ISSN 2007-9737

Time Evolution of the 3-Tangle of a System of 3-Qubit Interacting through a XY Hamiltonian

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Abstract. We consider a pure 3-qubits system interacting through a XY-Hamiltonian with antiferromagnetic constant J. We employ the 3-tangle as an efficient measure of the entanglement between such a 3-qubit system. The time evolution of such a 3-tangle is studied. In order to do the above, the 3-tangle associated to the pure 3-qubit state $|\psi(t)\rangle = c_0(t)|000\rangle + c_1(t)|001\rangle + c_2(t)|010\rangle + c_3(t)|011\rangle + c_4(t)|100\rangle + c_5(t)|101\rangle + c_6(t)|110\rangle + c_7(t)|111\rangle$ is calculated as a function of the initial coefficients $\{c_i(t=0)\}$ $(i = 0,1,...,7)$, the time t and the antiferromagnetic constant J. We find that the 3-tangle of the 3-qubit system is periodic with period $t = 4\pi/J$. Furthermore, we also find that the 3-tangle as a function of the time t and J has maximal and minimum values. The maximal values of the 3-tangle can be employed in Quantum Information Protocols (QIP) that use entanglement as a basic resource. The pattern found for the 3-tangle of the system of three qubits interacting through a XY Hamiltonian as a function of J and the time t resembles to a quantized physical quantity.

Keywords. 3-qubits; non-classical communications; quantum information processing; entanglement.

1 Introduction

Entanglement of multipartite pure states has been object of many studies both theoretical and experimental [1, 3]. The reason for the above is that multipartite entanglement is a basic ingredient for Quantum Information Protocols (QIP). Although certainly there have been advances in the study of multipartite entanglement [4, 11], it is not yet understood the time evolution of the initial entanglement of a system of several qubits. In particular, it arises the question about the characteristics of the time evolution of the 3-tangle of a system of 3-qubit interacting mutually through a XY Hamiltonian.

As it has been pointed out in Ref. [4] the 3-tangle can be an important quantity for measuring the entanglement of a 3-qubit system. In the present paper we study the time evolution of the 3-tangle associated to a 3-qubit system in a pure state. In order to do the above we employ the 3-tangle introduced in Ref. [4] and also the quantum Heisenberg XY-Hamiltonian [12] for a system of 3-qubit.

Thus, given an initial 3-qubit state $|\psi(t=0)\rangle = c_0(t=0)|000\rangle + c_1(t=0)|001\rangle + c_2(t=0)|010\rangle + c_3(t=0)|011\rangle + c_4(t=0)|100\rangle + c_5(t=0)|101\rangle + c_6(t=0)|110\rangle + c_7(t=0)|111\rangle$, the time evolution of such a state is given by the Heisenberg operator i.e. $|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle = c_0(t)|000\rangle + c_1(t)|001\rangle + c_2(t)|010\rangle + c_3(t)|011\rangle + c_4(t)|100\rangle + c_5(t)|101\rangle + c_6(t)|110\rangle + c_7(t)|111\rangle$ where H is the XY-Hamiltonian of the 3-qubit system. In our approach, we derive an analytic expression for the Heisenberg operator e^{-iHt} with which if the initial 3-tangle ($\tau(t=0)$) is known in terms of the initial coefficients $\{c_i(t=0)\}$ $(i = 0,1,...,7)$ then the final tangle $\tau(t)$ will be known in terms of the final coefficients $\{c_i(t)\}$ $(i = 0,1,...,7)$, the value of J and the time t.

As a result we find noticeable harmonic-like time behavior for the 3-tangle. The later seemingly suggests that the entanglement of a 3-qubit system

ISSN 2007-9737

interacting through a XY Hamiltonian is a quantized quantity. The paper is organized as follows: in Section 2 we derive the formalism for a 3-qubit system interacting through a XY-Hamiltonian. In Section 3 we find an expression for the 3-tangle as a function of time. Finally, we conclude the work by giving a discussion of our results in a section of Conclusions.

