher singularities, with more than one loss of freedom, of manipulators, has provided an advance in the subject such as the characterization of singular configurations, (Karger, 1996).

It is interesting to say that, while some topics such as the analysis and characterization of singularities of manipulators have received and appreciable amount of contributions, see for instance Litvin et al (1986), Kieffer (1990, 1992), Narasimhan and Kumar (1994), in contrast there are few contributions dealing with the escapement from singular configurations.

In this contribution, an algorithm to eliminate the loss of freedom due to higher singularities in manipulators is introduced. The proposed algorithm, which is based in screw theory, is applicable to both redundant and non-redundant manipulators, and is easily implemented on special software like Maple V©. Finally, the algorithm is applied in the analysis of a manipulator with seven degrees of freedom, which is at a singular configuration.

2 Fundamentals of the method

This Section provides the mathematical fundamentals of the method. A careful analysis of the set of screws associated to the kinematic pairs of the manipulator, at a singular configuration, and its corresponding subsets, yields valuable information about the screw or screws responsible for the singularity.

2.1 Forward kinematics

Consider the manipulator showed in Figure 1. The main elements of the manipulator are the base link, which is considered fixed to an inertial reference frame, and the end-effector. These elements are labelled 0 and m respectively, where $m > 0$. Clearly, the workspace, namely an m-dimensional task space, is defined by m joints.

Assume that two consecutive links, i and $i+1$, are connected by a helical pair $\$_i$ which is given, using Plücker co-ordinates, by

$$\$_i = \begin{bmatrix} \vec{s}_i \\ \vec{s}_{oi} \end{bmatrix}$$

where, $\vec{s}_i$ is a unit vector along the direction of the instantaneous screw axis, ISA, of the helical pair and $\vec{s}_{oi}$ is the moment part of a point O, fixed in body $i+1$, with respect to an arbitrary reference frame whose origin is located at point O. Moreover, the moment part is calculated, in terms of the pitch h, as follows

$$\vec{s}_{oi} = h\vec{s}_i + \vec{s}_i \times \vec{r},$$

where, $\vec{r}$ is the vector joining a point fixed to the ISA of the helical pair with O.

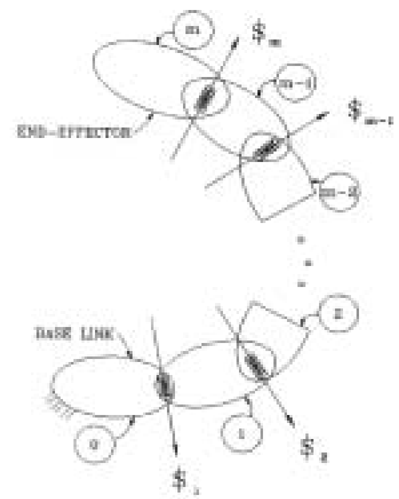

Figure 1: A typical serial redundant manipulator.

It is convenient to describe the manipulator by an **ordered screw set**, S, as follows

$$S = \{\$_1, \$_2, \ldots, \$_{m-1}, \$_m\},$$

where $\$_i$ $(i = 1, \ldots, m)$ are the consecutive infinitesimal screws, which are equivalent to helical pairs, that represent the kinematic pairs of the manipulator.

It is known that the velocity state, $\vec{V}$, of the end-effector, body m, with respect to the base link, body 0, can be expressed in terms of the joint rate velocities, ω_i, and the helical pairs, $\$_i$, as follows

$$\begin{aligned} \vec{V} &= \begin{bmatrix} \vec{\omega}_{m/0} \\ \vec{v}_{m/0} \end{bmatrix} \\ &= \omega_1\$_1 + \omega_2\$_2 + \ldots + \omega_{m-1}\$_{m-1} + \omega_m\$_m \\ &= [J]\begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{m-1} \\ \omega_m \end{bmatrix}, \end{aligned} \tag{1}$$

where $[J]$ is the Jacobian matrix whose column space is the subspace generated by the ordered screw set S, denoted by $[\$_1 \quad \$_2 \quad \ldots \quad \$_m]$.

