On Line Optimal Control of Robots for Tracking without Inverse Kinematics
Control Optimo en Línea de Robot para Seguimiento sin Cinemática Inversa

Alejandro Rodríguez Ángeles, Carlos. A. Cruz Villar and David Muro Maldonado

Departamento de Ingeniería Eléctrica, Cinvestav,
Av. Instituto Politécnico Nacional, San Pedro Zacatenco, C.P. 07360,
A.P. 14-740, México, D.F. 07000, México.
`aangeles@cinvestav.mx; cacruz@cinvestav.mx; dmuro@cinvestav.mx`

Abstract

This article presents a novel on-line optimal control for tracking tasks on robot manipulators for which inverse kinematics is not required. The controller is composed by a stabilization Cartesian PID control plus a joint space optimal control, which is in charge of improving tracking performance. The joint space dynamic optimal control is based on the gradient flow approach with the robot dynamics as a constraint. The combination of both controllers is implemented in joint space, by considering the robot Jacobian, nonetheless for design of both controllers only direct kinematics and Cartesian errors are taken into account. Joint space controllers which are based on Cartesian errors commonly require the inverse kinematics of the robot, in this proposal the joint space optimal controller determines on line the required joint variables to achieve the Cartesian task, without using the inverse kinematics of the robot, thus an explicit inverse kinematics model of the robot is not needed. The paper presents experimental results with a two degree of freedom (dof) planar manipulator, showing that the optimal control part highly improves the tracking performance of the closed loop system.

Keywords: Gradient flow, direct kinematics, sensitivities, Cartesian control.

Resumen

Este trabajo presenta un control óptimo en línea para tareas de seguimiento de trayectoria en robots manipuladores, el cual no requiere de la cinemática inversa. El control está compuesto por un control PID Cartesiano para fines de estabilidad y un control optimizante en espacio articular para mejorar el desempeño en seguimiento. El control optimizante se basa en el flujo gradiente considerando la dinámica del robot como restricción. La combinación de ambas estrategias de control se implementa en espacio articular a través del Jacobiano del manipulador, sin embargo para el diseño de ambos controles no se requiere del modelo cinemático inverso del robot. El controlador propuesto considera errores Cartesianos, pero a diferencia de controladores en espacio articular que requieren del modelo cinemático inverso. El control aquí propuesto determina de forma implícita las variables articulares requeridas para la tarea cinemática, sin hacer usado del modelo cinemático inverso. El artículo presenta resultados experimentales con un robot planar de dos grados de libertad, donde se muestra que el control óptimo mejora el desempeño del robot en tareas de seguimiento.

Palabras clave: Flujo gradiente, cinemática directa, sensitividad, control Cartesiano.

1 Introduction

The widespread use of robot manipulators in industry has become possible due to the variety of tasks that can be accomplished with them. Task programming of manipulators is divided into two major steps, *trajectory generation* and *trajectory control*.

Trajectory generation is usually off-line performed, by using the inverse kinematics model of the manipulator and considering both analytical [Hwang and Ahuja, 1992] and heuristic optimizing methods, such as pattern search [Ata and Myo, 2005], or genetic algorithms [Hammour and Mirza, 2002]. In [Hammour and Mirza, 2002] for example, the inverse kinematics problem is formulated as an optimization problem based on the concept of the minimization of the accumulative path deviation and is then solved using continuous genetic algorithms.

Concerning the off line path generation approach, once the trajectory is generated; a trajectory control is on-line applied to fulfill the task. The main drawback of this approach consists on that nonconsidered events can become non optimal the generated trajectory. Therefore, several real time algorithms for trajectory generation have been proposed

[Mcfarlane and Croft, 2003], which still use the inverse kinematics model. From a practical point of view, this approach can lead to nonfeasible solutions as the dynamic model of the manipulator is not considered for trajectory generation.

There have been some attempts in considering the dynamics of the robot while designing off line path trajectory. In [Lee, Kim, Park, and Kim, 2005] an optimization Newton type algorithm for path generation is proposed, being one of the few papers, to the best of our knowledge, in solving an optimization problem based on the dynamic model of the robot in order to solve robot trajectory generation.

A different approach considers solving the path trajectory problem simultaneously with the control problem, thus both are on line solved. In [Ding, Li and Tso, 2000] a procedure for the optimization of dynamic performance for redundant robots is proposed, based on recurrent neural networks. The robot configurations obtained with their approach yields minimum joint driving torques. Also in this context [Zhang, Ge and Lee, 2004] proposed an on line joint torque optimization strategy based on quadratic programming for control of redundant robots subject to physical constraints.