2 3-qubits XY Hamiltonian

In order to facilitate our calculations it is employed the decimal notation, which is defined as follows:

$$\begin{aligned} |0\rangle &= |000\rangle, \\ |1\rangle &= |001\rangle, \\ |2\rangle &= |010\rangle, \\ |3\rangle &= |011\rangle, \\ |4\rangle &= |100\rangle, \\ |5\rangle &= |101\rangle, \\ |6\rangle &= |110\rangle, \\ |7\rangle &= |111\rangle. \end{aligned} \quad (1)$$

Then, a general pure 3-qubits state can be defined in terms of a superposition of the above basis as follows:

$$|\psi\rangle = \sum_{i=0}^{7} c_i |i\rangle, \quad (2)$$

where:

$$\sum_{i=0}^{7} |c_i|^2 = 1. \quad (3)$$

With the decimal notation it is possible to associate a matrix with a Hamiltonian operator. The respective associated matrix elements to the Hamiltonian operator H become:

$$H_{ij} = \langle i|H|j\rangle. \quad (4)$$

The so called XY-Hamiltonian for n qubits is: [12]

$$H = J \sum_{i=0}^{N-1} (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y), \quad (5)$$

where $N = 2^n$, J is the coupling constant, and S_i^a is the a ($a = x, y$) component of the spin of the $i-th$ qubit. In the present case we have $n = 3$ qubits (i.e. $N = 8$).

Let us observe that the states $|0\rangle$ and $|7\rangle$ are annihilated by the action of the operator H of Eq. (5), that is:

$$\begin{aligned} H|0\rangle &= 0, \\ H|7\rangle &= 0. \end{aligned} \quad (6)$$

Furthermore, the action of the XY Hamiltonian H of Eq. (5) on the rest of the decimal states is:

$$\begin{aligned} H|1\rangle &= \frac{J}{2}\Big[|2\rangle + |4\rangle\Big], \\ H|2\rangle &= \frac{J}{2}\Big[|1\rangle + |4\rangle\Big], \\ H|3\rangle &= \frac{J}{2}\Big[|5\rangle + |6\rangle\Big], \\ H|4\rangle &= \frac{J}{2}\Big[|2\rangle + |1\rangle\Big], \\ H|5\rangle &= \frac{J}{2}\Big[|6\rangle + |3\rangle\Big], \\ H|6\rangle &= \frac{J}{2}\Big[|5\rangle + |3\rangle\Big]. \end{aligned} \quad (7)$$

Through the use of the Eqs. (4)-(7) and the orthonormality of the decimal basis, the construction of the matrix associated to H yields:

$$H = \frac{J}{2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

On the other hand, the time evolution operator can be expanded in powers of H as follows:

$$\begin{aligned} \mathcal{U}(t) &= exp\left[-iHt\right] \quad (9) \\ &= 1 - iHt + \frac{(-i)^2}{2}\left[Ht\right]^2 + \frac{(-i)^3}{3!}\left[Ht\right]^3. \end{aligned}$$

We observe that the several different powers of H of Eq. (8) behave peculiarly. For instance the quadratic power is:

$$\begin{aligned} H^2 &= \frac{J^2}{4}\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&2&1&0&1&0&0&0\\ 0&1&2&0&1&0&0&0\\ 0&0&0&2&0&1&1&0\\ 0&1&1&0&2&0&0&0\\ 0&0&0&1&0&2&1&0\\ 0&0&0&1&0&1&2&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \\ &= \frac{J^2}{4}\left[2\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}\right. \\ &\left.+\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&1&0&1&0&0&0\\ 0&1&0&0&1&0&0&0\\ 0&0&0&0&0&1&1&0\\ 0&1&1&0&0&0&0&0\\ 0&0&0&1&0&0&1&0\\ 0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}\right] \\ &\equiv \left(\frac{J}{2}\right)^2 2I_{\{2-7\}} + \frac{J}{2}H. \end{aligned}$$