Note that the components of the velocity state, $\vec{V}$, also called a twist on a screw (Ball, 1900) are two three-dimensional vectors which are referred with respect to the base link, therefore the velocity state results in a six-dimensional vector. The first vector, $\vec{\omega}_{m/0}$, is the angular velocity of body m, with respect to body 0, and the second vector, $\vec{v}_{m/0}$, is the linear velocity of a point coincident with point O but fixed to body m, with respect to body 0. These vectors, $\vec{\omega}_{m/0}$ and $\vec{v}_{m/0}$, are independent of each other and it is always possible to relate them by means of well-known expressions of elementary kinematics. Equation (1) allows to solve both the forward and reverse velocity analyses of manipulators.

2.2 The velocity state as an indicator of a singular configuration

Assume that a manipulator is at singular configuration. A brief inspection of equation (1) reveals that the velocity state of the end-effector is such that at least one joint rate velocity, ω_i, has an unnatural value. Thus, at a singular configuration there are elements of the Lie algebra, $e(3)$, such that $\$ \notin [\$_1, \ldots, \$_m]$ and the end-effector cannot have velocity states of the form

$$\vec{V} = \$_i + \lambda \$,$$

where $\$_i \in [\$_1 \quad \$_2 \quad \ldots \quad \$_m]$ and $\lambda \in \Re$ with $\lambda \neq 0$.

Thus it is evident that, in a singular configuration,

$$rank(J) = rank(J^T) = dim[\$_1 \quad \$_2 \quad \ldots \quad \$_m] < 6.$$

Furthermore, it can be proved that the matrix (JJ^T), where the T denotes transpose, becomes singular, see Bentley and Cooke (1973). Then, it follows that

$$rank(JJ^T) \leq Min\{rank(J), rank(J^T)\} \leq rank(J) < 6.$$

Therefore

$$|\ (JJ^T)\ | = 0.$$

At the outset, this results seems to indicate that the process outlined in Hunt (1986) may be used to determine the screw or screws responsible for the singularity of redundant manipulators. However, the drawbacks already indicated and the additional multiplication of the Jacobian matrix J by its transpose is a motivation for exploring another alternatives that shed light into the subject.

The Lie algebra, $e(3)$, which is isomorphic to screw theory and is considered as the algebra of the infinitesimal screws of the Euclidean group, $E(3)$, is, without doubt, a powerful tool to elucidate singularity problems of manipulators.

If the velocity state, $\vec{V}$, is a six-dimensional vector, then the rank of the Jacobian matrix is at most six. Thus, conveniently, at a singular configuration the ordered screw set, S, can be splitted in several ordered subsets whose cardinality will be at most six.

Let $S_s = \{\$_j, \ldots, \$_k\}$ be a subset of S, whose cardinality is at most six, where $1 \leq j < k \leq m$. If the elements of S_s are adjacent, then $S_s \subset S$ is called an **ordered subset** of S, and is denoted as S^i_{os}.

It is straightforward to demonstrate that at a singular configuration one or more of these ordered subsets are linearly dependent.

An ordered subset, S^i_{os}, is said to be a **minimal singular ordered subset**, S^i_{msos}, if the subset has one loss of freedom,

$$cardinality[S^i_o] - dim[S^i_o] = 1.$$

Finally, each minimal singular ordered subset has a subset linearly independent which is called a **minimal subset**, $S^i_{msos,mss}$.

Summarizing, at a singular configuration the velocity state, $\vec{V}$, of the end-effector, with respect to the base link, requires an impossible or extremely high valued solution of equation (1). This condition yields at least one minimal singular ordered subset, S^i_{msos}, of the ordered screw set, S. It is straightforward to demonstrate that the screw or screws responsible for a singularity is an element of at least one minimal singular ordered subset.