Most of the above papers deal with path generation taking into account energy performance or control effort. However, when related to on line tracking tasks it results natural to consider movement related performances. However such approaches have been largely unsuccessful due to the complexity of the robot dynamic equations.

In this paper an on-line optimal controller is proposed, where a convex function of the tracking Cartesian error is considered as performance index. As the error is defined on the task space, only the direct kinematics model of the manipulator is required. The proposed controller is composed of a task space PID controller plus an optimizing controller, yielding an on line controller which simultaneously performs trajectory generation and trajectory control. The controller design problem is considered as a dynamic optimization problem, which is on-line solved by using the gradient flow approach [Helmke and Moore, 1996], with the robot dynamics as an equality constraint. Therefore, the state derivatives with respect to the optimizing controller input (hereinafter referred to as sensitivities), must be on-line computed by solving a set of adjoined differential equations.

The rest of the paper is organized as follows. In Section 2, the model of open chain rigid manipulators is introduced. Section 3 presents the optimal controller for simultaneous trajectory generation and control, some remarks about the controller are provided. In Section 4, experimental results on a two degree of freedom planar manipulator are presented and discussed. Finally, Section 5 closes the paper with some conclusions.

2 Kinematic and dynamic models of the robot manipulator

Consider a n-joint fully actuated rigid robot, i.e. $q \in R^n$, and without loss of generality frictionless, since friction can be independently compensated. Then, the kinetic energy is given by $T(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q}$, with $M(q) \in R^{n \times n}$ the symmetric, positive-definite inertia matrix, and the potential energy is denoted by $U(q)$. Hence, applying the Euler-Lagrange [Lewis, 1993] formalism the joint space dynamic model of the robot is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

Where $g(q) = \frac{\partial}{\partial q}U(q) \in R^n$ denotes the gravity forces, $C(q,\dot{q})\dot{q} \in R^n$ represents the Coriolis and centrifugal forces, and $\tau \in R^n$ is the vector of input torques.

To describe the robot system, it is necessary to characterize its kinematic models [Spong and Vidyasagar, 1989]. In general terms the direct kinematics relates the joint $q \in R^n$ and Cartesian $X \in R^m$ variables, this is

$$X = F_{DK}(q) \tag{2}$$

while for most trajectory designing and for some control implementations the inverse kinematics, which gives the inverse relationship, is required, this is

$$q = F_{IK}(X) \tag{3}$$

notice that the inverse kinematics problem implies, in general, multiple solutions or even singular solutions, depending on the robot architecture. Thus being the most difficult kinematic model to obtain.

Finally to fully relate the joint and Cartesian spaces, it is required to relate the joint torques τ and Cartesian forces F, for this, the Jacobian $J(q) = \dfrac{\partial F_{DK}(q)}{\partial q} \in R^{m \times n}$ of the robot is considered, thus

$$\tau = J(q)^T F \tag{4}$$

3 Joint/Cartesian optimal control

In this section a joint space controller that does not require the inverse kinematics model is developed. For stability purposes a Cartesian PID control is introduced, while for improving the closed loop performance a joint space dynamic optimization controller is designed. The PID control yields small tracking errors, and then the optimal part reduces the tracking errors improving the performance of the system. Since the PID control is Cartesian based, it is mapped through the Jacobian of the manipulator to the torques at the joint space. Thus, from (4) the control torque τ in (1) is proposed as

$$\tau = J(q)^T F_{PID} + \tau_o \tag{5}$$

where $F_{PID} \in R^m$ is the PID Cartesian control, and $\tau_o \in R^n$ corresponds to the optimal control part.

PID Cartesian control

The PID control F_{PID} is Cartesian type and thus it is based on Cartesian space variables, then by considering the direct kinematics model (2), it follows that

$$F_{PID} = K_{p,c}e_c + K_{d,c}\dot{e}_c + K_{i,c}\int e_c dt \tag{6}$$

where $K_{p,c}$, $K_{d,c}$, $K_{i,c} \in R^{m \times n}$ are the proportional, derivative, and integral diagonal gain matrices, $e_c \in R^m$ denotes the Cartesian tracking error, $\dot{e}_c \in R^m$ corresponds to the Cartesian velocity tracking error, and they are given by

$$e_c = X_d - X = X_d - F_{DK}(q) \tag{7}$$

$$\dot{e}_c = \dot{X}_d - \dot{X} = \dot{X}_d - J(q)\dot{q} \tag{8}$$

with X_d, $\dot{X}_d \in R^m$ the desired Cartesian position and velocity variables respectively.