In a similar way, for the other powers we obtain that:

$$\begin{aligned} H^3 &= \left(\frac{J}{2}\right)^3 2I_{\{2-7\}} + \left(\frac{J}{2}\right)^2 3H, \\ H^4 &= \left(\frac{J}{2}\right)^4 2*3I_{\{2-7\}} + \left(\frac{J}{2}\right)^3 (3+2)H, \\ H^5 &= \left(\frac{J}{2}\right)^5 2*5I_{\{2-7\}} + \left(\frac{J}{2}\right)^4 (5+6)H, \\ H^6 &= \left(\frac{J}{2}\right)^6 2*11I_{\{2-7\}} + \left(\frac{J}{2}\right)^5 (11+10)H, \end{aligned} \tag{10}$$

where $I_{\{2-7\}}$ has been defined in Eq. (11). In general for the $n-th$ power we find that:

$$H^n = \left(\frac{J}{2}\right)^n a_n I_{\{2-7\}} + \left(\frac{J}{2}\right)^{n-1} b_n H. \tag{11}$$

However, we can see that $a_n = 2b_{n-1}$ and $b_n = b_{n-1} + a_{n-1} = b_{n-1} + 2b_{n-2}$, then the above equation can be expressed as:

$$\begin{aligned} H^n &= \left(\frac{J}{2}\right)^n \frac{2}{3}\left[-(-1)^{n-1} + 2^{n-1}\right] I_{\{2-7\}} \\ &+ \left(\frac{J}{2}\right)^{n-1} \frac{\left[-(-1)^n + 2^n\right]}{3} H, \quad n \geq 1. \end{aligned} \tag{12}$$

We observe from the above equation that for $n = 0$, the second term will be equal to zero and that the first one is equal to 1. However, in this case, $H^0 = I_{\{2-7\}}$ and this is not the identity I_8 as can be seen from Eq. (11). Such a problem can be solved as follows:

$$\begin{aligned} H^n &= I_{\{1,8\}}\delta_{0n} \\ &+ \left(\frac{J}{2}\right)^n \frac{2}{3}\left[-(-1)^{n-1} + 2^{n-1}\right] I_{\{2-7\}} \\ &+ \left(\frac{J}{2}\right)^{n-1} \frac{\left[-(-1)^n + 2^n\right]}{3} H, \quad n \geq 0, \end{aligned} \tag{13}$$

where:

$$I_{\{1,8\}} \equiv \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}. \tag{14}$$

From the above equation we find that the time evolution operator will always be linear on H, and the time evolution operator can be written as:

$$
\begin{aligned}
\mathcal{U}(t) &= \sum_{n=0} \frac{(-iHt)^n}{n!} \\
&= \sum_{n=0} \frac{(-it)^n}{n!} \Big\{ I_{\{1,8\}} \delta_{0n} \\
&\quad + \left(\frac{J}{2}\right)^n \frac{2}{3} [-(-1)^{n-1} + 2^{n-1}] I_{\{2-7\}} \\
&\quad + \left(\frac{j}{2}\right)^{n-1} \frac{[-(-1)^n + 2^n]}{3} H \Big\} \\
&= I_{\{1,8\}} \\
&\quad + \frac{2I_{\{2-7\}}}{3} \sum_{n=0} \frac{1}{n!} \left(\frac{-itJ}{2}\right)^n [-(-1)^{n-1} \\
&\quad + 2^{n-1}] \\
&\quad + \frac{2H}{3J} \sum_{n=0} \frac{1}{n!} \left(\frac{-itJ}{2}\right)^n [-(-1)^n \\
&\quad + 2^n]. \qquad (15)
\end{aligned}
$$

It is worth to observe that the last expression can be written in terms of exponentials with which the time evolution operator takes a simple form:

$$
\begin{aligned}
\mathcal{U}(t) &= I_{\{1,8\}} + \frac{2I_{\{2-7\}}}{3} \left(e^{\frac{iJt}{2}} + \frac{1}{2} e^{-iJt} \right) \\
&\quad + \frac{2H}{3J} \left(e^{-iJt} - e^{\frac{iJt}{2}} \right). \qquad (16)
\end{aligned}
$$

