Sliding Mode Control Applied to a Mini-Aircraft Pitch Position Model

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Abstract. Normally, mini-aircraft must be able to perform tasks such as aerial photography, aerial surveillance, remote fire and pollution sensing, disaster areas, road traffic and security monitoring, among others, without stability problems in the presence of many bounded perturbations. The dynamical model is affected by blast perturbations. Based on this, it is possible to design, evaluate and compare the real result with respect to pitch control law based on reference trajectory in the presence of external disturbances (blasts) or changes in the aircraft controller model. The model has non-linear properties but, with soft perturbations through the aircraft trajectory, allows a linear description without losing its essential properties. The Laplace description is a transfer function that works to develop the state space, with unknown invariant parameters using a wind tunnel. Control law is based on a feedback sliding mode with decoupled disturbances, and the output result is compared with the real pitch position measured in the real system. The control law applied to the system has a high convergence performance.

Keywords. Sliding modes, integral and proportional control, mini-aircraft models.

Control de posición de cabeceo por modos deslizantes para un avión pequeño

Resumen. Comúnmente un avión pequeño debe ser capaz de realizar tareas tales como la de fotografía aérea, vigilancia, detección de incendios a distancia, detectar los niveles de contaminación, monitorear las zonas de desastre, ver el tránsito y brindar seguridad a través de la video-vigilancia, entre otras aplicaciones considerando que no tiene problemas de estabilidad en presencia de perturbaciones acotadas. El modelo dinámico de esa aeronave se ve afectado por las perturbaciones, y que con base en ellas fue posible diseñar un controlador por modos deslizantes. Aplicable a los diferentes movimientos longitudinales que hace hacia arriba o hacia abajo con respecto a la trayectoria de referencia, el modelo de avión tiene propiedades no lineales; pero con perturbaciones suaves a través de su trayectoria; lo que permite una descripción lineal sin perder muchas de sus propiedades esenciales. La descripción de Laplace permitió obtener su función de transferencia y así desarrollar el espacio de estados, con parámetros invariantes y desconocidos. Los cuales fueron descritos utilizando un túnel de viento. La ley de control se basó en la técnica de modos deslizantes con perturbaciones desacopladas. Sus resultados se compararon con el movimiento de cabeceo medido dentro de la aeronave. La ley de control aplicada al sistema real tuvo un desempeño con alta convergencia.

Palabras clave. Modos deslizantes, control proporcional e integral, modelo para aviones pequeños.

1 Introduction

Throughout aviation history, aircrafts were statically modeled for design proposes, specifically in traditional aeronautical industry. Currently it has become necessary to satisfy features such as aircraft dynamic responses and tracking trajectories. The dynamic airplane behavior concepts were introduced in [1-6]. Before applying any control algorithm it was necessary to satisfy controllable and observable conditions in agreement with [2] and [7-11].The dynamic responses to input variations are directed to the control mechanism adjusting its trajectories to a predefined objective [12]. A real flight design control system has great problems mainly in the real-time action control surfaces, whereas the time-delay control algorithm and measurement systems (known as avionics) impact the effectiveness and general performance [13]. This paper does not

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consider real applications, only the feedback law control to the reference trajectory based on a specific model [14-16]. In this sense, the common proportional integral derivative (PID) condition control law adjusts the aerodynamics control surfaces to a specific trajectory [15].

The motion force equation is based on the second Newton's Law, where the atmospheric flight force vector consists of the forces caused by the aerodynamic aircraft reactions and weights [3-6]. This equation is:

$$F_w = AC_w + mg + mgC_w \qquad (1)$$

where A, C_w are forces caused by aircraft aerodynamic reactions elements and Mg stays for plane weights. The force components are given by:

$$Fx_w = m\dot{v}, \qquad Fy_w = mVr_w, \qquad Fz_w = -mVq_w \qquad (2)$$

1.1 Longitudinal Non-linear Equations

From (1) and (2), the longitudinal force applied to aircraft [10] and [11] has the form

$$\begin{gathered} Tx_w - D - mgsin(\theta w) = m\dot{v}, \\ Ty_w - C - mgcos(\theta w)\,sin(\varphi w) = mVr_w, \\ Tz_w - L - mgcos(\theta w)\,cos(\varphi w) = -mVq_w. \end{gathered} \qquad (3)$$

where in agreement with [3-6] Tx_w, Ty_w, Tz_w are traction or propulsion force components, and D, L, C are Drag, Lift and Wind forces.