The Lie product —an operation of the Lie algebra, $e(3)$— provides information about which screw or screws are responsible for causing singularities in redundant and non-redundant manipulators. The next Section shows the concept of the Lie product.

2.3 The Lie product

The Lie product, also known as the dual vector product or motor product, is a fundamental operation of the Lie algebra, $e(3)$. Given two elements of the Lie algebra,

$$\$_1 = (\vec{s}_1, \vec{s}_{O1}), \$_2 = (\vec{s}_2, \vec{s}_{O2}) \in e(3);$$

the Lie product, $[\$_1 \quad \$_2]$, is defined as a special arrangement of the above-mentioned screws as follows

$$[\$_1 \quad \$_2] \equiv \begin{bmatrix} \vec{s}_1 \times \vec{s}_2 \\ \vec{s}_1 \times \vec{s}_{O2} - \vec{s}_2 \times \vec{s}_{O1} \end{bmatrix} \qquad (2)$$

Note that the Lie product is anticommutative and nilpotent.

Another interesting definition of the Lie product is given by (Hausner and Schwartz, 1968),

$$[\$_1 \quad \$_2] \equiv \frac{d}{dt}\left\{m_1(t)\$_2[m_1(t)^{-1}]\right\}|_{t=0}, \qquad (3)$$

where, $\$_1$ is a tangent vector to the Euclidean motion $m_1(t)$ at $t = 0$.

It is known, (Chevallereau, 1996), that at a singular configuration the trajectory of the end-effector can be perfectly tracked if the planned trajectory is both orthogonal an tangent to the singular direction. Further, in some cases if the time parametrization of the trajectory is selected properly, the trajectory can be tracked with finite joint rates, see Chevallereau (1998).

By inspection of equations (2) and (3), the Lie product, $[\$_1 \quad \$_2]$, yields a screw whose direction is orthogonal to the plane generated by the directions of $\$_1$ and $\$_2$ and tangent to the direction of $\$_2$. Thus, the Lie product provides the necessary conditions to eliminate singularities.

3 Kinematic rules to recover from the loss of motion

Assume that a manipulator is at a singular configuration. Before do any further, it is pertinent to keep in mind that not all the elements of the ordered screw set, S, can be responsible for the singularity. Effectively, it is reported in the literature that:

1. The extreme screws, $\$_1$ and $\$_m$, never affects the relative positions of the remaining screws (Waldron et al., 1985); consequently, $\$_1$ and $\$_m$ can never be either responsible for causing a manipulator to fall into, or able to produce an escape from, a singular configuration and therefore can be disregarded from the procedure.
2. Hunt (1978) presented a table, which is repeated in Hunt (1986), under the heading "Screw systems that 'guarantee' full-cycle mobility". If a group of this table is present at a singular configuration, and contains another element which is linearly dependent on two or more other elements of the ordered screw set S, then the elements of this group cannot effect an escape from the singular configurations. That is, in order to avoid permanent singular configurations, a minimal singular ordered subset of S will do not generate a subalgebra of the Lie algebra $e(3)$.

From now on it will be assumed that the singular configuration are not permanent.

The following rules for the escapement from singular configurations, an adaptation of those obtained by (Rico et al., 1995) for non-redundant manipulators, can be applied to restore the rank of the subspace generated by each minimal singular ordered subset, S^i_{msos}.

