

Trajectory Tracking of Complex Dynamical Network for Chaos Synchronization Using Recurrent Neural Network

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Abstract. In this paper the problem of trajectory tracking is studied. Based on the Lyapunov theory, a control law that achieves the global asymptotic stability of the tracking error between a recurrent neural network and a complex dynamical network is obtained. To illustrate the analytic results we present a tracking simulation of a dynamical network with each node being just one Lorenz's dynamical system and three identical Chen's dynamical systems.

Keywords. Trajectory tracking, recurrent neural network, complex dynamical network, Lyapunov analysis.

1 Introduction

Since the most famous random graph model was proposed by Erdős and Rényi [1], the complex network has attracted much attention in many fields of research, such as biology, physics, computer networks, the World Wide Web (WWW) [2], and so on. Network synchronization has obvious advantages, it has great application value in practice. Therefore, Atay et al. [3] studied synchronization of complex network when delays exist among the nodes; Motter et al. [4] studied the influence of coupling strength on the synchronizing ability of a complex network; Timme et al. [5] studied the web synchronization law of pulse-coupled dynamical systems; Checco et al. [6] studied the synchronization of random web. Lü et al. [7] constructed general complex dynamical networks and studied the synchronization; Lu and Chen [8] studied synchronization analysis of linearly coupled networks of discrete time systems; Han and Lu [9] studied the changes of synchronization ability of coupled networks from ring networks to chain networks;

He and Yang [10] studied adaptive synchronization in nonlinearly coupled dynamical networks.

The analysis and control of complex behavior in complex networks, which consist of dynamical nodes, has become a point of great interest in recent studies, [11, 12, 13]. The complexity in networks comes from their structure and dynamics but also from their topology, which often affects their function. Recurrent neural networks have been widely used in the fields of optimization, pattern recognition, signal processing and control systems, among others. They have to be designed in such a way that there is one equilibrium point that is globally asymptotically stable. In biological and artificial neural networks, time delays arise in the processing of information storage and transmission. Also, it is known that these delays can create oscillatory or even unstable trajectories. Trajectory tracking is a very interesting problem in the field of theory of systems control; it allows the implementation of important tasks for automatic control such as: high speed target recognition and tracking, real-time visual inspection, and recognition of context sensitive and moving scenes, among others.

The motivation in this paper lies in the complex network synchronization and chaos control importance. Network synchronization is one of the most practical and valuable issues. A synchronization of network means the situation in which the output of all nodes in the study of the complex network is consistent with any given external input signal under a certain condition. Numerical simulations are used to verify the effectiveness of the proposed techniques. We present the results of the design of a control law that guarantees the tracking of general complex dynamical networks.

2 Mathematical Models

2.1 General Complex Dynamical Network

Consider a network consisting of N linearly and diffusively coupled nodes, with each node being an n -dimensional dynamical system, described by

$$\dot{x}_i = f_i(x_i) + \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma(x_j - x_i), \quad i = 1, 2, \dots, N, \quad (1)$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in \mathbb{R}^n$ are the state vectors of node i , $f_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents the self-dynamics of node i , constants $c_{ij} > 0$ are the coupling strengths between node i and node j , with $i, j = 1, 2, \dots, N$. $\Gamma = (\tau_{ij}) \in \mathbb{R}^{n \times n}$ is a constant internal matrix that describes the way of linking the components in each pair of connected node vectors $(x_j - x_i)$: that is to say for some pairs (i, j) with $1 \leq i, j \leq n$ and $\tau_{ij} \neq 0$ the two coupled nodes are linked through their i th and j th sub-state variables, respectively, while the coupling matrix $A = (a_{ij}) \in \mathbb{R}^{N \times N}$ denotes the coupling configuration of the entire network: that is to say if there is a connection between node i and node j ($i \neq j$), then $a_{ij} = a_{ji} = 1$; otherwise $a_{ij} = a_{ji} = 0$.