Let us note that according to Eqs. (9) and (10) the time evolution of the state $|\psi(t=0)\rangle$ is given by:

$$
\begin{aligned}
|\psi(t)\rangle &= \mathcal{U}|\psi(t=0)\rangle \\
&= \mathcal{U}\Big[c_0(t=0)|0\rangle + c_1(t=0)|1\rangle \\
&\quad + c_2(t=0)|2\rangle + c_3(t=0)|3\rangle \\
&\quad + c_4(t=0)|4\rangle + c_5(t=0)|5\rangle \\
&\quad + c_6(t=0)|6\rangle + c_7(t=0)|7\rangle \Big] \\
&= c_0(t)|0\rangle + c_1(t)|1\rangle + c_2(t)|2\rangle \\
&\quad + c_3(t)|3\rangle + c_4(t)|4\rangle + c_5(t)|5\rangle \\
&\quad + c_6(t)|6\rangle + c_7(t)|7\rangle. \qquad (17)
\end{aligned}
$$

It can be observed from the above equation that we can calculate the coefficients at any time $\{c_j(t)\}$ ($j = 0, 1, ..., 7$) if the initial coefficients $\{c_j(t = 0)\}$ ($j = 0, 1, ..., 7$) are known and if it is also known the action of the time evolution operator on each of the decimal states, that is, $\mathcal{U}(t)|i\rangle$ for $i = 0, ..., 7$. Through the use of Eqs. (6), (7), (11), (16), and (18) it is found that:

$$
\begin{aligned}
\mathcal{U}(t)|0\rangle &= |0\rangle, \qquad (18) \\
\mathcal{U}(t)|1\rangle &= \frac{2}{3}\left(e^{\frac{iJt}{2}} + \frac{1}{2} e^{-iJt} \right)|1\rangle \qquad (19) \\
&\quad + \frac{1}{3}\left(e^{-iJt} - e^{\frac{iJt}{2}} \right)[|2\rangle + |4\rangle], \\
\mathcal{U}(t)|2\rangle &= \frac{2}{3}\left(e^{\frac{iJt}{2}} + \frac{1}{2} e^{-iJt} \right)|2\rangle \qquad (20) \\
&\quad + \frac{1}{3}\left(e^{-iJt} - e^{\frac{iJt}{2}} \right)[|1\rangle + |4\rangle], \\
\mathcal{U}(t)|3\rangle &= \frac{2}{3}\left(e^{\frac{iJt}{2}} + \frac{1}{2} e^{-iJt} \right)|3\rangle \qquad (21) \\
&\quad + \frac{1}{3}\left(e^{-iJt} - e^{\frac{iJt}{2}} \right)[|5\rangle + |6\rangle], \\
\mathcal{U}(t)|4\rangle &= \frac{2}{3}\left(e^{\frac{iJt}{2}} + \frac{1}{2} e^{-iJt} \right)|4\rangle \qquad (22) \\
&\quad + \frac{1}{3}\left(e^{-iJt} - e^{\frac{iJt}{2}} \right)[|2\rangle + |1\rangle], \\
\mathcal{U}(t)|5\rangle &= \frac{2}{3}\left(e^{\frac{iJt}{2}} + \frac{1}{2} e^{-iJt} \right)|5\rangle \qquad (23) \\
&\quad + \frac{1}{3}\left(e^{-iJt} - e^{\frac{iJt}{2}} \right)[|6\rangle + |3\rangle], \\
\mathcal{U}(t)|6\rangle &= \frac{2}{3}\left(e^{\frac{iJt}{2}} + \frac{1}{2} e^{-iJt} \right)|6\rangle \qquad (24) \\
&\quad + \frac{1}{3}\left(e^{-iJt} - e^{\frac{iJt}{2}} \right)[|5\rangle + |3\rangle], \\
\mathcal{U}(t)|7\rangle &= |7\rangle. \qquad (25)
\end{aligned}
$$

To substitute Eqs. (20)-(27) into Eq. (19), we find the coefficients at any time $\{c_j(t)\}$ ($j = 0, 1, ..., 7$) in terms of both the above exponentials and the initial coefficients $\{c_j(t = 0)\}$ ($j = 0, 1, ..., 7$) where $\sum_{j=0}^{7} |c_j(t = 0)|^2 = 1$.

