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Spatial Dynamics Behavioral Analysis of Motivational Operations Using Weighted Voronoi Diagrams

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Abstract. This paper introduces a new method for analyzing how spatial behavior is organized, using weighted Voronoi diagrams. The objective is to track and understand where an experimental subject moves and distributes time within an experimental space, helping to identify specific areas of highest behavioral interest. The method involves dividing the space into a grid, selecting generator points, and assigning weights based on how long the subject stays in each area. Data for the analysis came from several experimental sessions where subjects were exposed to different conditions, such as food deprivation, water deprivation, both, or no deprivation at all. These varying conditions led to distinct spatial patterns. The weighted Voronoi diagrams offered a clear and detailed representation of the areas of interest, making it easier to study changes in behavior under different circumstances, such as varying motivational conditions. This approach provides a valuable perspective for the dynamic study of spatial behavior in changing experimental environments.

Keywords. Weighted Voronoi diagrams, behavior analysis, spatial dynamics behavioral analysis, motivational operations.

1 Introduction

Spatial Dynamics Behavioral Analysis (SDBA) explores how individuals, whether animals or humans, move within the context of behavioral processes such as motivation, learning, and fear. This analysis aims to understand how environmental features and individual factors, such as motivational states, interact to shape movement and behavior. One of the biggest challenges in SDBA is finding a clear and effective way to visually represent these complex interactions. To address this, SDBA often involves dividing space into zones that reflect the subject's own movement patterns, helping to identify what we call Regions of Behavioral Relevance (RBR), areas where key behaviors occur. Tracking and analyzing these RBRs under different conditions is essential for gaining insight into the dynamics of these behaviors. In this study, we propose using weighted Voronoi diagrams as a tool to improve the representation and analysis of RBRs under different conditions of deprivation, like food deprivation, water deprivation, food and water deprivation, and no deprivation whatsoever.

A Voronoi diagram is a mathematical method to divide a given space into regions based on a finite set of points, known as generators. Each region belongs to the generator point closest to it [12, 22, 23]. When weights are added to these points, a weighted Voronoi diagram is generated, where the size and shape of each region adjust according to the weight of its generator. Our study uses this approach, applying multiplicative weighting, to create a more accurate representation of the influence of spatial and behavioral factors under different conditions.

Voronoi diagrams have been widely used to model and analyze spatial problems in various fields. In geography, they've been helpful for urban planning and resource distribution [6], and spatial visualization [10]. They've also been valuable in predicting fire spread and optimizing patrol routes with the use of drones [13]. Furthermore, in image reconstruction, Voronoi diagrams using natural neighbor interpolation have made considerable improvements in image quality [9]. In sports science, they've been used to analyze team behavior, pinpoint key moments in games, and study the spatial interactions between players [8, 11, 14]. More closely related to SDBA, these diagrams have been used in urban mobility studies to track population density, commuting patterns, and the speed and direction of movement [19, 24, 25, 2]. Recently, they have been used for modelling animal territorial behaviour [15]. Despite their usefulness, Voronoi diagrams have not been fully explored in the experimental analysis of the spatial dynamics of organisms' behavior and behavioral processes.

This paper suggests using Voronoi diagrams, particularly its multiplicatively weighted version, to study behavioral patterns in response to different Motivational Operations [18, 20] under varying deprivation conditions. This method would allow for a functional analysis of how zones of influence shift, revealing how behavioral dynamics evolve, depending on such deprivation conditions. We believe this will offer a powerful tool for understanding changes in RBRs and provide a more comprehensive picture of how behavior is organized spatially in response to different Motivational Operations.

2 Weighted Voronoi Diagrams

The definitions of this section can be found in [3, 22], also in the second reference some applications and algorithms for computing Voronoi diagrams can also be consulted.

Consider a set of generator points $P = \{p_1, p_2, \ldots, p_n\} \in \mathbb{R}^2$, which we will call the generator set, and a set of weights $W = \{w_1, w_2, \ldots, w_n\} \in \mathbb{R}^+$, where each parameter w_i is the weight associated with the point p_i, which represents the ability of p_i to influence the space. In order to reflect this influence, the value of w_i is used to define a weighted distance relative to p_i, which is denoted as $d_w(x, p_i)$. The specific manner in which d_w is defined depends on the type of weighted Voronoi diagram that is being considered.