2.2 Recurrent Neural Network

Consider a recurrent neural network in the following form:

$$\begin{aligned} \dot{x}_{n_i} &= A_{n_i} x_{n_i} + W_{n_i} \sigma(x_{n_i}) + u_{n_i} + \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{n_i n_j} a_{n_i n_j} \Gamma(x_{n_j} - x_{n_i}), \quad (2) \\ i &= 1, 2, \dots, N, \end{aligned}$$

where $x_{n_i} = (x_{n_i1}, x_{n_i2}, \dots, x_{n_in})^T \in \mathbb{R}^n$ is the state vector of neural network i , $u_{n_i} \in \mathbb{R}^n$ is the input of neural network i , $A_{n_i} = -\lambda_{n_i} I_{n \times n}$, $i = 1, 2, \dots, N$, is the state feedback matrix, with λ_{n_i} being a positive constant, $W_{n_i} \in \mathbb{R}^{n \times n}$ is the connection weight matrix with $i = 1, 2, \dots, N$, and $\sigma(\cdot) \in \mathbb{R}^n$ is a Lipschitz sigmoid vector function [14], [15], such that $\sigma(x_{n_i}) = 0$ only at $x_{n_i} = 0$, with Lipschitz constant L_{σ_i} , $i = 1, 2, \dots, N$ and neuron activation functions $\sigma_i(\cdot) = \tanh(\cdot)$, $i = 1, 2, \dots, n$.

3 Trajectory Tracking

The objective is to develop a control law such that the i th neural network (2) tracks to the trajectory of the i th dynamical system (1). We define the tracking error as $e_i = x_{n_i} - x_i$, $i = 1, 2, \dots, N$ whose time derivative is

$$\dot{e}_i = \dot{x}_{n_i} - \dot{x}_i, \quad i = 1, 2, \dots, N. \quad (3)$$

Substituting (1) and (2) in (3), we obtain

$$\begin{aligned} \dot{e}_i &= A_{n_i} x_{n_i} + W_{n_i} \sigma(x_{n_i}) + u_{n_i} - f_i(x_i) + \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{n_i n_j} a_{n_i n_j} \Gamma(x_{n_j} - x_{n_i}) - \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma(x_j - x_i), \quad i = 1, 2, \dots, N. \end{aligned} \quad (4)$$

Adding and subtracting $W_{n_i} \sigma(x_i)$, $\alpha_i(t)$, $i = 1, 2, \dots, N$, to (4), where α_i to be determined below, and taking into account that $x_{n_i} = e_i + x_i$, $i = 1, 2, \dots, N$, then

$$\begin{aligned} \dot{e}_i &= W_{n_i} (\sigma(e_i + x_i) - \sigma(x_i)) + \\ &\quad (u_{n_i} - \alpha_i(t)) + A_{n_i} e_i + \\ &\quad (A_{n_i} x_i + W_{n_i} \sigma(x_i) + \alpha_i(t)) - f_i(x_i) + \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{n_i n_j} a_{n_i n_j} \Gamma(x_{n_j} - x_{n_i}) - \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma(x_j - x_i), \quad i = 1, 2, \dots, N. \end{aligned} \quad (5)$$

In order to guarantee that the i th neural network (2) tracks the i th reference trajectory (1), the following assumption has to be satisfied:

Assumption 1. There exist functions $\rho_i(t)$ and $\alpha_i(t)$, $i = 1, 2, \dots, N$, such that

$$\begin{aligned} \frac{d\rho_i(t)}{dt} &= A_{n_i} \rho_i(t) + W_{n_i} \sigma(\rho_i(t)) + \alpha_i(t) \\ \rho_i(t) &= x_i(t), \quad i = 1, 2, \dots, N. \end{aligned} \quad (6)$$

