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PH. D. THESIS ABSTRACT

A Genetic Algorithm Application for Individual and Group Multicriteria Decision Making

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Introduction

The classical theory of multicriteria decision making lies on the total comparability axiom. The preferential binary relations, strict and indifference, have the transitive property and the alternative pairs are comparable under one of those two relations. This axiom implies the existence of a value function; it is a model that demands ideal conditions to the behavior of the "Decision Maker" (DM), hard to realize in practice, but once constructed and analyzed its consistency results easy to exploit and no objections can be made to the rationality of its recommendation. However, other problems exist for which the decision analysis does not adjust to a value function. A possible preference modeling option is based on the "outranking" relations (Roy, 1990), which works with a weak questionable preference relation thus gives up transitivity and comparability as prioristic properties. We are interested in situations where the preference modeling is based in a fuzzy preference relation; the outranking relation is established with a certain degree of credibility; the model is easier to construct, it needs less information and imposes requisites that are less strict to the actor of the decision process, but its recommendation is much more questionable. We think that in the exploitation of the fuzzy preference relation is its fundamental weakness as an option to the normative theory, for which we propose, as a first contribution, a new ranking method based in a robust search technique. Even more complicated is the group decision making, when the contradictory preferences of different DM must in some way be included in one global model that reaches the accepted conclusions and accepts them as valid by the group consensus. For the construction of this representation we decided to use a flexible focus of preference modeling based on fuzzy outranking relations, applying genetic algorithms to exploit the preference integration model of the obtained group, and obtain as a result a recommendation to arrange the alternative set in decreasing preference.

2 Research Project

Without any doubt, it has been important for the advancement of science to experiment and enrich a theory with ideas, concepts and procedures of other theories and methodologies that allow generating new hybrid methods, in order to become more robust. The European school of multicriteria analysis experimented this when they introduced in some of their methods,ideas and concepts of the theory of fuzzy sets. Our research project experiments with a new relation between the multicriteria analysis from the European school methodology and genetic algorithms. In the last years genetic algorithms have had significant achievements and important advancements on diverse disciplines of human knowledge and day by day are becoming one of the most robust optimization techniques that help to resolve difficult problems.

2.1 Problem Formulation

Let A be a finite set of alternatives or potential actions considered a group and evaluated with multiple criteria. The group is composed of a set $M=\{1,2,...,N\}$ of members, whose work is somehow controlled by the «Supra Decision Maker». Let $\sigma_i:AxA \Rightarrow [0,1]$ be a fuzzy binary relation which integrates the preferences of the i-th member over the multiple criteria that describe the elements of A. The problem is:

i. For each one of the individuals and according to their appreciation of the alternatives, obtain a complete ranking O_i derived from exploiting the relation σ_i in decreasing preferential order.

ii. Based on the N pairs (σ_i,O_i) establish an integration model and a group preference balance that determines a complete ranking O that reflects the best group compromise according to the preferences of the "Supra Decision Maker".

2.2 General Objective

To create a new solution method that integrates robust searching techniques and flexible modeling preferential methods, in a unique procedure that contributes to the solution of the individual and group ranking problem of a set of alternatives characterized by multiple attributes.

2.3 Hypothesis

The use of genetic algorithms to exploit a fuzzy binary relation together with the preferential integration focus based on the concordance-discordance principle makes it possible to obtain a method to derive a consistent ranking in multicriteria and group decision problems.

2.4 Specific Objectives

To develop a genetic algorithm to obtain a final ranking from a fuzzy binary relation.

2 To create a solution method for multicriteria and group decision problems with better properties than other reported focuses.

3 Proposals that Characterize the Formulated Problem

Group decision making covers a wide range of situations Our interest is reduced to modeling a situation of group decision making with the following characteristics.

Each group member has different value systems. Every group member participates in the decision making process and is partially responsible of the final decision. Each group member considers the same alternative set. The alternative set boundary is generally fuzzy. The alternatives are weighted by multiple criteria, some of them are in conflict between them. The group has more recourses than each of its members and the potential to realize an effective decision is greater. Even when the group members are physical people, all their preferences are rarely well established. In this problem, the person who really takes the decision does not exist like so. There is an instance of authority to assign consensus rules and preferential information to the group members. We call this imaginary person the "Supra Decision Maker" (SDM) which will integrate in an altruist form the preferences of the group members. We suppose that the SDM will always be able to create a prescription of one of the alternatives of the set and that it will always create the same prescription under the same conditions. The prescription consists of a complete group ranking of the set of alternatives derived from each of the individual ranking associated to each group member.