3 3-tangle as a Measure of Multipartite Entanglement of a 3-qubit System

The measure of entanglement for a 3-qubit system can be is obtained through the 3-tangle which is defined as [4]

$$\tau_3 = 4|d_1 - 2d_2 + 4d_3|, \tag{26}$$

with:

$$\begin{aligned} d_1 &= c_0^2c_7^2 + c_1^2c_6^2 + c_2^2c_5^2 + c_4^2c_3^2, && (27)\\ d_2 &= c_0c_7c_3c_4 + c_0c_7c_5c_2 + c_0c_7c_6c_1 && (28)\\ &\quad + c_3c_4c_5c_2 + c_3c_4c_6c_1 + c_5c_2c_6c_1, \\ d_3 &= c_7c_6c_5c_3 + c_7c_1c_2c_4, && (29) \end{aligned}$$

where c_i represents the coefficient of basic state $|i\rangle$. Thus, by calculating the coefficients c_i ($i = 0, 1, ..., 7$) as a function of time, in the way it was explained at the end of the above section, we shall be able of finding the 3-tangle of Eq. (28) as a function of time. That is to find $\tau_3(t) = 4|d_1(t) - 2d_2(t) + 4d_3(t)|$ providing the coefficients $c_i(t)$ are known. It is worth to observe from Eqs. (18) and (19) that the coefficients $c_i(t)$ ($i = 0, 1, ..., 7$) will depend on the initial coefficients $c_j(t = 0)$ ($j = 0, 1, ..., 7$), the antiferromagnetic constant J and the time t. By the way, in the present work the initial coefficients $c_j(t = 0)$ ($\sum_{j=0}^{7} |c_j|^2 = 1$) are found in a random way with which the coefficients $c_i(t)$ ($i = 0, 1, ..., 7$) at time t will result a two variables function namely J and t.

Before of considering a general state we are focusing on the so called W and GHZ states which are defined as:

$$\begin{aligned} |W\rangle &= \frac{1}{\sqrt{3}} \left(|4\rangle + |2\rangle + |1\rangle\right), && (30)\\ |GHZ\rangle &= \frac{1}{\sqrt{2}} \left(|0\rangle + |7\rangle\right). && (31) \end{aligned}$$

The respective initial 3-tangle for the GHZ-state is unit while for the W-state the initial 3-tangle is zero. Now, the W-state time evolution is only over the phase. Therefore the 3-tangle of the W-state does not change in time. Thus, the XY Hamiltonian keeps constant the entanglement of the W-state which is an important result. On the other hand, the GHZ-state also is not modified by the time evolution operator of Eq. (19) hence its associated 3-tangle keeps constant in time. We conclude that the XY Hamiltonian assures that the entanglement of the GHZ-state does not change in time.

Let us now consider an arbitrary initial 3-qubit state at $t = 0$ denoted by $|\psi(t = 0)\rangle = c_0(t = 0)|000\rangle + c_1(t = 0)|001\rangle + c_2(t = 0)|010\rangle + c_3(t = 0)|011\rangle + c_4(t = 0)|100\rangle + c_5(t = 0)|101\rangle + c_6(t = 0)|110\rangle + c_7(t = 0)|111\rangle$ where $\sum_{i=0}^{7} |c_i(t = 0)|^2 = 1$. In order to evaluate the 3-tangle at time t from Eqs. (28)-(31), we employ eqs. (19)-(27) where the initial coefficients $c_i(t = 0)$ are found in a random way. We perform the above procedure in three different cases and calculate the respective 3-tangle in each one of the three different cases. In the Appendix we write the three different random initial 3-qubit states employed in the present work. In figure 6, we show the time evolution of the 3-tangle as a function of both J and t associated to each of the three different random initial 3-qubit states employed in the present work.

4 Relevance of Entanglement for Technological Applications

Quantum entanglement is essential not only for technological applications such as quantum computation [13], data base search algorithm [14] or quantum cryptography [15] and quantum secret sharing [16] but also for non-artificial systems. For instance for photosynthesis [17]-[18], navigational orientation of animals [19], the imbalance of matter and antimatter in the universe [20] and evolution itself [21].