Let define

$$\begin{aligned} \tilde{u}_{n_i} &= (u_{n_i} - \alpha_i(t)) \\ \phi_\sigma(e_i, x_i) &= \sigma(e_i + x_i) - \sigma(x_i), \quad (7) \\ i &= 1, 2, \dots, N. \end{aligned}$$

Considering (6) and (7), the equation (5) is reduced to

$$\begin{aligned} \dot{e}_i &= A_{n_i} e_i + W_{n_i} \phi_\sigma(e_i, x_i) + \tilde{u}_{n_i} + \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{n_i n_j} a_{n_i n_j} \Gamma(x_{n_j} - x_{n_i}) - \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma(x_j - x_i), \\ i &= 1, 2, \dots, N. \end{aligned} \quad (8)$$

Rewriting the summations as

$$\begin{aligned} &\sum_{\substack{j=1 \\ j \neq i}}^N c_{n_i n_j} a_{n_i n_j} \Gamma(x_{n_j} - x_{n_i}) \\ &= \Gamma \left(\sum_{\substack{j=1 \\ j \neq i}}^N c_{n_i n_j} a_{n_i n_j} x_{n_j} - x_{n_i} \sum_{\substack{j=1 \\ j \neq i}}^N c_{n_i n_j} a_{n_i n_j} \right) \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma(x_j - x_i) \\ &= \Gamma \left(\sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} x_j - x_i \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \right), \\ i &= 1, 2, \dots, N. \end{aligned} \quad (9)$$

also taking into account that $c_{n_i n_j} = c_{ij}$ and $a_{n_i n_j} = a_{ij}$, then, using the equations above (8) becomes

$$\begin{aligned} \dot{e}_i &= A_{n_i} e_i + W_{n_i} \phi_\sigma(e_i, x_i) + \tilde{u}_{n_i} + \\ &\quad \Gamma \left(\sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} e_j - e_i \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \right) \\ &= A_{n_i} e_i + W_{n_i} \phi_\sigma(e_i, x_i) + \tilde{u}_{n_i} + \\ &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma(e_j - e_i), \\ i &= 1, 2, \dots, N. \end{aligned} \quad (10)$$

It is clear that $e_i = 0$, $i = 1, 2, \dots, N$ is an equilibrium point of (10), when $\tilde{u}_{n_i} = 0$, $i = 1, 2, \dots, N$. In this way, the tracking problem can be restated as a global asymptotic stabilization problem for system (10).

4 Tracking Error Stabilization and Control Design

In order to establish the convergence of (10) to $e_i = 0$, $i = 1, 2, \dots, N$, which ensures the desired tracking, first, we propose the following candidate Lyapunov function

$$\begin{aligned} V_N(e) &= \sum_{i=1}^N V(e_i) \\ &= \sum_{i=1}^N \left(\frac{1}{2} \| e_i \|^2 \right) \\ e &= (e_1^T, \dots, e_N^T)^T. \end{aligned} \quad (11)$$

The time derivative of (11), along the trajectories of (10), is

$$\begin{aligned} \dot{V}_N(e) &= \frac{\partial V_N(e)}{\partial e} \dot{e} = \sum_{i=1}^N \frac{\partial V_N(e)}{\partial e_i} \dot{e}_i \\ &= \sum_{i=1}^N e_i^T (A_{n_i} e_i + W_{n_i} \phi_\sigma(e_i, x_i) + \\ &\quad \tilde{u}_{n_i} + \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma(e_j - e_i)). \end{aligned} \quad (12)$$

Reformulating (12), we get

$$\begin{aligned} \dot{V}_N(e) &= \sum_{i=1}^N (-\lambda_{n_i} \| e_i \|^2 + e_i^T W_{n_i} \phi_\sigma(e_i, x_i) + e_i^T \tilde{u}_{n_i} \\ &\quad \sum_{i=1}^N \left(\sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} e_i^T \Gamma e_j - e_i \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} e_i^T \Gamma e_i \right). \end{aligned} \quad (13)$$