For a given point $p_i \in P$, the weighted Voronoi polygon associated with this point is defined as follows:

$$Vor(p_i) = \{x \in \mathbb{R}^2 : d_w(x, p_i) \leq d_w(x, p_j), j \neq i\}.$$

The weighted Voronoi diagram generated by the set P is a partition of $\mathbb{R}^2$ into n regions defined by the Voronoi polygons associated with all $p_i \in P$, and is denoted by:

$$Vor(P) = \{Vor(p_1), \ldots, Vor(p_n)\}.$$

In the case of two distinct points, $p_i, p_j \in P$, if the intersection of the polygons associated with these points contains more than one point, it will be the weighted edge associated with p_i and p_j, that is:

$$e(p_i, p_j) = Vor(p_i) \cap Vor(p_j).$$

We will call the union of the weighted Voronoi edges a Voronoi lattice. We also define the weighted bisector between these points as:

$$b(p_i, p_j) = \{x \in \mathbb{R}^2 : d_w(x, p_i) = d_w(x, p_j)\}.$$

If we define the domain region of p_i over p_j as the set:

$$Dom(p_i, p_j) = \{x \in \mathbb{R}^2 : d_w(x, p_i) \leq d_w(x, p_j), \},$$

then we have:

$$Vor(p_i) = \bigcap_{j \neq i, j=1}^{n} Dom(p_i, p_j),$$
$$b(p_i, p_j) = Dom(p_i, p_j) \cap Dom(p_j, p_i).$$

Thus, the bisector divides the space into the two domain regions p_i and p_j over p_j and p_i, respectively.

There are several variants of weighted Voronoi diagrams, such as multiplicatively weighted, additively weighted, compoundly weighted, and power weighted. The choice of one variant or another depends on how the values of the set of weights W are used to define the function d_w (see [22]). In this study, we will focus on multiplicatively weighted Voronoi diagrams, and thus the weighted distance will be considered as follows:

$$d_w(x, p_i) = \frac{1}{w_i} \|x - p_i\|,$$

where $\|x - p_i\|$ is the Euclidean distance between points x and p_i.

The shape of the resulting domains is contingent upon the relationship between the weights associated with two points, $p_i, p_j \in P$. In consideration of the weighted distance previously defined, if $w_i < w_j$, the domain region of the point p_i over point p_j can be expressed as the closed ball:

$$Dom(p_i, p_j) = \left\{x \in \mathbb{R}^2 : \|x - o\| \leq r\right\},$$

where

$$o = \frac{w_j^2}{w_j^2 - w_i^2} p_i - \frac{w_i^2}{w_j^2 - w_i^2} p_j,$$
$$r = \frac{w_i w_j}{w_j^2 - w_i^2} \|p_i - p_j\|.$$

Conversely, if $w_i > w_j$, the domain region of p_i over p_j is given by the following equation:

$$Dom(p_i, p_j) = \left\{x \in \mathbb{R}^2 : \|x - o\| \geq r\right\}.$$

This equation defines the complement of the open ball with center at o and radius r. When $w_i = w_j$, the domain region corresponds to the classical unweighted definition. Thus, we have the following result, see its proof in [22].

Theorem 1 *The edges of a multiplicatively weighted Voronoi region are circular arcs if and only if the weights of two adjacent regions are not equal and are straight lines in the plane if and only if the weights of two adjacent regions are equal.*

In this type of diagram, the largest weights associated with the corresponding generating point determine more important regions and geometrically correspond to larger regions with greater area. As illustrated in Fig. 1, two distinct Voronoi diagrams are presented for the same generator set, but with different sets of weights. It should be noted that the Voronoi regions are not necessarily convex.

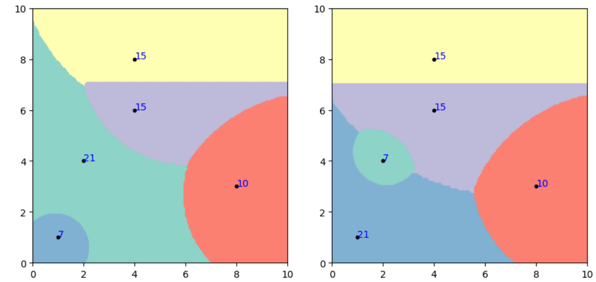

Fig. 1. Multiplicatively weighted Voronoi diagrams, the numbers represent the corresponding weights

A variety of algorithms have been developed for the construction of Voronoi diagrams, including Domain Intersection, Incremental Algorithm, and Divide and Conquer method, among others [4, 7, 22]. In particular, for this work, the Domain intersection algorithm was adapted for weighted Voronoi diagrams, because it is conceptually simple [21].