- **Rule 1.** Let S be the ordered screw set of a manipulator at time $t = 0$, with $\$_j(0), \$_k(0) \in S$ and $j < k$. Then, the screw $\$_k(\Delta t)$ is given by

$$\begin{aligned} \$_k(\Delta t) &\approx \$_k(0) + [\$_j(0) \quad \$_k(0)](\Delta t) \\ &= \$_k + [\$_j \quad \$_k](\Delta t), \quad \text{with} \quad \Delta t \neq 0. \end{aligned}$$

- **Rule 2.** There are elements $\$^i_f, \$^i_h \in S^i_{msos}$, with $\$^i_h \in S^i_{msos,mss}$ and $h > f$ such that

$$[\$^i_f \quad \$^i_h] \notin [S^i_{msos}].$$

- **Rule 3.** The dimension of the subspace generated by the minimal singular ordered subset S^i_{msos} obtained by substituting $\$^i_h + [\$^i_f \quad \$^i_h]\Delta t$ instead of $\$^i_h$ is equivalent to the cardinality of S^i_{msos}. Thus, with this substitution, the subset S^i_{msos} is no longer singular. The screw $\$_f$ is said to be the **actuator screw** and $\$_h$ is called the **receptor screw**. Moreover, the dimension of the subsets of the remaining screws is not affected by displacements around the screws involved at the singularity.
- **Rule 4.** Given a minimal singular ordered subset, S^i_{msos}, the Lie product $[\$_u \quad \$_v]$, where $\$_u, \$_v \in S^i_{msos}$ with $u < v$ which do not lies to the subspace generated by the minimal singular ordered subset, contains the screw responsible of the singularity. Thus, by inspection of the Lie products that satisfy this condition, the screw responsible of the singularity, namely the actuator screw, can be isolated.
- **Rule 5.** An actuator screw can restore the dimension of more than one minimal singular ordered subset.

These rules will be applied to each one of the minimal singular ordered subsets.

4 An algorithm that restores the dimension of a minimal singular ordered subset

Given a manipulator at a singular configuration, this Section provides an algorithm that restores the dimension of each one of the different minimal singular ordered subsets associated to the singularity. It is clear that once the global loss of freedom is eliminated, the manipulator is out of the singular configuration. The algorithm, which has been implemented on Maple V©, has been subdivided in several procedures. A brief description of each procedure is given now.

4.1 Reading the ordered screw set S

The numerical information of the ordered screw set, S, is stored in a database. This database can be created using a simple editor or by means of a routine in Maple

V©. Afterwards, the information of the database is introduced in the main program by means of the following procedure,

```
readoss := proc(i, j)
    local k;
    global ordered_screw_set;
     ordered_screw_set := vector(
      readdata('C : /jirs/problem2/S.tex', 6));
     for k from i to j do
      S(k) := ordered_screw_set_k ;
      print(S(k)) ; SS(k) := S(k)
     od
    end
```

Thus, with the instruction **readoss(i,j)**, where **i** and **j** denotes the limits of the ordered screw set, the numerical information of the ordered screws associated to the kinematic pairs of the manipulator is introduced into the main program. It is important to take into account that this procedure do not accept information using rational numbers, therefore the data are stored using real numbers.

4.2 Computing the Lie product of two screws

Given two screws, $\$_i$ and $\$_j$, their Lie product, $[\$_i \quad \$_j]$, is calculated by means of the following procedure,

```
Lie := proc(x, y)
    local xd, yd, xm, ym, a, b;
     xd := vector([x1, x2, x3]) ;
     yd := vector([y1, y2, y3]) ;
     xm := vector([x4, x5, x6]) ;
     ym := vector([y4, y5, y6]) ;
     a := crossprod(xd, yd) ;
     b := evalm(crossprod(xd, ym)
       - crossprod(yd, xm));
     vector([a1, a2, a3, b1, b2, b3])
    end
```

The instruction $\mathbf{Lie}(\mathbf{N_i}, \mathbf{N_j})$ performs the Lie product between $\$_i$, labelled $\mathbf{N_i}$, and $\$_j$, labelled $\mathbf{N_j}$. Conveniently, for further operations, this operation is carried out in a six-dimensional vector form.