Next, let consider the following inequality, proved in [16, 17]:

$$X^T Y + Y^T X \leq X^T \Lambda X + Y^T \Lambda^{-1} Y, \quad (14)$$

which holds for all matrices $X, Y \in \mathbb{R}^{n \times k}$ and $\Lambda \in \mathbb{R}^{n \times n}$ with $\Lambda = \Lambda^T > 0$. Applying (14) with $\Lambda = I_{n \times n}$ to the term $e_i^T W_{n_i} \phi_\sigma(e_i, x_i)$, $i = 1, 2, \dots, N$ we get

$$\begin{aligned} &e_i^T W_{n_i} \phi_\sigma(e_i, x_i) \\ &\leq \frac{1}{2} e_i^T e_i + \frac{1}{2} \phi_\sigma^T(e_i, x_i) W_{n_i}^T W_{n_i} \phi_\sigma(e_i, x_i) \\ &= \frac{1}{2} \| e_i \|^2 + \frac{1}{2} \phi_\sigma^T(e_i, x_i) \\ &\quad x W_{n_i}^T W_{n_i} \phi_\sigma(e_i, x_i), \\ &i = 1, 2, \dots, N. \end{aligned} \quad (15)$$

Then we have that, and taking into account that ϕ_σ is Lypchitz:

$$\begin{aligned} & \| \phi_\sigma(e_i, x_i) \| = \| \sigma(e_i + x_i) - \sigma(x_i) \| \\ & \leq L_{\phi_{\sigma_i}} \| e_i + x_i - x_i \| \\ & = L_{\phi_{\sigma_i}} \| e_i \|, \quad i = 1, 2, \dots, N, \end{aligned} \tag{16}$$

with Lipschitz constant $L_{\phi_{\sigma_i}}$. Applying (16) to $\frac{1}{2}\phi_\sigma^T(e_i, x_i)W_{i_n}^T W_{i_n}\phi_\sigma(e_i, x_i)$ we obtain

$$\begin{aligned} & \frac{1}{2}\phi_\sigma^T(e_i, x_i)W_{i_n}^T W_{i_n}\phi_\sigma(e_i, x_i) \\ & \leq \frac{1}{2} \| \phi_\sigma^T(e_i, x_i)W_{i_n}^T W_{i_n}\phi_\sigma(e_i, x_i) \| \\ & \leq \frac{1}{2}(L_{\phi_{\sigma_i}})^2 \| W_{i_n} \|^2 \| e_i \|^2, \quad i = 1, 2, \dots, N. \end{aligned} \tag{17}$$

By simplifying (15), we obtain

$$\begin{aligned} & e_i^T W_{n_i} \phi_\sigma(e_i, x_i) \\ & \leq \frac{1}{2} \| e_i \|^2 + \frac{1}{2}(L_{\phi_{\sigma_i}})^2 \| W_{i_n} \|^2 \| e_i \|^2 \\ & = \frac{1}{2}(1 + L_{\phi_{\sigma_i}}^2 \| W_{i_n} \|^2) \| e_i \|^2, \quad i = 1, 2, \dots, N. \end{aligned} \tag{18}$$

Then we have that:

$$\begin{aligned} \dot{V}_N(e) & \leq - \sum_{i=1}^N \left(\lambda_{n_i} \| e_i \|^2 + \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} e_i^T \Gamma e_j \right) + \\ & \frac{1}{2} \sum_{i=1}^N \left((1 + L_{\phi_{\sigma_i}}^2 \| W_{i_n} \|^2) \| e_i \|^2 + 2 \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} e_i^T \Gamma e_j \right) + \\ & e_i^T \tilde{u}_{n_i}. \end{aligned} \tag{19}$$