2 Development

Given the aircraft non-linear equation complexity, it is necessary to find a linear mathematical model whose structure is easy to handle and which approaches the performance of real movements, considering small perturbations around an operating point [12-16]. The Taylor linearization method is considered and the results are shown in the following equations:

$$\begin{aligned} &\Delta Tcos(\alpha T) - \Delta\alpha Tsin(\alpha T) - \Delta D \\ &\qquad - mgcos(\gamma e\Delta\gamma) = m\dot{v}, \\ &\Delta Tcos(\alpha T) - Fy_w = mVr_w, \\ &\qquad Fz_w = -mVq_w, \\ &\Delta T\cos(\alpha T)\text{ -}\Delta\alpha Te\sin(\alpha T)\text{ - }\Delta D - mg\cos(\gamma e), \\ &\Delta\gamma = m\,\dot{v}, \\ &\Delta T\sin\,(\alpha T)\,\Delta\alpha\,Te\cos(\alpha T) + \Delta L + mg\sin(\gamma e), \\ &\Delta\gamma = m\,\dot{v}\,e\,\gamma, \\ &\Delta M = I_y\,\dot{q}. \end{aligned} \qquad (4)$$

Once the aircraft system was modeled, all perturbations *ΔV, Δα, Dp,* etc. are small, and their squares and products can be ignored; with small pitch movements $cos\,(\Delta\gamma) \approx 1$, $sin\,(\Delta\gamma) \approx \Delta\gamma$ and *A* as constant for uniform velocity, where

$$I_y = (x_2 + z_2)dm\,,$$

$$\dot{q} = \dot{\gamma} + \dot{\alpha}\,.$$

Using Taylor linearization with respect to (4) the following linear longitudinal descriptions are given in the following equations:

$$\begin{aligned} &(Tvcos(\alpha_T) - Dv)\Delta V \\ &\quad - (Tesin(\alpha_T) + D\alpha)\Delta\alpha \\ &\quad - mgcos(\gamma_e)\Delta\gamma \\ &\quad + (T_z cos(\alpha_T) - D_z)\Delta z_\varepsilon \\ &\quad + \Delta T_c cos(\alpha_T) - AD_c \\ &\quad = m\dot{V} \\ &(Te\sin\alpha_T + L_V)\Delta V \\ &\quad + \left(Te\cos(\alpha_{T)} + L_\alpha\right)\Delta\alpha \\ &\quad + \left(L_q - mVe\right)q \\ &\quad + \mathrm{mg}\sin(\gamma_e)\Delta\gamma \\ &\quad + (Tz\sin(\alpha_T) + Lz)\Delta z_E \\ &\quad + \Delta\,T_c\sin(\alpha_T) + \Delta Lc \\ &\quad = (mVe + L_{\dot{\alpha}})\,\dot{\alpha} \\ &M_V\,\Delta V + M_\alpha\,\Delta\alpha + M_q\,q + M_z\,\Delta Z_E \\ &\quad + \Delta Mc = I_y\,\dot{q} \end{aligned} \qquad (5)$$

Minimizing (5) in agreement with [17], the form (6) is obtained:

$$\begin{gathered} (\hat{G}\,TV - 2\mu s)\,\Delta V + (CLe\text{-}\hat{G}\,D\alpha)\Delta\alpha\text{-}Cwe\,\Delta\theta=0, \\ \text{-}2Cwe\,\Delta V\text{-}(\,GL\alpha + CDe + 2\mu s)\,\Delta\alpha + 2\mu\,q = \hat{G}\,L\boldsymbol{\delta}\,\Delta\boldsymbol{\delta}, \\ m\alpha\,\Delta\alpha + (\hat{I}\,y\,s\text{-}\hat{G}\,mq)q = \hat{G}\,L\boldsymbol{\delta}\,\Delta\boldsymbol{\delta}, \\ q\text{ -}s\Delta\theta=0, \end{gathered} \qquad (6)$$

where α is replaced by $\Delta\alpha = -(Cm\delta\,/\,(\,Cm\alpha))\Delta\delta e$.

The transfer function with respect to (6) in the symbolic form is described in (7).

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$$Ga(s) = \frac{M(s)}{\delta(s)} = \frac{b_0 s^2 + b_1 s + b_2}{a_0 s^2 + a_1 s + a_2} \qquad (7)$$

The five coefficients have the nominal values shown in Table 1.