3 Application for SDBA

Experimental Behavior Analysis (EBA) examines the behavior of individuals by exploring the environmental variables that influence it [1]. EBA studies the factors that affect behavior, traditionally focusing on single-discrete responses. However, studying organism movement has become crucial in understanding various behavioral phenomena within ecological contexts. This approach is known as Spatial Dynamics Behavior Analysis (SDBA) (see [16]).

SDBA can be applied to the study of specific behavioral phenomena, such as Motivational Operations (MOs), by analyzing the effect of different deprivation conditions (food, water, and combined food-and-water deprivation) on the emergence of different spatial dynamics when rats are given a choice between consuming water or food in a concurrent delivery arrangement. While

SDBA represents an important advancement in EBA, it presents several significant challenges, one of which is visualizing complex spatial behavioral patterns. For example, understanding how water or food deprivation affects the distribution of stays in specific zones where these resources are found is a primary focus.

To address this challenge, we will explore using weighted Voronoi diagrams in the context of MOs. First, we describe the experimental setting and method, followed by the data collected. Finally, we analyze the data, focusing on the spatial organization of behavior under different deprivation conditions using weighted Voronoi diagrams.

3.1 Data Acquisition

The study was conducted by members of the Comparative Psychology Laboratory at the Universidad Veracruzana. The experimental subjects were six Wistar rats subjected to a schedule of water or food restriction, depending on the corresponding experimental phase, as described below.

A single-case experimental design was implemented following established behavior analysis protocols, incorporating a detailed behavioral record for each rat (7200 data points of the rat's location per session). For data collection, we used a modified open field chamber with an experimental space measuring $92cm \times 92cm \times 33cm$ in length (width and height). A limited-availability water dispenser was placed in the middle of one wall of the chamber at coordinates $(46, 0)$, and a limited-availability food dispenser was placed on the opposite wall at coordinates $(46, 92)$. We used a tracking system (EthoVision XT) to monitor the displacement of rats inside the chamber throughout 20 minutes in a potential food-or water-seeking behavior. The data provided indicate the trajectory of the subjects in coordinates (x, y) over time, recorded at a resolution of 5 frames per second.

Twenty-four sessions, each lasting 20 minutes, were conducted under one of four deprivation conditions (6 sessions per condition), while the food and water delivery schedule during the experimental sessions remained constant. Different deprivation conditions could involve different Motivational Operations (MOs). The deprivation conditions were: a) Water Deprivation (WD) b) Food Deprivation (FD) c) Food and Water Deprivation (FWD) d) No Deprivation (ND).

For each deprivation condition, food and/or water consumption was restricted for 22 hours before each experimental session. Each deprivation condition consisted of three days with the corresponding restriction. Each experimental session consisted of concurrently presenting food and water in each dispenser every 30 seconds, with a limited availability of 3 seconds. A yellow light above both dispensers was turned on with every delivery and remained on during 3 seconds of availability.

3.2 Generation of Weighted Voronoi Diagrams

In order to construct weighted Voronoi diagrams based on the collected experimental data, it is first necessary to determine a generator set and then assign appropriate weights to each of its points.

To this end, the experimental space was partitioned into a uniform grid of $n \times m$ regions, which defined an initial set Q of $n \times m$ points, comprising the centers of these regions. From the data obtained from the EthoVision XT system relating to the subject's trajectory in each experimental space session, the time accumulated by the subject in each region during the session was determined. To perform the aforementioned computations, the MOTUS software was employed, which enables the generation of various graphical representations of data derived from the displacement of an individual, recorded as spatial changes in the (x, y) coordinates over time (see reference [17]).