We define $\tilde{u}_{n_i} = \tilde{u}_i + \tilde{u}_{ij}, i = 1, 2, \dots, N$, then (19) becomes in

$$\begin{aligned} \dot{V}_N(e) & \leq - \sum_{i=1}^N \left(\lambda_{n_i} \| e_i \|^2 + \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} e_i^T \Gamma e_j \right) + \\ & \frac{1}{2} \sum_{i=1}^N \left(e_i^T ((1 + L_{\phi_{\sigma_i}}^2 \| W_{i_n} \|^2) e_i + 2\tilde{u}_i) \right) + \\ & \sum_{i=1}^N \left(e_i^T \left(\sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma e_j + \tilde{u}_{ij} \right) \right). \end{aligned} \tag{20}$$

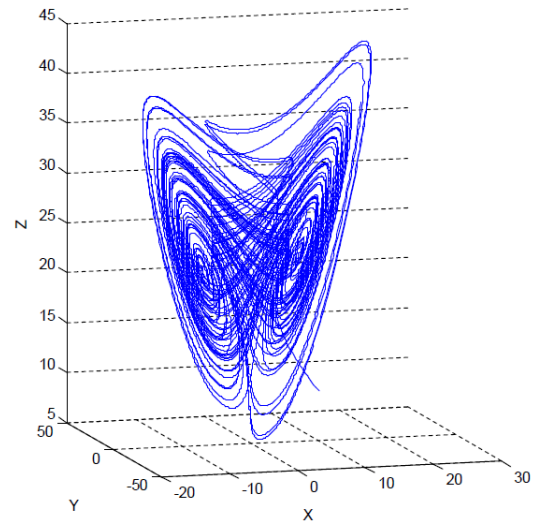


Fig. 1. Sub-State of Lorenz's attractor with initial condition $X_1(0) = (10, 0, 10)^T$

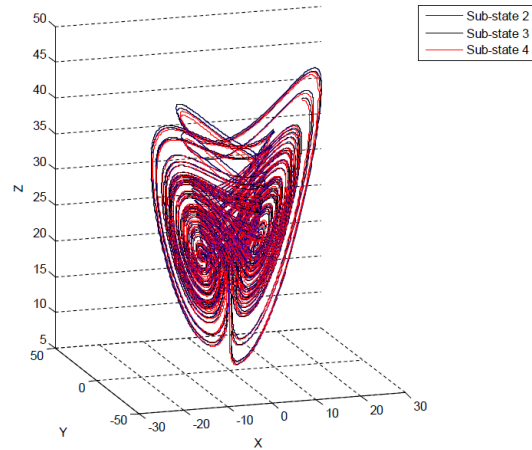


Fig. 2. Sub-States of Chen's attractor with initial condition $X_{2;3;4}(0) = (-10, 0, 37)^T$

Now, we propose to use the following control law:

$$\begin{aligned} \tilde{u}_{n_i} & = \tilde{u}_i + \tilde{u}_{ij} \\ & = -\frac{1}{2}(1 + L_{\phi_{\sigma_i}}^2 \| W_{n_i} \|^2) e_i - \\ & \quad - \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma e_j, \\ & \quad i = 1, 2, \dots, N, \end{aligned} \tag{21}$$

then $\dot{V}_N(e) < 0$ for all $e \neq 0$.

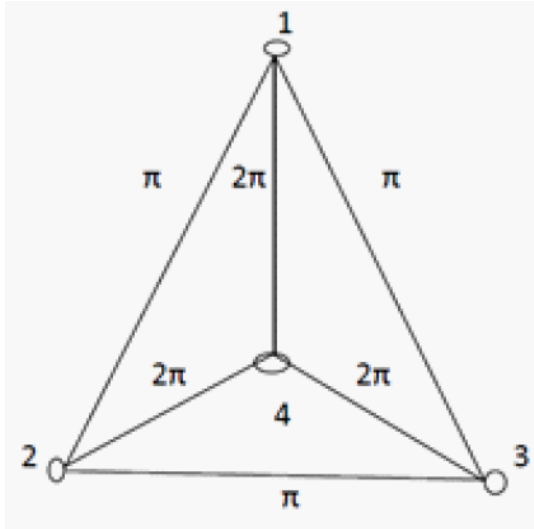


Fig. 3. Structure of the network with each node being a Lorenz and Chen’s system

This means that the proposed control law (21) can globally and asymptotically stabilize the i th error system (10), thereby ensuring the tracking of (1) by (2). Finally, the control action driving the recurrent neural networks is given by