On-line Optimal Control

To improve the performance of the closed loop system it is considered a dynamic optimization problem, which is related to the Cartesian errors. The optimization based controller works on line and depends on the dynamics of the robot through the state sensitivities of the system with respect to τ_o. The dynamic optimization problem is formulated as

$$\min_{\tau_o} I = \frac{1}{2} K_o (\alpha e_c + \dot{e}_c)^T (\alpha e_c + \dot{e}_c) \tag{9}$$

subject to the dynamic constraint given by the closed loop equation of the robot (1) and the controller (5), this is

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = J(q)^T F_{PID} + \tau_o \tag{10}$$

with $\alpha \in R^{m \times m}$ a diagonal gain matrix, and K_o a scalar gain.

Notice that the optimization problem (9) is a dynamic one, which can be solved on-line by using the gradient flow approach. Also notice that the optimization problem is subjected to dynamic constraints. Nonetheless any mechanical system such as a robot, always presents limitations and constraints such as limited power, bounded motions, etc.. These static constraints would be considered in further extensions of the proposed controller.

To solve the dynamic optimization problem (9) the gradient flow approach is considered [Helmke and Moore, 1996]. Obtaining the gradient of the performance index (9) with respect to the optimization variable τ_o, it follows that

$$\dot{\tau}^T = -\gamma \frac{\partial I}{\partial \tau_o} \tag{11}$$

where $\gamma \in R^{n \times n}$ is a gain diagonal matrix related to convergence properties of the gradient flow approach, and from (7, 8)

$$\frac{\partial I}{\partial \tau_o} = -K_o (\alpha e_c + \dot{e}_c)^T \left(\alpha \frac{\partial F_{DK}}{\partial \tau_o} + \frac{\partial J(q)\dot{q}}{\partial \tau_o} \right) \tag{12}$$

such that from chain rule differentiation, it follows that $\dfrac{\partial F_{DK}}{\partial \tau_o}$ and $\dfrac{\partial J(q)\dot{q}}{\partial \tau_o}$ depends on the sensitivity functions

$\left[\dfrac{\partial q}{\partial \tau_o}, \dfrac{\partial \dot{q}}{\partial \tau_o} \right]$, which are obtained from partial differentiation of (10), this is

$$\frac{\partial}{\partial \tau_o}\left\{M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=J(q)^T F_{PID}+\tau_o\right\} \tag{13}$$

Since the controller depends on the Jacobian of the robot, it is sensible to singularities. Nonetheless, the inverse of the Jacobian is not required for the proposed control. Singularities of the Jacobian reduce the solution space of the optimization problem, since the Jacobian looses rank. In case of redundant robots the desired Cartesian task can still be achieved if there remain enough degrees of freedom (as many as the Cartesian task requires), otherwise the optimization problem results on suboptimal solutions.

Stability and convergence properties of the proposed controller are important issues, however this article focus only on the presentation of the control architecture. Convergence of the optimal control strategy and stability can be addressed by Lyapunov theory and will be considered in a future article.

Remarks on the optimal control design
- Notice that the controller (5) is in general form, so that, redundant robots with $n > m$ can be considered.
- The controller is based on measured joint variables $(q,\dot{q})$ and Cartesian variables to determine the task space error. These Cartesian variables are obtained by means of the direct kinematics, the Jacobian models, and joint measurements. Thus the inverse kinematics model is avoided. In fact, the proposed controller directly computes the joint torques τ that minimizes the functional I, given by (9). But desired joint variables $(q_d,\dot{q}_d)$ are never required by the controller, only desired task space (Cartesian) variables are needed.
- The complexity of the optimization control design relies on the computation of the sensitivities $\left[\dfrac{\partial q}{\partial \tau_o},\dfrac{\partial \dot{q}}{\partial \tau_o}\right]$, however there exist techniques for approximation of such functions as presented in [Maly and Petzold, 1996].