The graphs (a) and (b) depicted in Fig. 2 were generated by this software. This information was then incorporated into a matrix of dimensions $n \times m$ (see matrix (c) in Fig. 2). The generator set, denoted by P, was defined as the set of points in Q corresponding to the regions with nonzero cumulative time. The set of weights, represented by W, consists of positive cumulative times in these regions. Finally, the set P is ordered in descending

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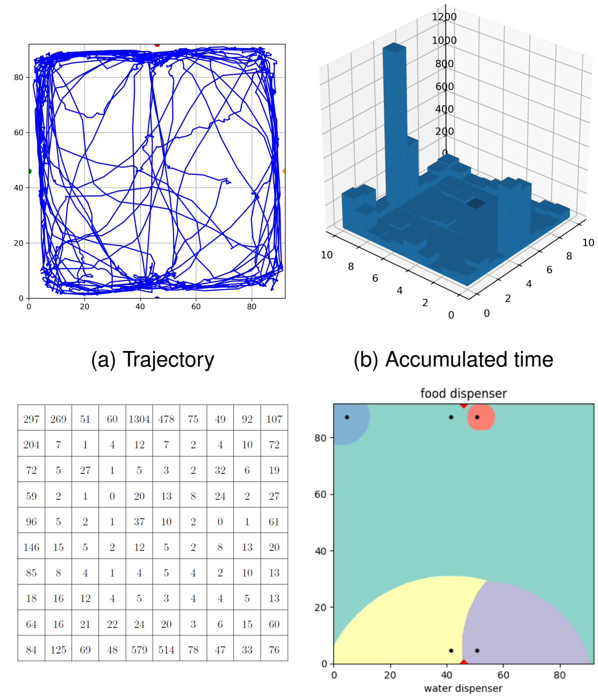
Algorithm 1 Voronoi Diagram

Require: P (generator set), W (weights set) and cl (color set)
Ensure: Voronoi regions
  for $p_i \in P$ and $w_i \in W$ **do**
    if $p_i = p_0$ **then**
      paint the background with color $cl[p_i]$
    else
      $D \leftarrow []$ ▷ empty list
      for $p_k \in P$ **do**
        if $p_k \neq p_i$ **then**
          add $Dom(p_i, p_k)$ to D ▷ calculate the domain of the region
        end if
      end for
    end if
    $c \leftarrow []$ ▷ empty list
    for point in $Dom(p_i, p_0)$ **do**
      $InArea(point, D)$ add to c
    end for
    paint c with color $cl[p_i]$
  end for
  draw the points of P



(a) Trajectory (b) Accumulated time

297	269	51	60	1304	478	75	49	92	107
204	7	1	4	12	7	2	4	10	72
72	5	27	1	5	3	2	32	6	19
59	2	1	0	20	13	8	24	2	27
96	5	2	1	37	10	2	0	1	61
146	15	5	2	12	5	2	8	13	20
85	8	4	1	4	5	4	2	10	13
18	16	12	4	5	3	4	4	5	13
64	16	21	22	24	20	3	6	15	60
84	125	69	48	579	514	78	47	33	76

(c) Cumulative time matrix (d) Voronoi diagram

Fig. 2. Trajectory, accumulated time in regions and the corresponding multiplicatively weighted Voronoi diagram

order with respect to the weight associated with its points.

The Algorithm 1 illustrates the method for obtaining the Voronoi region associated with a given point. The functions $Dom(p_i, p_k)$ and $InArea(point, D)$ perform the following operations: the function $Dom(p_i, p_k)$ identifies the domain region of the generator point p_i over p_k, while the function $InArea(point, D)$ determines whether a point belongs to $Vor(p_i)$.

The Algorithm 1 can be summarized as follows: Given that the generating points are ordered in descending order of weight, the $Vor(p_1)$ region will be the largest, thus determining the background color. Subsequently, for a specific generator point $p_i \neq p_1$, all domains $Dom(p_i, p_j)$ for $i \neq j$ are saved in a list D. Thereafter, the set of points that are in all domains of the list D, that is, all points in $Vor(p_i)$, is constructed using the $InArea(point, D)$ function. The aforementioned points are then stored in a list, designated as c, along with the corresponding color assigned to the $Vor(p_i)$ region. The points are subsequently rendered. At the conclusion of this process, the generating points are then rendered. This approach allows us to paint over the previous color layer. In the absence of an ordering of the points, the generator with the highest weight would eventually cover the Voronoi regions of the preceding generators.

Fig. 2 provides a visual representation of the process described above for generating the Voronoi diagram of a session of Subject 1 under the WD condition. In graph (a), the trajectory is shown within the experimental space of $92cm \times 92cm$, which was divided into a grid of 10×10, as shown in the accumulated time graph (b), generating an initial set of 100 points. Using the information presented in matrix (c), the generator set and the associated set of weights were determined; to simplify the visualization, only the five points with the highest weights were included. As shown in Fig. 2 (d), the multiplicatively weighted Voronoi diagram includes red triangles indicating

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the locations of the food and water dispensers in the experimental space; and colored regions representing the weighted Voronoi regions of the generator points, marked with black dots.