$$\begin{aligned}
 u_{n_i} &= \tilde{u}_{n_i} + \alpha_i \\
 &= -\frac{1}{2}(1 + L_{\phi, \sigma_i}^2 \|W_{n_i}\|^2)e_i - \\
 &\quad \sum_{\substack{j=1 \\ j \neq i}}^N c_{ij} a_{ij} \Gamma e_j + f_i(x_i) + \lambda_{i_n} x_{i-W_{n_i}} \sigma(x_i), \\
 i &= 1, 2, \dots, N.
 \end{aligned} \tag{22}$$

5 Simulations

In order to illustrate the applicability of the discussed results, we consider a dynamical network with just one Lorenz’s node and three identical Chen’s nodes. The single Lorenz’s system is described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 10x_2 - 10x_1 \\ -x_2 - x_1x_2 + 28x_1 \\ x_1x_2 - (8/3)x_3 \end{pmatrix}, \tag{23}$$

$$x_i(0) = (10, 0, 10)^T, \quad i = 1,$$

and the Chen’s oscillator is described by Eq. 24:

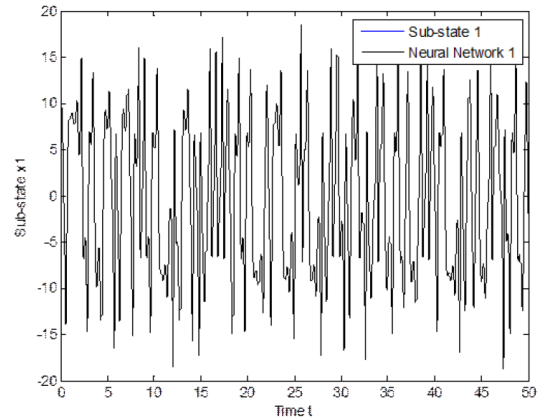


Fig. 4. Time evolution for sub-states 1 with initial state $X_{n_1}(0) = (10, 0, 10)^T$

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} p_1(x_{i2} - x_{i1}) + \sum_{\substack{j=1 \\ j \neq i}}^4 c_{ij} a_{ij} (x_{j1} - x_{i1}) \\ (p_3 - p_2)x_{i1} - x_{i1}x_{i3} + p_3x_{i2} + \sum_{\substack{j=1 \\ j \neq i}}^4 c_{ij} a_{ij} (x_{j2} - x_{i2}) \\ x_{i1}x_{i2} - p_2x_{i3} + \sum_{\substack{j=1 \\ j \neq i}}^4 c_{ij} a_{ij} (x_{j3} - x_{i3}) \end{pmatrix}$$

$$x_i(0) = (-10, 0, 37)^T, \quad i = 1, 2, 3, 4. \tag{24}$$

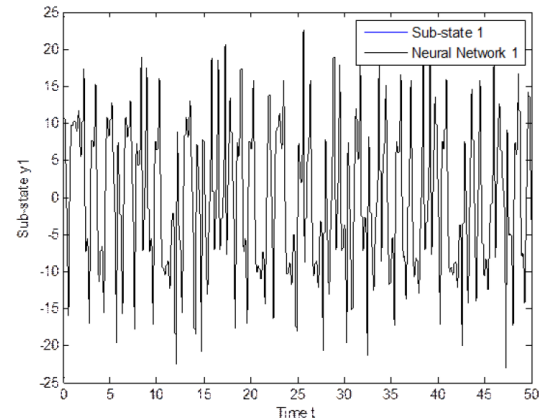


Fig. 5. Time evolution for sub-states 1 with initial state $X_{n_1}(0) = (10, 0, 10)^T$

If the system parameters are selected as $p_1 = 35$, $p_2 = 3$, $p_3 = 28$, then the Lorenz’s system and Chen’s system are shown in Fig. 1 and Fig.2 respectively. In this set of system parameters, one

unstable equilibrium point of the oscillator (24) is $x = (7 : 9373; 7 : 9373; 21)T$.