4 General Focus Used to Integrate the Group Preferences

The focus used here consists in that each DM expresses its own preferences, integrates them in individualy manner, later integrating the preferences of all the DM. This focus has the advantage of taking into account the preferences of all the actors (Macharis et al, 1998).

5 The New Procedure of Solution (Preference Model of SDM)

Firstly we propose a method for solving possible conflicts that from the point of view of the SDM judgement can appear between a fuzzy preference relation and the ranking derived from it for each individual. We later define a fuzzy preference relation for the SDM.

5.1 Conflict Treatment: The preference Matrix

Let σ_i be a weighted binary relation that integrates the preferences of the with group member over the base of the weights that make multiple criteria on the elements of A.

Let O_i be a complete ranking of A made with a procedure that uses a σ_i. The basic idea is to consider each k member of the group as a criteria of a new multicriteria problem, in this case the group problem. The comparison of the alternatives by the new criteria considers the incoming information from two significant elements: a) The values $\sigma_k(a,a')$ y $\sigma_k(a',a)$, y b) the relative position of projects a and a' in O_k. For modelling the SDM preferences, we introduce two parameters λ and β that serve as limits. We assume that λ $(0<\lambda<1)$ exists, such that if $\sigma_k(a,a')\geq\lambda$ the SDM conforms with the statement "In the absence of any other information, this is a sufficient reason to think that a is as good as a' from the point of view of the k member." We assume that β exists, such that if $\sigma_k(a,a')\leq\lambda-\beta$ the SDM conforms with the statement "In the absence of any other information, action a is not as good as a' from the point of view of the k member." In the interval of $(\lambda-\beta\leq\sigma_k\leq\lambda)$ the SDM has doubts about the outranking. Analyzing the dimension of σ_k, 9 different zones were observed:

$\sigma_k(a,a')\geq\lambda$, $\sigma_k(a',a)\geq\lambda$

II. $\sigma_k(a,a')\geq\lambda$, $\lambda-\beta<\sigma_k(a',a)<\lambda$

III. $\sigma_k(a,a')\geq\lambda$, $\sigma_k(a',a)\leq\lambda-\beta$

IV. $\sigma_k(a',a)\geq\lambda$, $\lambda-\beta<\sigma_k(a,a')<\lambda$

V. $\lambda-\beta<\sigma_k(a,a')<\lambda$, $\lambda-\beta<\sigma_k(a',a)<\lambda$

VI. $\sigma_k(a',a)\leq\lambda-\beta$, $\lambda-\beta<\sigma_k(a,a')<\lambda$

VII. $\sigma_k(a',a)\geq\lambda$, $\sigma_k(a,a')\leq\lambda-\beta$

VIII. $\sigma_k(a,a')\leq\lambda-\beta$, $\lambda-\beta<\sigma_k(a',a)<\lambda$

IX. $\sigma_k(a',a)\leq\lambda-\beta$, $\sigma_k(a,a')\leq\lambda-\beta$

Let $u_k : A\rightarrow N$ be a function defined as $u_k(a_i)=card(B)+1$ where $B=\{a_j \in A : a_j$ is ranked worse than a_i in $O_k\}$. In (b), the SDM considers by default the following 5 different situations:

- $u_k(a) >> u_k(a')$: This representation is for the case where action a is ranked in the first positions while a' is one of the worst ranked actions (this classification is defined by the SDM which can take into account the opinion of the k member.
- $u_k(a) > u_k(a')$: This representation is for the case where action a is ranked better than a', but the previous situation does not happen.
- $u_k(a) = u_k(a')$: Actions a and a' are ranked in the same position or their difference is minimal.
- $u_k(a') > u_k(a)$.
- $u_k(a') >> u_k(a)$.