3.3 Data Analysis

To illustrate the use of Voronoi diagrams in SDBA, two subjects were selected from the six experimental subjects, with each subject exposed to the four aforementioned conditions. Fig. 3 display the subject and four sessions, one for each deprivation condition, in which the weighted Voronoi diagram of the five generators with the highest weight was generated. The Voronoi diagrams show the behavioral segmentation of the experimental space. The sessions were chosen to depict the differences on behavioral spatial segmentation as an effect of the deprivation condition under the same food and water delivery schedule. This analysis of behavioral spatial organization serves as an example of SDBA under different motivational operations.

Each plot represents the area of the modified open field chamber ($92cm \times 92cm$), with the location of both dispensers marked with red triangles (food at the top; water at the bottom), where commodities were delivered simultaneously. The location of the five generators is marked with black points, and the Voronoi diagram with five regions is differentiated by color. Each generator point, together with its weight, defines a Region of Behavioral Relevance (RBR).

The first criterion for comparing the diagrams is the location of the generators and their proximity to the dispensers, with closer proximity indicating a stronger influence of the dispenser on the generator region.

The second criterion is the extension of the RBR, which is related to the relevance of each region given the multiplicative weight and location of the generators, with larger areas indicating greater behavioral relevance.

Finally, the type of arcs in each region illustrates the spatial influence between regions.

Straight-line arcs indicate equal relevance and influence between adjacent regions, suggesting similar weights and distances.

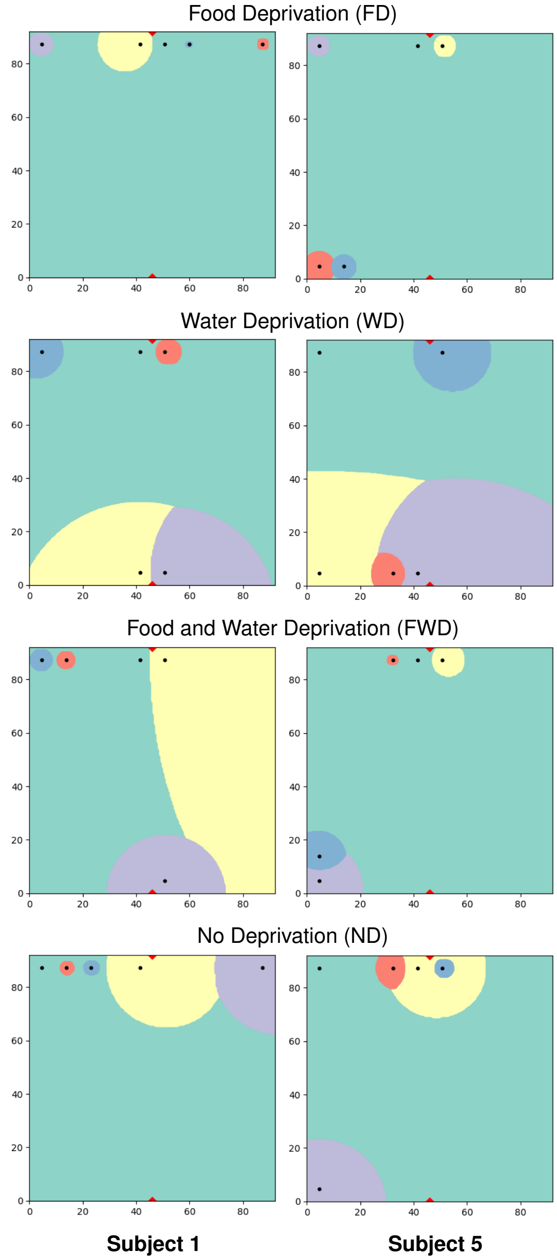

Fig. 3. Voronoi diagrams for subjects 1 (left column) and 5 (right column), illustrating a specific session under different conditions, indicated on the corresponding row

Curved arcs show a subordinate influence, where one region's generator has a significantly higher weight or closer proximity, resulting in a curved boundary due to the dominance in spatial influence.