Table 1. Experimental results

Coefficient	Wind tunnel experimental results
b_0	2.628238218 (dimensionless)
b_1	$4927.325304\ (\mathrm{N/m.s.})^{-1}$
b_2	$1125059746.00\ (\mathrm{N/m.s.})^{-2}$
a_0	1 (dimensionless)
a_1	$2835.2929 (\mathrm{N/m.s.})^{-1}$
a_2	$417643437.5\ (\mathrm{N/m.s.})^{-2}$

Moreover, according to [7] and [14-18], it was proposed that the model represents the wind blast, where *r* affects the system that has a minimal form (8) described illustratively in Fig. 1:

$$Ga(s) = \frac{\theta(s)}{\delta(s) - r(s)} = \frac{1}{s^2 + a_1 s + a_2}. \qquad (8)$$

In (8), *r(s)* is the wind blast, $\Theta(s)$ is the pitch angle, $\delta(s)$ is the elevator angle. The transfer function (7) in the state space considering [8], [18-23] has the form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_1 & -a_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta_4 \end{bmatrix} r$$
$$y = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \beta_0 \mu \qquad (9)$$

with coefficients $\beta_0 = b_0,\ \beta_1 = b_1 - a_1\beta_0,$
$\beta_2 = b_2 - a_1\beta_0,\ \ \beta_4 = b_2$

Let us consider the system (9) expressed in the symbolic form:

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bu + Pr(t), \\ Y(t) &= CX(t) + Du(t), \end{aligned} \qquad (10)$$