Using the binary relations: Strict Preference (P), Weak Preference (Q), Indifference (I) and Incomparability (R) defined by Roy (Roy, 1996) the SDM expresses its preferences in the following matrix:

Zona	u(a) >> u(a')	u(a) > u(a')	u(a)=u(a')	u(a') > u(a)	u(a') >> u(a)
I	aPa'	aQa'	aIa'	a'Qa	a'Pa
II	aPa'	aPa'	aQa'	aIa'	a'Pa
III	aPa'	aPa'	aQa'	aIa'	a'Qa
IV	aPa'	aIa'	a'Qa	a'Pa	a'Pa
V	aPa'	aQa'	aIa'	a'Qa	a'Pa
VI	aPa'	aPa'	aQa'	aIa'	a'Pa
VII	aQa'	aIa'	a'Qa	a'Pa	a'Pa
VIII	aPa'	aIa'	a'Qa	a'Pa	a'Pa
IX	aPa'	aQa'	aIa'	a'Qa	a'Pa

Preferences Matrix

5.2 A fuzzy outranking relation that integrates the SDM preferences

We define a weighted binary relation in a similar manner as ELECTRE III (Roy, 1990) built it.

<u>Preliminary definitions</u>.- Action *a* outrank action a' from the point of view of actor k (constrained to the outranking relation $aS_k a'$) if and only if $aP_k a'$, $aQ_k a'$ or $aI_k a'$, according to the corresponding preference matrix element. Actor *k* is in concordance with the statement $aS_G a'$ (where S_G is the group outranking) if and only if $aS_K a'$. $\mathfrak{C}(aS_G a')$ denotes the concordance coalition, the set of actors that are in concordance with $aS_G a'$. Actor k is in discordance with the statement with $aS_G a'$. Actor k belongs to the veto coalition $V(aS_G a')$ if and only if the following two conditions are satisfied: (i) $u_k(a') >> u_k(a)$ (ii)$\sigma_k(a',a)-\sigma_k(a,a')\geq\beta$. Actor k belongs to the imcomparability coalition $\mathfrak{C}(aR_G a')$ if $aR_k a'$.

<u>The concordance index</u>

The role of the different actor (criteria) is not necessarily the same from the point of view of the SDM. The importance of the j-th criteria is taken into account through two independent factors: its importance coefficient $w_j>0$ and its veto capacity.

$$C(a,a') = \frac{1}{W} \sum_{j \in C(aS_G a')} w_j$$

donde $W = \sum_{j \in M'} w_j$, $M' = \{k \in M : aS_k a' \text{ o } a'S_k a\}$

The power of the veto coalition

The veto condition is given by the number and/or the importance of the actors that belong to $V(aS_G a')$. Let v_j be the number of votes that the SDM assigns to the j-th group member. Assume that i denotes a common member not related to the group; therefore, without generality loss, v_i is equal to 1. Let N_v be the number of votes that the SDM considers necessary to make valid the statement $aS_G a'$. The discordance index is defined as:

$$d(a,a') = \begin{cases} \sum_{V(aS_G a')} v_j \,/\, N_v & \text{si } \sum_{V(aS_G a')} v_j \le N_v \\ 1 & \text{the other mode} \end{cases} \quad (2)$$

The incomparability coalition

The decision is considered valid only if one important part of the group votes in an effective manner. We propose the comparability index in the following manner:

$$r(a,a') = \begin{cases} \left(\sum_{M'} w_j - \sum_{C(aR_G a')} w_j\right) \Big/ \sum_{M'} w & \text{si } \sum_{M'} w_j \ge \sum_{C(aR_G a')} w_j \\ 0 & \text{si } \sum_{M'} w_j < \sum_{C(aR_G a')} w_j \end{cases} \quad (3)$$

The fuzzy relation ELECTRE-GD

The preceding indexes allow us to define a fuzzy outranking relation for group decision making in the following manner:

$\sigma_G : AXA \rightarrow [0,1]$

$\sigma_G(a,a') = C(a,a').\ (1-d(a,a')).\ r(a,a')$

where C(a,a'), d(a,a'), r(a,a') are given by (1), (2) and (3).

σ_G must be interpreted as a credibility value for the group of the statement "a is at least as good as a'" in reference to the SDM preferences.