5 Random Initial 3-qubit States

We write the three different random initial 3-qubit states that we have employed in the present work.

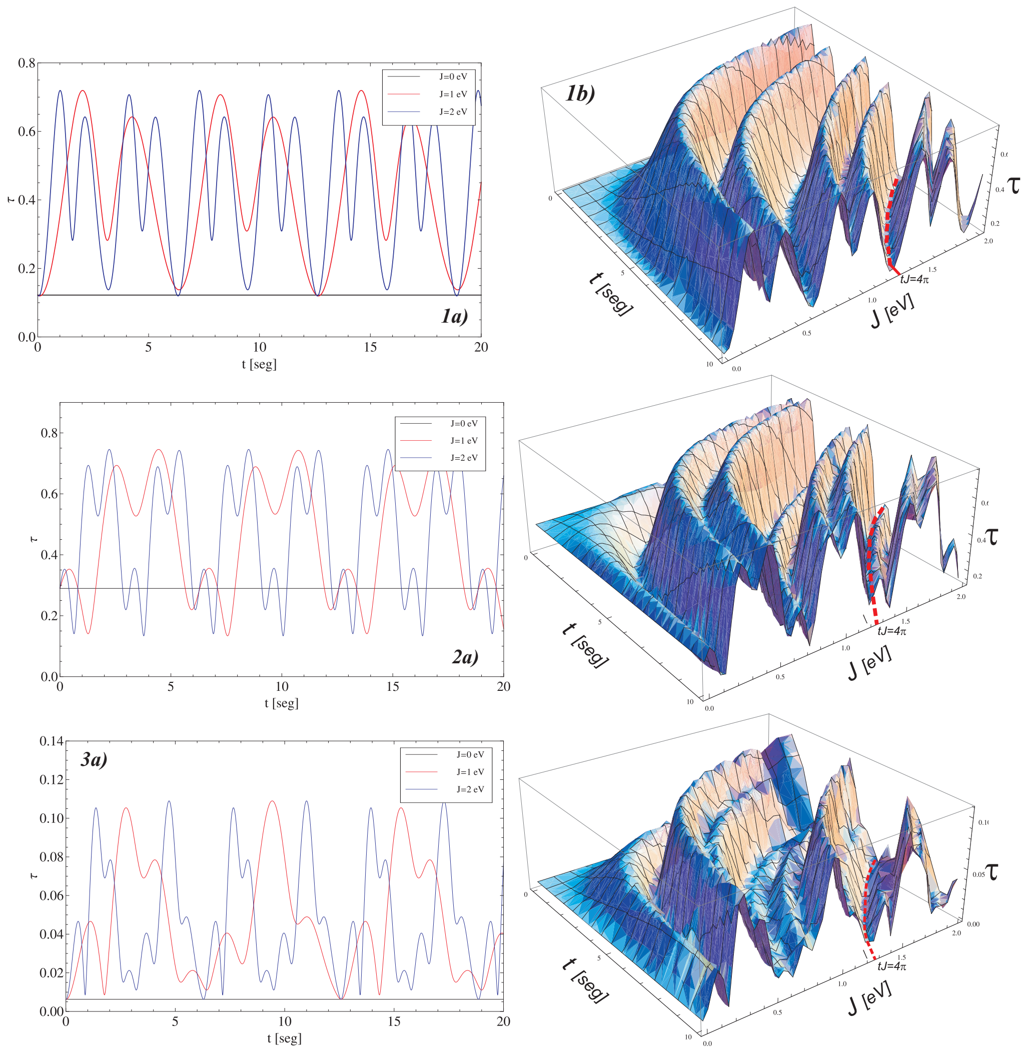

Fig. 1. The 3-tangle as a function of both the time t and the antiferromagnetic factor J for a three different states which their respective initial coefficients $\{c_i(t=0)\}$ are found in a random way. Eqs. (28)-(31) and (19)-(27) are used. Concerning to the label, the number represent the state while the letter expresses the kind of graphic