4 Experimental case study

The proposed optimal controller (5) is tested on a two degree of freedom planar robot, which diagram is shown in Figure 1. The two dof robot is built with aluminum (alloy 6063 T-5) of 9.525 mm thickness, and the joints are driven by DC brushless servomotors of the brand Micromo Electronics Inc., part number 2444-024B. The servomotors are provided with planetary gearboxes and optical encoders part number HEDS 5540A, their characteristics are listed in Table 1.

The direct kinematics model of the two dof robot is obtained by the Denavit-Hartenberg method [Craig, 1989], and it is given by (14 - 15)

$$x = l_1\cos(q_1)+l_2\cos(q_1+q_2) \tag{14}$$

$$y = l_1\sin(q_1)+l_2\sin(q_1+q_2) \tag{15}$$

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Fig. 1. Picture of the two dof robot

Table 1 Servomotors technical data

Technical data	
Nominal voltage	24 [V]
No load speed	23 000 rpm
Reduction ratio	159 : 1
Maximum torque	4.5 [Nm]
Resolution	500 ppr

The dynamic model is obtained by the Euler-Lagrange formalism, and it is of the form given by (1), with the matrix entries

$$M_{11} = I_1 + l_{xc1}^2 m_1 + I_2 + l_{xc2}^2 m_2 + 2m_2 l_{xc2} l_1 \cos(q_2) + l_1^2 m_2$$

$$M_{12} = M_{21} = I_2 + l_{xc2}^2 m_2 + m_2 l_{xc2} l_1 \cos(q_2)$$

$$M_{22} = I_2 + l_{xc2}^2 m_2$$

$$C_{11} = -m_2 l_{xc2} l_1 \sin(q_2) \dot{q}_2$$

$$C_{12} = -m_2 l_{xc2} l_1 \sin(q_2)(\dot{q}_1 + \dot{q}_2)$$

$$C_{21} = m_2 l_{xc2} l_1 \sin(q_2) \dot{q}_1$$

$$C_{22} = 0$$

$$G_{11} = m_2 g \cos(q_1 + q_2) l_{xc2} + m_2 q \cos(q_1) l_1 + m_1 g \cos(q_1) l_{xc1}$$

$$G_{12} = m_2 g \cos(q_1 + q_2) l_{xc2}$$

where I_i represents the inertial moment, m_i is the mass, and l_{xci} the distance from the i-th joint to the i-th mass center position, l_i is the length of the i-th link. By using CAD tools all the parameters of the dynamic model were estimated and their values are listed in Table 2.

Table 2 Robot physical parameters

	m_i [Kg]	I_i [Kg m^2]	l_{xci} [m]	l_i [m]
i=1	0.7	16.84x10^{-3}	0.144	0.175
i=2	0.6	8.40x10^{-3}	0.108	0.130

The proposed controller is applicable to any Cartesian task that is properly defined at the working space of the robot. In this section and for the sake of clarity of the results a simple trajectory is considered. The desired Cartesian trajectory $X_d(t)$ in (7), runs for 20 [sec] and follows a sinusoidal wave along a vertical plane. The desired trajectory is given by

$$X_d(t) = \begin{cases} x_d = 0.15[m] \\ y_d = 0.1\sin(\omega t) - 0.1[m] \end{cases} \tag{16}$$

With the desired trajectory frequency $\omega = 1$.

Experimental results

For comparison purposes, first the PID Cartesian control (6) is tested alone. Then the optimization control strategy (5) is considered. Both controllers are tuned with the same PID control gains, which have been selected by simulations and trial and error methods. The main diagonal elements of the PID gain matrices are listed in Table 3, for each one of the axes on the task space.

**Table 3** Cartesian PID control gains

	$K_{p,c}$	$K_{d,c}$	$K_{i,c}$
x	600	90	-4.8
y	900	90	-4.8

At the initial conditions the robot is pointing downward, thus the joint initial values are $q_1(0) = -90^o$ and $q_2(0) = 0^o$, which implies the Cartesian initial position $x = 0[m]$ and $y = -0.305[m]$.