Suppose that each pair of two connected Lorenz and Chen’s oscillators are linked together through their identical sub-state variables, i.e., $\Gamma = \text{diag}(1, 1, 1)$, and the coupling strengths are $c_{12} = c_{21} = \pi$, $c_{13} = c_{31} = \pi$, $c_{23} = c_{32} = \pi$, $c_{14} = c_{41} = 2\pi$, $c_{24} = c_{42} = 2\pi$, $c_{34} = c_{43} = 2\pi$. Fig. 3 visualizes this entire dynamical network:

The neural network was selected as

$$\begin{aligned}
 A_{n_i} &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \\
 W_{n_i} &= \begin{pmatrix} 1 & 2 & 0 \\ -3 & 4 & 0 \\ 0 & 2 & 3 \end{pmatrix}, \\
 \sigma(x_{n_i}(t - \tau)) &= \begin{pmatrix} \tanh(x_{n_i1}) \\ \tanh(x_{n_i2}) \\ \tanh(x_{n_i3}) \end{pmatrix}, \\
 L_{\phi_{\sigma_i}} &\triangleq n = 3, \\
 x_{n_i}(0) &= (20, 20, -10)^T, \quad i = 1, 2, 3, 4.
 \end{aligned} \tag{25}$$

Theorem 1 For the unknown nonlinear system modeled by (1), the on-line learning law $\text{tr}\{W^T W\} = -e^T W \sigma(x)$ and the control law (22) ensure the tracking of to the nonlinear reference model (2).

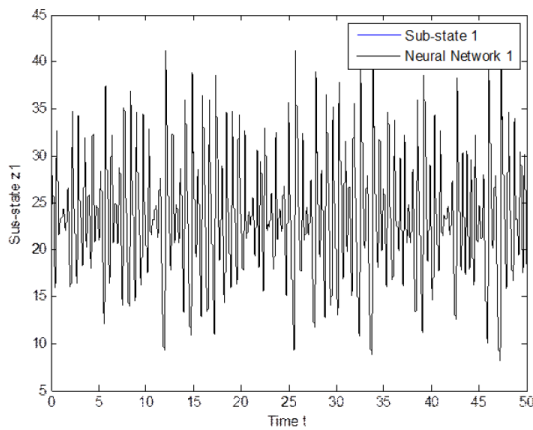


Fig. 6. Time evolution for sub-states 2 with initial state $X_{n1}(0) = (10, 0, 10)^T$

Remark 2 From (20) we have $\dot{V}_N(e) \leq \sum_{i=1}^N e_i^T (-\lambda_{n_i} e_i - \sum_{j=1, j \neq i}^N c_{ij} a_{ij} \Gamma e_j + (\frac{1}{2} + L_{\phi_{\sigma_i}}^2 \|W_{n_i}\|^2) e_i + \tilde{u}_{n_i}^{(1)} + \sum_{j=1, j \neq i}^N c_{ij} a_{ij} \Gamma e_j + \tilde{u}_{n_i}^{(2)}) < 0, \forall e \neq 0, \forall W$, and therefore V is decreasing and bounded from below by $V(0)$. Since $V_N(e) = \sum_{i=1}^N (\frac{1}{2} \|e_i\|^2)$, then we conclude that $e, W \in L_1$; this means that the weights remain bounded.