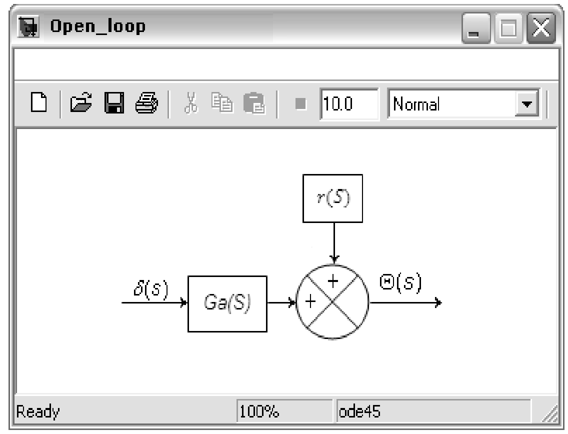

Fig. 1. Open loop aircraft model block diagram

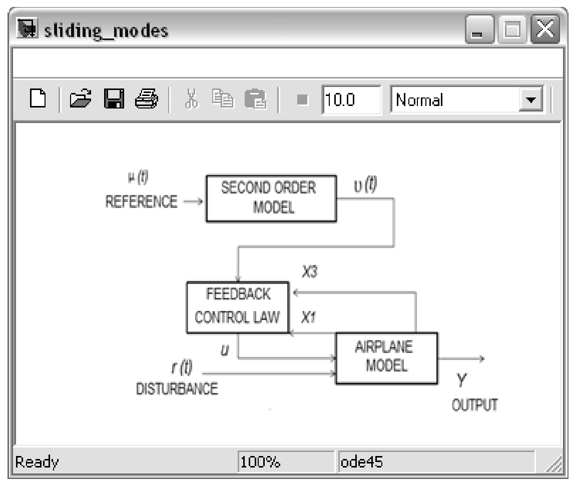

Fig. 2. Mini-aircraft control block diagram

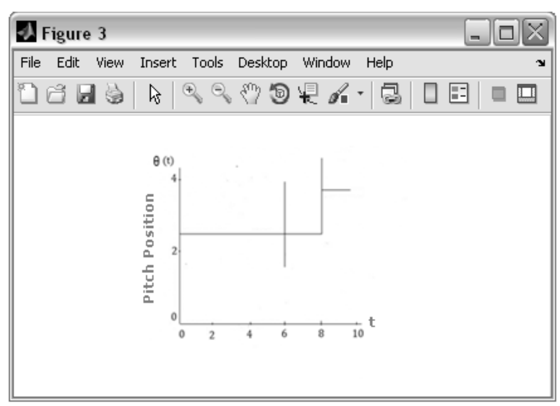

Fig. 3. Open loop pitch position in mini-aircraft model

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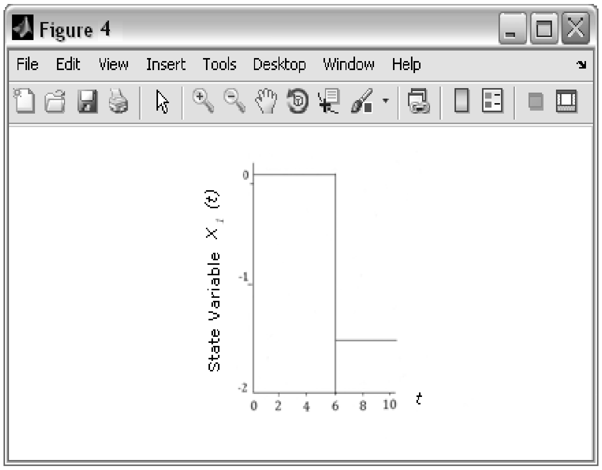

Fig. 4 State variable $X_1(t)$

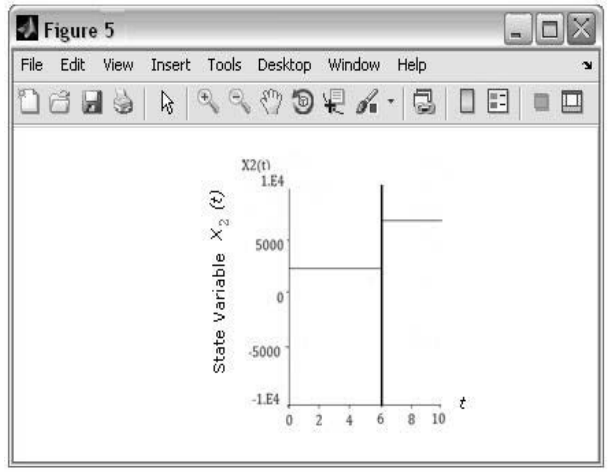

Fig. 5 State variable $X_2(t)$ in open loop through time

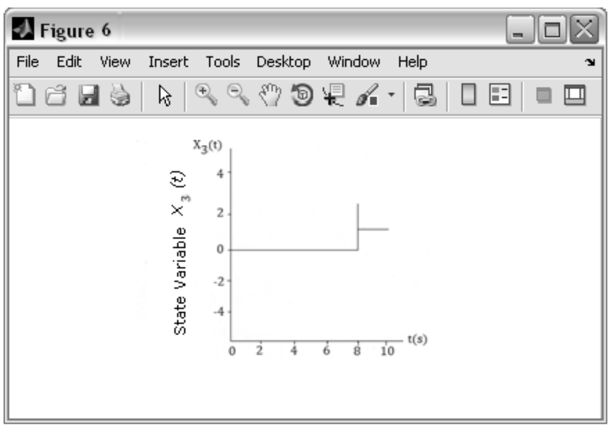

Fig. 6 State variable $X_3(t)$ in open loop

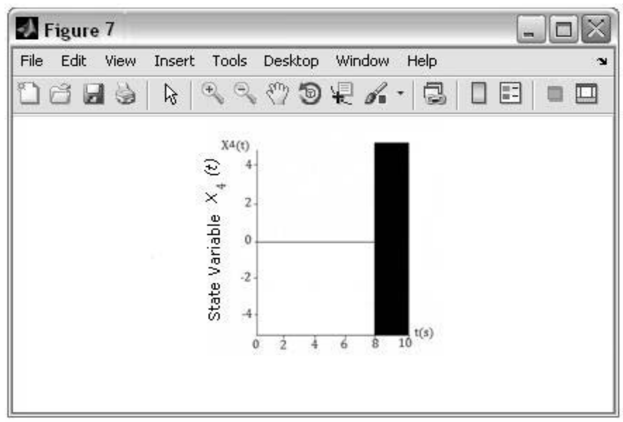

Fig. 7 State variable $X_4(t)$ in open loop through time

with $\dot{X}, X \in \Re^{[4\times1]}_{[-1,1]}$; $u, y \in \Re^{[1]}$; $B,P,C^T \in \Re^{4\times1}_{[-1,1]}$; $A \in \Re^{[4\times4]}_{[-1,1]}$; $\{r(\tau)\}_t \subseteq N(\mu, \sigma^2 < \infty_{\aleph_1})$ and $D \in \Re$. The error is defined as $e(t) := Y(t) - \hat{Y}(t)$, where $\hat{Y}(t)$ is the reference signal choosing the sliding surface described in (11) in agreement with [17-26]:

$$S(e(t)) = \int_0^t e(\tau)\, d\tau. \tag{11}$$

The surface in a finite time considers that

$$\hat{Y}(t) \cong Y(t), \tag{12}$$

for *S(e)* to have an attracting region [7] accomplishing with

$$S\dot{S} < 0. \tag{13}$$

Therefore, a smooth trajectory is proposed:

$$\dot{S} = -F(S), \tag{14}$$

where *F(S)* is a discontinuous function of S. Considering its derivate in (11) and applying (14), we get:

$$-F(t) = Y(t) - \hat{Y}(t). \tag{15}$$

Based on (10) in (15), the region is described as:

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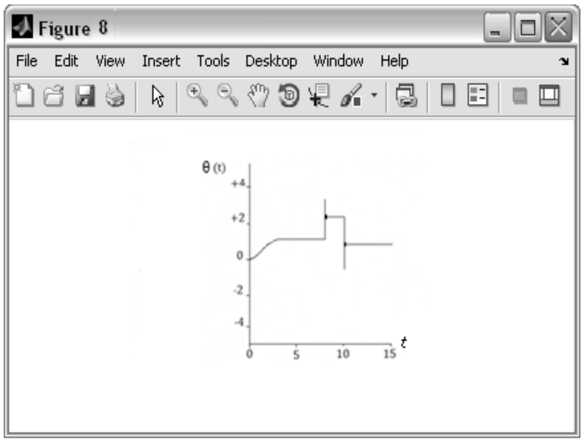

Fig. 8. Output signal θ measured in the mini-aircraft system

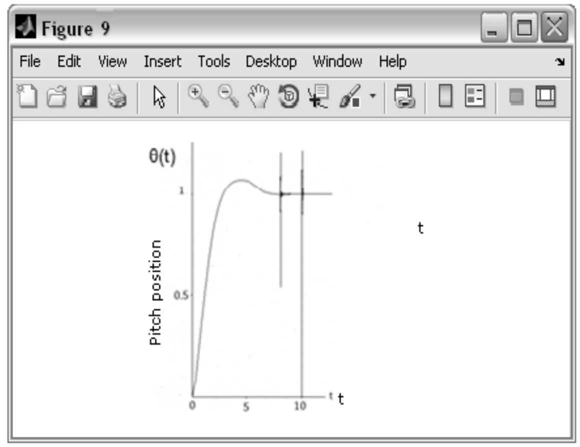

Fig. 9. Output signal θ using sliding mode feedback control law

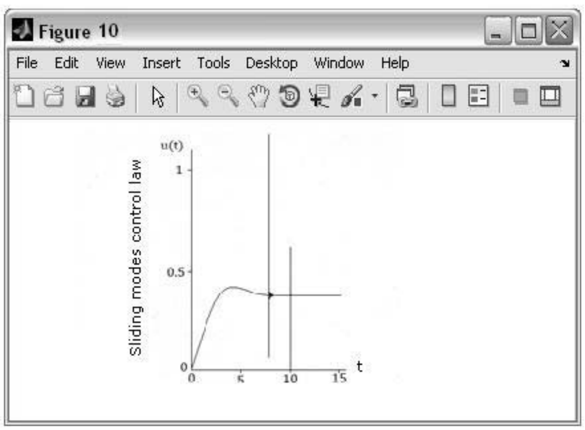

Fig. 10. Sliding mode control law

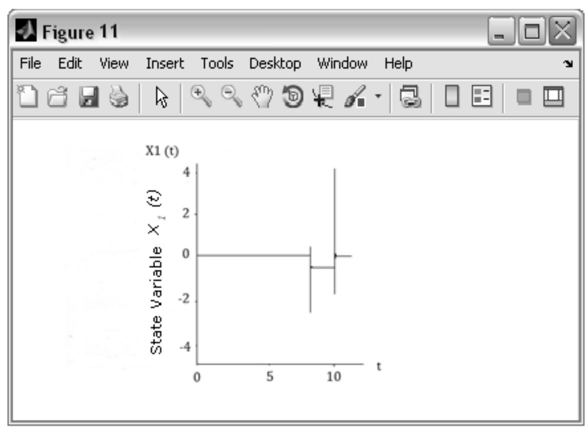

Fig. 11. Sliding mode control law applied to state variable $X_1(t)$ in close loop