4.3 Searching a minimal singular ordered subset

The following procedure tests the existence of a $i-th$ minimal singular ordered subset, S^i_{msos}. The skeleton of the procedure is as follows,

```
msos := proc(i, j)
     local k, A, Lof, indicator;
     if 6 < j - i then print('The ca\
      rdinalyity of the subset must \
      be smaller than 6. Please try \
      with another subset.')
     else
      A := rank(augment(seq(S(k), k = i..j)));
      print('Dimension = ', A) ;
      Lof := j - i + 1 - A ;
      print('Lost of freedom = ', Lof) ;
      if Lof = 1 then
       indicator := it_is_a_msos
      else indicator :=
       please_try_with_another_subset
      fi
     fi
    end
```

Given an ordered subset of S, $\{\$_i, \ldots, \$_j\}$ with $i < j$, defined by an interval, $(\$_i, \$_j)$, the procedure shows how many loss of freedom, if any, has the chosen subset. Besides, the procedure indicates if the ordered subset is a minimal singular ordered subset.

The corresponding instruction of this procedure is given by $\mathbf{msos}(\mathbf{N_i}, \mathbf{N_j})$, where $\mathbf{N_i}$ and $\mathbf{N_j}$ are the labels of the first and the last screws that define the chosen interval.

It is necessary to keep in mind that this procedure do not decide by itself what could be the ordered subset whose rank is deficient. Thus, in order to shorten the search of a minimal singular ordered subset, this part of the computer procedure requires an intuitive participation of the user.

4.4 Isolating the screw or screws responsible for causing a singularity

Once a minimal singular ordered subset is found. The next step is to isolate the screw or screws responsible for the singularity. It is important to note that the Lie product among the screw or screws responsible of the singularity with each one of the remaining screws of the minimal singular ordered subset do not belong to the subspace generated by the chosen minimal singular ordered subset. The sketch of this procedure is as follows,

```
subspace := proc(i, j)
     local k, screw1, A, count, count2;
```

```
for screw1 from i to j do
 if screw1 < j then
  count2 := j - screw1
 else count2 := 1
 fi;
 count := 0 ;
 for k from screw1 + 1 to j do
  A := rank(augment(
   seq(S(kl), kl = i..j),
   Lie(S(screw1), S(k))));
  if A = j - i + 1 then
   posibility(screw1, k) := no ;
   count := count + 1
  else posibility(screw1, k) := yes
  fi;
  print('[', screw1, k, ']',
   posibility(screw1, k))
 od;
 if count = count2 thenprint(
  'Actuating the screw', screw1,
  'it is possible to restore the di\
  mension of the subset(', i, j, ')')
 else
 fi
od
end
```

Given a minimal singular ordered subset, $\{\$_i, \ldots, \$_j\}$ with $i < j$, this procedure computes the following Lie products

$$[\$_u \quad \$_v], u = i, \ldots, j-1; v = i+1, \ldots, j.$$

Afterwards, the procedure shows whether the computed Lie products belong to the subspace generated by the chosen minimal singular ordered subset or not. Finally, the procedure indicates the possible screw or screws that must be actuated to eliminate the linear dependence of the chosen minimal singular ordered subset.

The instruction **subspace**($\mathbf{N_i}, \mathbf{N_j}$) performs the abovementioned operations.

4.5 Computing the new ordered screw set

Given a minimal singular ordered subset $\{\$_i, \ldots, \$_j\}$, with an element, $\$_s \in \{\$_i, \ldots, \$_j\}$, such that $\$_s$ is the screw responsible for causing the singularity. This procedure, in order to modify the ordered screw set, computes the following screws

$$\$_i = \$_i + [\$_s \quad \$_i], \quad i = s+1, \ldots, j.$$

The skeleton of this procedure is as follows,

```
newscrews := proc(screwact, j)
 local k;
 global SS;
  for k from screwact + 1 to j do
   SS(k) :=
    evalm(S(k) + Lie(S(screwact), S(k)));
   S(k) := SS(k) ;
   print('S(', k, ') = ', S(k))
  od
 end
```

The instruction **newscrews**($\mathbf{N_s}, \mathbf{N_j}$), where $\mathbf{N_s}$ is the label of the screw responsible of the singularity, replace the indicated screws with the new Plücker coordinates.