Such a states are the following:

$$
\begin{aligned}
|\psi_1(t=0)\rangle \simeq\ & (0.0649682 + 0.480244i)|0\rangle \\
& + (0.0820031 + 0.0744268i)|1\rangle \\
& + (0.157695 + 0.567361i)|2\rangle \\
& + (0.00990613 + 0.30057i)|3\rangle \\
& + (0.159286 + 0.122371i)|4\rangle \\
& + (0.136861 + 0.0406154i)|5\rangle \\
& + (0.00576077 + 0.267818i)|6\rangle \\
& + (0.424509 + 0.054595i)|7\rangle,
\end{aligned} \quad (32)
$$

$$
\begin{aligned}
|\psi_2(t=0)\rangle \simeq\ & (0.254723 + 0.452791i)|0\rangle \\
& + (0.205806 + 0.3656i)|1\rangle \\
& + (0.119695 + 0.452655i)|2\rangle \\
& + (0.10712 + 0.095714i)|3\rangle \\
& + (0.000551918 + 0.408866i)|4\rangle \\
& + (0.0713835 + 0.0732269i)|5\rangle \\
& + (0.0279197 + 0.0993365i)|6\rangle \\
& + (0.316043 + 0.161424i)|7\rangle,
\end{aligned} \quad (33)
$$

$$
\begin{aligned}
|\psi_3(t=0)\rangle \simeq\ & (0.228717 + 0.66739i)|0\rangle \\
& + (0.124412 + 0.62744i)|1\rangle \\
& + (0.0241769 + 0.16416i)|2\rangle \\
& + (0.00878132 + 0.0690814i)|3\rangle \\
& + (0.0589419 + 0.165814i)|4\rangle \\
& + (0.0255238 + 0.105097i)|5\rangle \\
& + (0.0946251 + 0.0750734i)|6\rangle \\
& + (0.00977502 + 0.0581965i)|7\rangle.
\end{aligned} \tag{34}
$$

We observe that all of the above three 3-qubit states are normalized to unit.

6 Conclusions

We have studied the behavior in time of the 3-tangle associated to a 3-qubit system interacting through the XY Hamiltonian given by Eqs. (5) and (8). The 3-tangle associated to the state $|\psi(t)\rangle = c_0(t)|000\rangle + c_1(t)|001\rangle + c_2(t)|010\rangle + c_3(t)|011\rangle + c_4(t)|100\rangle + c_5(t)|101\rangle + c_6(t)|110\rangle + c_7(t)|111\rangle$ is given by Eqs. (28)-(31) where each one of the coefficients $\{c_i(t)\}$ $(i = 0, 1, ..., 7)$ depend on the random initial coefficients $\{c_j(t = 0)\}$ $(j = 0, 1, ..., 7)$, J and the time t as it can be seen from Eqs. (18)-(27).

An important result obtained in the present work is that the entanglement of both the W-state and the GHZ-state keeps constant in time providing the three qubits interact through the XY Hamiltonian given by Eq. (5).

Such a result could have important experimental advantages whereas both the W-state and the GHZ-state can be used on solid basis for testing different QIP protocols.

In Figure we have plotted the 3-tangle of Eq. (28) as a function of both the time t and the antiferromagnetic factor J for three different random 3-qubit states. It is worth to point out that the 3-tangle shows a noticeable periodic behavior as it is appreciated from Figure being the respective period $t = 4\pi/J$. Such a behavior in time is a consequence of the harmonic structure of the time evolution operator of Eq. (18).

Our results invoke to the present experimental facilities to measure the 3-tangle for a system of 3-qubits by taking into account that for certain times the entanglement disappears and that for other values of both the time and the antiferromagnetic constant J such a quantity is maximal. The maximal values of the 3-tangle can be used for implementing Quantum Information Processing protocols where entanglement is a resource. Our results might indicate that the 3-tangle associated to a 3-qubit system resembles to a quantized physical quantity providing the three qubits interact through a XY Hamiltonian.

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Article received on 27/10/2015; accepted on 20/05/2016.
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