$$-F(t) = CX(t) + Du(t) - \hat{Y}(t). \qquad (16)$$

Defining $\boldsymbol{F}(s)$ as $-F(t) + \hat{Y}(t)$, the control law action is described as:

$$u(t)^* = F(s) - CX(t)/d. \qquad (17)$$

The mini-aircraft model output (9) is considered in (15); the control law is given as:

$$u(t)^* = F(s) - (X_1(t) + X_3(t))/\beta_0. \qquad (18)$$

3 Simulation

The control law action applied to the mini-aircraft model system is shown in Figs. 3 to 7, developing the desired output signal, pitch position, control law, and state variables from X_1 to X_4.

Fig. 4 shows the state variable X_1 in an open loop.

Fig. 8 describes the real pitch position without considering the model properties and the ideal

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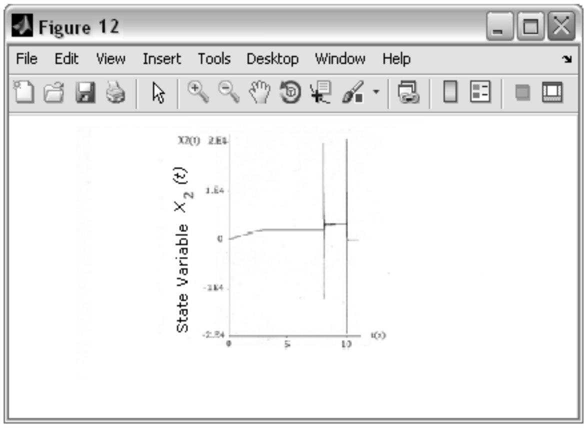

Fig. 12. Sliding mode control law applied to state variable $X_2(t)$ in close loop

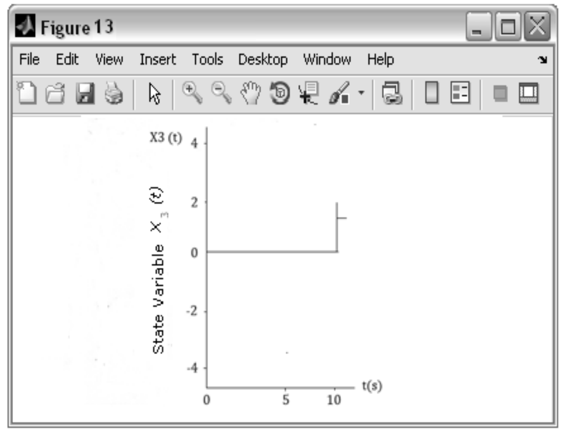

Fig. 13. Sliding mode control law applied to state variable $X_3(t)$ in close loop

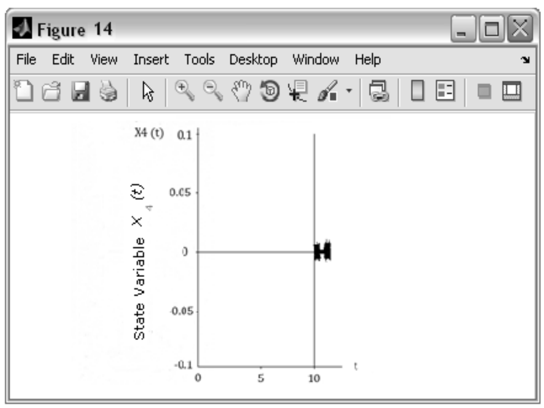

Fig. 14. Sliding mode control law applied to state variable $X_4(t)$ in close loop

conditions measured inside the mini-aircraft system.

Now, Figs. 9 to 10 present the results considering the sliding mode properties based on (18) and Fig. 2, where the control law directly affects the pitch position, the input as a law and the internal states.

4 Conclusions

Mini-aircraft is typically able to perform different complex tasks without loss of stability in the presence of wind perturbations known as blast. The model used in this paper as a base is a fourth order described in state space with four internal variables presented in Equation (9) and in the symbolic form in Equation (10).

In agreement with trajectories, the control law using sliding modes presented in Equation (18) was suggested, which directly affects the internal state model, allowing the pitch position to be automatically independent of visibility and distance conditions.

Figures 3 to 7 describe the model internal states and the pitch position developed, Fig. 8 presents the real angular position measured. Fig. 9 shows the sliding mode control law, and Figures 10 to 14 describe the four controlled internal states.

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