4.6 Computing the dimension of an ordered subset

Given an ordered subset of S, $\{\$_i, \ldots, \$_j\}$, defined by an interval $(\$_i, \$_j)$, this procedure computes the dimension of the chosen ordered subset. The skeleton of the procedure is as follows

```
dimension := proc(i, j)
 local A;
  A := rank(augment(seq(SS(k), k = i..j)));
  print(A)
 end
```

The dimension of the chosen interval is calculated with the instruction **dimension**($\mathbf{N_i}, \mathbf{N_j}$).

5 Numerical example, a manipulator with seven degrees of freedom

In what follows, in order to exemplify the methodology showed through the previous Sections and the efficiency of the proposed algorithm, in this Section a redundant manipulator is analyzed.

Consider the serial redundant manipulator showed in figure 2. The ordered screw set consists of seven elements; five of them, labelled $\$_1, \$_4, \$_5, \$_6$ and $\$_7$, correspond to revolute pairs and the remaining two, labelled $\$_2$ and $\$_3$, are associated to the only cylindrical pair[1]

Moreover, the screws $\$_5, \$_6$ and $\$_7$ are concurrent and therefore can be considered as the representation of a spheric pair.

Furthermore, note that the screw labelled 1 connects the serial chain with the base link, which acts as an inertial reference frame, and the screw labelled 7 connects the end-effector with the serial chain.

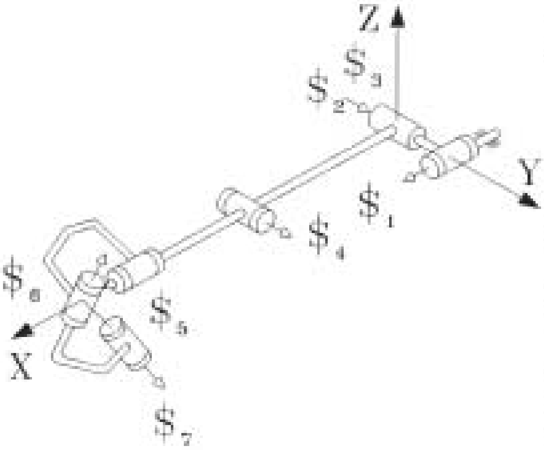

Figure 2: A RCRRRR manipulator.

The normalized screws that represent the kinematic pairs of the manipulator, with respect to the chosen coordinate-system XYZ shown in the figure, are given by

$$\begin{aligned}
\$_1 &= (1,0,0;0,0,-1),\\
\$_2 &= (0,0,0;0,1,0),\\
\$_3 &= (0,1,0;0,0,0)\\
\$_4 &= (0,1,0;0,0,5),\\
\$_5 &= (1,0,0;0,0,0),\\
\$_6 &= \left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}};0,\frac{-10}{\sqrt{2}},\frac{10}{\sqrt{2}}\right),\\
\$_7 &= \left(0,\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}};0,\frac{10}{\sqrt{2}},\frac{10}{\sqrt{2}}\right),
\end{aligned}$$

where the screws $\$_2$ and $\$_3$ are associated with the cylindrical pair.

The ordered screw set, S, of the manipulator is

$$S = \{\$_1, \$_2, \$_3, \$_4, \$_5, \$_6, \$_7\}.$$

[1]Clearly, a cylindrical pair can be considered as the combination of a revolute pair and a prismatic pair.

Thus, the Jacobian matrix, J, results in

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{10}{\sqrt{2}} & \frac{10}{\sqrt{2}} \\ -1 & 0 & 0 & 5 & 0 & \frac{10}{\sqrt{2}} & \frac{10}{\sqrt{2}} \end{bmatrix}$$

Then, the computation of the determinant of the matrix (JJ^T) is zero and consequently the manipulator is at a singular configuration.