6 Genetic Algorithms

The developed algorithm allows us to exploit a fuzzy outranking relation with the purpose of building a prescription for the problem of ranking a set of alternatives or potential actions. A potential solution for this problem is a ranking of the potential action in order of decreasing preferences. These actions (genes) are joined to form a chain of values (chromosomes). The chromosome is represented as a chain whose symbols belong to an n-th alphabet, where n is the number of action in the decision problem. A coded action with a value of a_{ki} in the i-th input of the chain means that the coded action with the value a_{ki} is ranked in the i-th position in order, in addition, a_{ki} is preferred to a_{kj} if i<j, where $a_{ki} \in A=\{a_1,a_2,...,a_n\}$, i=1,1,...,n, and $[k_1,k_2,...,k_n]$ is a permutation of [1,2,...,n]. Each individual is associated to a number λ ($0 \leq \lambda \leq 1$) that is directly associated with the credibility level of a crisp outranking relation defined upon the set of potential actions. The adaptability measure of an individual is divided in two; an adaptability function and an inadaptability function. The adaptability function f of an individual **p** with the credibility level λ is defined in the following manner: Let **p**=$a_{k1}\ a_{k2}\ ...\ a_{kn}$ be the schematic representation of the chromosome of an individual and assume that a_{ki} and a_{kj} are two actions such that $\sigma(a_{ki},a_{kj}) \geq \lambda$ and $\sigma(a_{kj},a_{ki}) \leq \lambda-\beta$ ($\beta>0$, representing a limit), therefore we agree that "a_{ki} outranks a_{kj}" ($a_{ki}\ S^{\lambda}\ a_{kj}$) and "$a_{kj}$ does not outrank a_{ki}" ($a_{kj}\ nS^{\lambda}\ a_{ki}$). In the case of the crisp outranking relation generated by λ, S_A^{λ}, a preference in favor of a_{ki} is assumed.

Therefore: $f(\mathbf{p}) = |\{(a_{ki},a_{kj}): a_{ki}\ nS\ a_{kj}$ and $a_{kj}\ nS\ a_{ki}$ $i=1,2,...,n-1, j=2,3,...,n, i<j\}|$ where $[k_1,k_2,...,k_n]$ is a permutation of [1,2,...,n].

f(**p**) is the number of incomparabilities between the pairs of actions (a_{ki},a_{kj}) in the individual **p**=$a_{k1}\ a_{k2}\ ...\ a_{kn}$ in reference to the crisp relation S_A^{λ}.

The inadaptability *u* of an individual **p** measures the amount of infactibility (in relative terms) and is defined in the following manner:

$u(\mathbf{p}) = |\{(a_{ki},a_{kj}): a_{ki}\ S\ a_{kj}$ and $a_{kj}\ nS\ a_{ki};\ i=1,2,...,n, j=1,2,...,n, i>j\}|$.

u(**p**) is the number of preferences between the actions of an individual **p** that are not "well ordered" in reference to S_A^{λ}. An individual **p** is plausible if u(**p**)=0 and not plausible if u(**p**)>0. It is clear and it appears natural to define that the inadaptability function takes a minimum value of 0 if and only if the solution is plausible. Each one of the individual **p**'s can be represented by a triad of values f, u and λ.

7 Examples and Applications of the Method

To prove that the genetic algorithm can be used as a method to obtain a ranking hypothetical examples where generated (fuzzy binary relations) and the results where compared with those obtained from other methods created for the same purpose. Decisive elements where found that allow claiming the superiority of our method (Leyva and Fernandez, 1999). The used method to obtain the group ranking was compared against PROMETHEE with an application presented in (Macharis et al., 1998); the results clearly benefit our proposes solution.

8 Conclusions and Recommendations

According to the empirical tests and comparisons performed with other methods, the work developed here supports a new reliable tool of multicriteria decision analysis based on genetic algorithms that help a group of decision makers reach a consensus. Possible future lines of research and development are the following:

i. Search of genetic algorithm properties as a ranking method,

ii. Analysis, design and development of a GDSS whose nucleus is the method proposed here to be used in a first phase in a "Computarized Room Decision" driven by a facilitator. In a second phase there is development of a client-server version that allows the Decision Makers to be placed in different places and to be installed in Internet o in an Intranet.

iii. Variants of the genetic algorithm that allow finding multicriteria information automatically for the problem of individual and group ranking.

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