The comparison of Voronoi diagrams was conducted both within subjects and between subjects. In the within-subject analysis, the difference in behavioral spatial organization between deprivation conditions in both subjects is noticeable. At the same time, the similarities between subjects under the different conditions are highlighted.

3.3.1 Subject 1

For Subject 1, under Food Deprivation (FD) condition, the five generators were close to the wall of the food dispenser. Three of them were near the dispenser, two were actually adjacent to it, and two were at the corners. Regarding the extension of RBRs, one of the generators adjacent to the dispenser was by far the most extended. The arcs of the regions were circular, showing a significant influence and dominance of a region close to the dispenser over the rest of the regions. It is noticeable that two of the five generators were located in the corners close to the dispenser, given that these locations usually have ecological relevance as a home base or safety zone for the Wistar rats.

Under Water Deprivation (WD) condition, two of the five generators were located close to the water dispenser. Nevertheless, the remaining three generators were close to the food dispenser wall, with two close to the food dispenser and one at the corner. Regarding the extension of the RBRs, the largest was associated with a generator close to the food dispenser, while the second and third largest were associated to the water dispenser. The arcs of the RBRs were curved, showing a dominance of the main RBRs over the rest.

Under Food and Water Deprivation (FWD) condition, four generators were located close to the food dispenser wall, and only one adjacent to the water dispenser. Regarding to the RBRs extension, the two largest regions were adjacent to the food dispenser, the third in size to the water dispenser, and the last two were associated with a corner close to food dispenser.

Under No Deprivation (ND) condition, again the five generators were close to the food dispenser, but this time the most extended region was associated with a corner. It is noticeable that among the possible 100 regions, there was consistency in the location of the generators, and only eight of them were among the five main regions. However, in all conditions of deprivation, the regions adjacent to the food dispenser were by far the most relevant. Only under WD were the regions adjacent to the water dispenser relevant, but not as much as the regions adjacent to the food dispenser. Under FWD, a lower relevance of the water dispenser emerge. Finally, under ND, the most relevant region was a corner, followed by a region adjacent to the food dispenser.

3.3.2 Subject 5

For Subject 5, under Food Deprivation Condition (FD), three of the five generators were located close to the food dispenser wall, two adjacent to it and one at the corner; additionally, two generators were located close to the opposite wall at the corner near the water dispenser. Regarding the extension of the RBRs, the largest by far was an adjacent RBR to the food dispenser. Noticeably, the RBRs near the water dispenser together form a significant region that could function as a home base.

Under Water Deprivation Condition (WD), three of the five generators were close to the wall of the water dispenser and two close to the wall of the food dispenser. Regarding the extension of the RBRs, it is highlighted that the RBR adjacent to the water dispenser is by far larger than the RBR adjacent to the food dispenser. Nevertheless, the largest RBR was associated with a corner near the food dispenser, and another large RBR was associated with a corner close to the water dispenser. The arcs of the RBRs associated with the generators at the corners tended to be straighter, indicating that both had a similar influence and weight, so they could function as a double home base. In this diagram, the RBRs were significantly more distributed and less concentrated than under the FD condition.

Under Food and Water Deprivation (FWD), the generators were located in a similar way to the FD condition: three generators close to the food dispenser wall and two in a corner close to the water dispenser. Nevertheless, the RBRs associated with the corner increased in comparison to the FD condition. One of the regions adjacent to the food dispenser was by far the largest and most relevant, with curved arcs indicating subordination of the rest of the regions to it.

Under No Deprivation (ND) condition, four of the generators were located close to the food dispenser wall, three of them adjacent to the dispenser and one at the corner. Finally, a generator was located at a corner near the water dispenser. In contrast with the FD and FWD conditions, by far the largest RBR was associated with a corner. The curved arcs show that all regions were subsumed by the largest region; nevertheless, the influence of this largest region is not as significant as those associated with the food dispensers under FD and FWD.

The similarity of the patterns between subjects, as shown in the Voronoi diagrams for each condition, is noticeable. First, under FD, the most salient RBR by far is adjacent to the food dispenser. Second, the salience of RBRs associated with the food dispenser was always present, even under the WD and ND conditions. Third, the RBRs adjacent to the water dispenser were salient only under the WD condition. Fourth, under all conditions, there was always at least one RBR associated with a corner, functioning as a home base, but only under ND were such RBRs more salient than the rest. On the other hand, it is highlighted that the different spatial segmentation is an effect of the type of deprivation, under the same reinforcement schedule.