Further it is straighforward to demonstrate that

$$dim[\$_1 \quad \$_2 \quad \$_3 \quad \$_4 \quad \$_5 \quad \$_6 \quad \$_7] = 5,$$

Thus, apparently, the manipulator has loses two degrees of freedom.

Before do any further it is convenient to discard the screws that have few possibilities to effect an escapement from the singularity.

The extreme screws of the manipulator are $\$_1$ and $\$_7$. Moreover, the elements of the group $\{\$_5, \$_6, \$_7\} \subset S$ are concurrent and generates a subalgebra of $e(3)$, namely the subgroup of rotations $SO(3)$ of the Euclidean group $E(3)$. Thus, the screws $\$_1$, $\$_5$, $\$_6$, and $\$_7$ have no possibility to effect an escape from the singular configuration.

The remaining screws will be analyzed now by applying the proposed algorithm. The application of the computer procedure yields the following results:

readoss(1,7);

$$[1., 0, 0, 0, 0, -1]$$

$$[0, 0, 0, 0, 1, 0]$$

$$[0, 1, 0, 0, 0, 0]$$

$$[0, 1, 0, 0, 0, 5]$$

$$[1., 0, 0, 0, 0, 0]$$

$$[0, .707106, .707106, 0, -7.07106, 7.07106]$$

$$[0, .707106, -.707106, 0, 7.07106, 7.07106]$$

msos(1,5);

Dimension =, 4

Lost of freedom =, 1

it_is_a_msos

subspace(1,5);

[, 1, 2,], *yes*

[, 1, 3,], *no*

[, 1, 4,], *no*

[, 1, 5,], *yes*

[, 2, 3,], *yes*

[, 2, 4,], *yes*

[, 2, 5,], *yes*

[, 3, 4,], *no*

[, 3, 5,], *no*

[, 4, 5,], *no*

Actuating the screw, 3, it is possible to restore the dimension of the subset(, 1, 5,)

Actuating the screw, 4, it is possible to restore the dimension of the subset(, 1, 5,)

newscrews(4,5);

$S(, 5,) =, [1., 0, -1., 0, 5., 0]$

dimension(1,5);

5

dimension(1,6);

5

readoss(4,5);

[0, 1., 0, 0, 0, 5.]

[1., 0, 0, 0, 0, 0]

newscrews(3,5);

$S(, 4,) =, [0, 1., 0, 5., 0, 5.]$

$S(, 5,) =, [1., 0, -1., 0, 0, 0]$

dimension(1,5);

5

dimension(1,6);

6

dimension(2,7);

6

It is interesting to note that, although both screws, $\$_3$ and $\$_4$, can restore the dimension of the minimal singular ordered subset $\{\$_1, \ldots, \$_5\}$, the screw $\$_4$ cannot restore the dimension of the singular ordered subset $\{\$_1, \ldots, \$_6\}$. Thus, the final conclusion is that actuating the third revolute, $\$_3$, the manipulator can overcome the singular configuration.

6 Conclusions

Given a manipulator at a singular configuration, this contribution provides an algorithm that restores the dimension of each one of the minimal singular ordered subsets which are associated to the singular configuration.

Most researchers consider that the study of higher singularities represents a tedious work. In this contribution the presence of higher singularities do not represent an obstacle to recover the lost of motion of the manipulator. Moreover, the algorithm is applicable to both redundant and non-redundant manipulators.

Finally, the proposed algorithm was successfully applied in the analysis of a manipulator with seven degrees of freedom. The computation of the determinant of the matrix (JJ^T) indicated that the manipulator is at a singular configuration. Further, the computation of the rank of the Jacobian matrix showed that, in the given position, the manipulator has lost one degree of freedom. The application of the computer procedure revealed that actuating only one revolute, the manipulator can escape from the singular configuration recovering its full mobility.

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