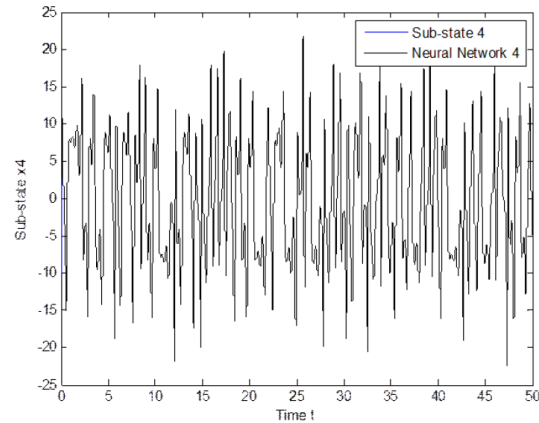


Fig. 7. Time evolution for sub-states 4 with initial state $X_{n4}(0) = (20, 20, -10)^T$

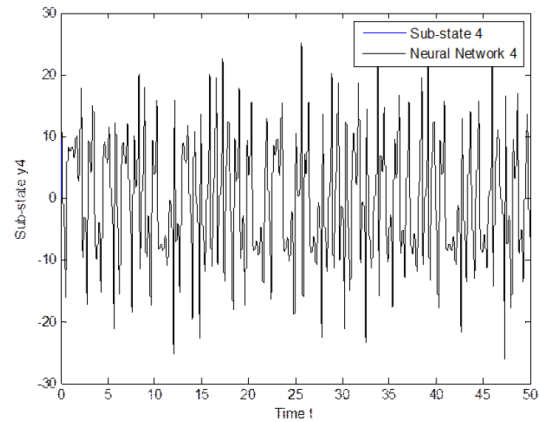


Fig. 8. Time evolution for sub-states 4 with initial state $X_{n4}(0) = (20, 20, -10)^T$

The experiment is performed as follows. Both systems, the recurrent neural network (2) and the dynamical networks (24) and (25), evolve

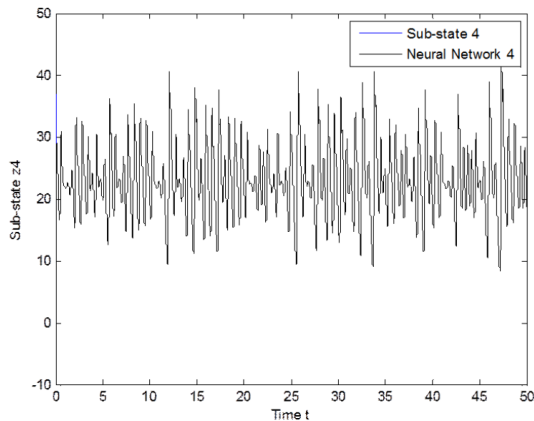


Fig. 9. Time evolution for sub-states 4 with initial state $X_{n,4}(0) = (20, 20, -10)^T$

independently; at that time, the proposed control law (23) is incepted. Simulation results are presented in Fig. 4 - Fig. 6 for sub-sates of node 1. As can be seen, tracking is successfully achieved and error is asymptotically stable, as it is shown in Fig. 7 -Fig. 9 for sub-states of node 4.

6 Conclusions

We have presented the controller design for trajectory tracking determined by a general complex dynamical network. This framework is based on dynamic recurrent neural networks and the methodology is based on Lyapunov theory. The proposed control is applied to a dynamical network with each node being a Lorenz and Chen's dynamical system, respectively, being able to also stabilize in asymptotic form the tracking error between two systems. The results of the simulation shows clearly the desired tracking. In future work, we will consider the stochastic case for the complex dynamical network.

Acknowledgement

The authors thank the support of CONACYT and the Dynamical Systems group of the Facultad de Ciencias Fisico-Matematicas, Universidad Autonoma de Nuevo Leon, Mexico.

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*Article received on 13/01/2015; accepted on 17/04/2017.
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