4 Conclusions

Each region in the Weighted Voronoi Diagram (WVD) defines a Region of Behavioral Relevance (RBR). The various organizations of these regions represent well-differentiated patterns of behavioral segmentation within the same experimental space, even under the same reinforcement schedule. In this work, they depict distinctive spatial behavioral patterns related to the deprivation conditions. These different Spatial Organization of Behavior (SOB) suggest that the different deprivation conditions functioned as different Motivational Operations (MOs), giving varying importance to the regions close to the dispenser and corners. Identifying these RBRs provides valuable information about behavior dynamics concerning the deprivation conditions.

The WVDs differed across deprivation conditions, allowing for the differentiation of spatial patterns known as RBRs. These patterns show how deprivation conditions affect the SOB associated with seeking and contacting food and water, offering crucial insights into a subject's behavioral adjustments under different MOs. One of the most impactful insights is the asymmetrical effect of deprivation conditions on behavioral spatial organization; that is, the spatial organization under Food Deprivation is not simply inverted or symmetrical concerning the spatial organization under Water Deprivation. In addition, when the rats were exposed to both Food and Water Deprivation, there was no symmetrical distribution of the RBRs between regions close to both dispensers; instead, the RBRs near the food dispenser were significantly more salient. Finally, under No Deprivation, only the regions close to the water dispenser were irrelevant. These asymmetrical effects of deprivation conditions on the spatial organization of behavior, particularly in regions where commodities were delivered, suggest that deprivation conditions function differently and asymmetrically as MOs [18, 20].

Voronoi diagrams (VDs) have proven their value in enhancing the understanding of spatial variables and features in various fields such as urbanism [6], sports science [11, 14], and ecology [15]. However, to our knowledge, this is one of the first works to implement Voronoi Diagrams in the field of Experimental Analysis of Behavior (EAB) [1], offering a highly useful representation tool for SDBA, where one of the main challenges is achieving a clear and effective visualization of behavioral patterns. In this context, the current paper extends previous work [5] by applying our approach to a new set of experiments. Specifically, it advances upon the previous study by shifting the focus from reinforcement schedules to the effect of different MOs on the SOB. In our earlier work, we applied WVD to analyze the SOB under four different spatiotemporal reinforcement schedules (i.e., four distinct arrangements of water delivery). In this paper, we compare SOB under four deprivation conditions (food deprivation, water deprivation, combined food and water deprivation, and no deprivation) while maintaining the same concurrent reinforcement schedule (a periodic delivery of both food and water).

In this sense, this study advances the previous one by extending the application of WVD to visualize the effects of organismic states —associated with different MOs (deprivation)— on SOB, whereas the earlier work focused on spatiotemporal reinforcement schedules. We believe this extension is significant for several reasons: a) It demonstrates that our approach is versatile and can be used to visualize the effects of both environmental variables and organismic states. b) It highlights the method's ability to identify SOB under various experimental conditions, such as different reinforcer arrangements and spatiotemporal schedules, as well as under both single and concurrent reinforcement schedules. c) It shows that our approach can be applied effectively to study and visualize a wide range of behavioral phenomena, from spatiotemporal learning to motivation. Nevertheless, further work is needed to expand the application of this tool, such as applying the proposed approach to the analysis of other relevant behavioral phenomena, such as the SOB under aversive stimulation.

Our primary goal was to demonstrate the application of WVD to identify Regions of Behavioral Relevance (RBRs) and the Spatial Organization of Behavior (SOB), rather than to present specific findings or behavioral data in a strict scientific sense. However, considering that within EAB, the Single-Case Design (SCD) methodology is commonly used—wherein the same subject serves as its control due to intrinsic individual behavioral variability—our study's inter-individual consistency (i.e., similar SOB patterns in both subjects across conditions) suggests generalizability regarding the effects of the MOs used in this study on SOB and its related RBRs.

Finally, as mentioned above, WVD is an extremely useful visualization tool in SDBA. However, from a computational point of view, other algorithms with lower computational costs could be explored in future work. In addition, there are several aspects regarding the mathematical developments to extend the application of VD to SDBA. Among them, the first is to analyze and compare different methods to weight the generator points. A second aspect is to analyze the evolution of the SOB over time using the evolution of VDs. Thus, future work is assured in exploring the implementation of WVD in the SOB.

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