1) PID Cartesian control: Figure 2 shows the trajectory of the end effector, which particularly for the x-axis, can be easily compared with the desired trajectory (16). The Cartesian errors (7) are shown in Figure 3.
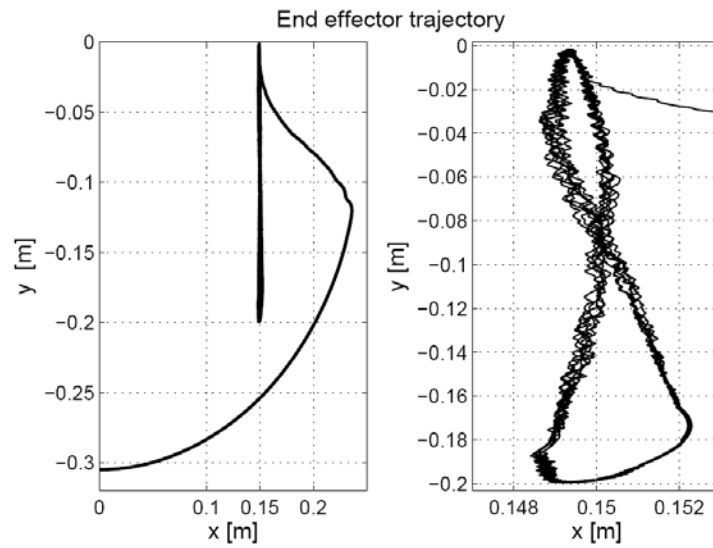
Fig. 2. End effector trajectory and its zoom for PID Cartesian control
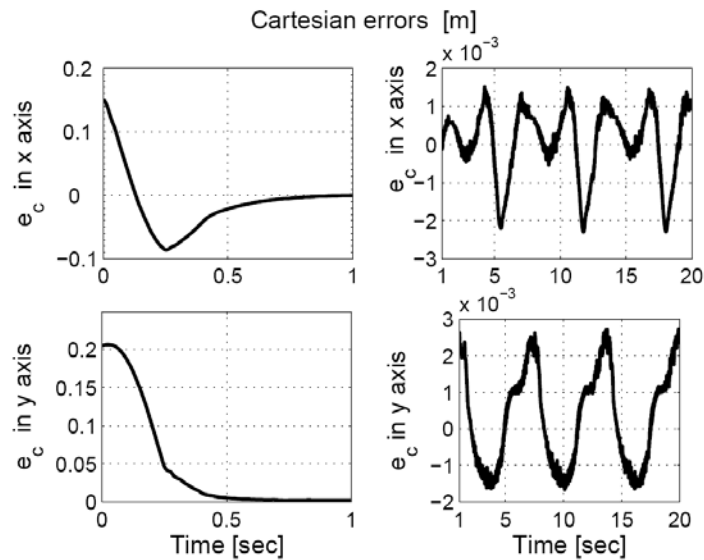
Fig. 3. Cartesian errors in [m] and their zoom for PID Cartesian control

The joint input torques, given by controller (5) with only the PID Cartesian control activated, are shown in Figure 4. Notice that although from Table 1 the maximum torque supplied by the servomotors is 4.5 [Nm], for safety reasons at the experimental setup the torques are bounded at 3.5 [Nm].
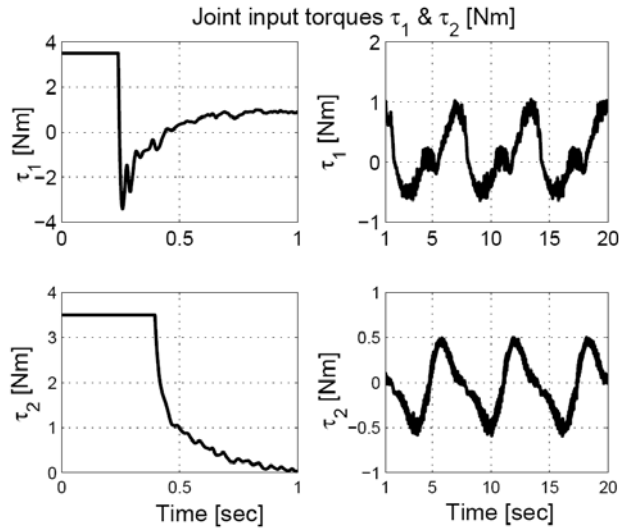
Fig. 4. Joint input torques with PID Cartesian control

2) On-line optimal control: For the sake of comparison the Cartesian PID gains correspond to the ones of Table 3, while the gains of the optimization part are given in Table 4.

Figure 5 shows the trajectory of the end effector, which can be easily compared with the desired trajectory (16). The Cartesian errors (7) are shown in Figure 6.

Table 4 Optimization control gains

	K_o	α	γ
τ_{o1}	17	17	0.3
τ_{o2}	4	4	0.5
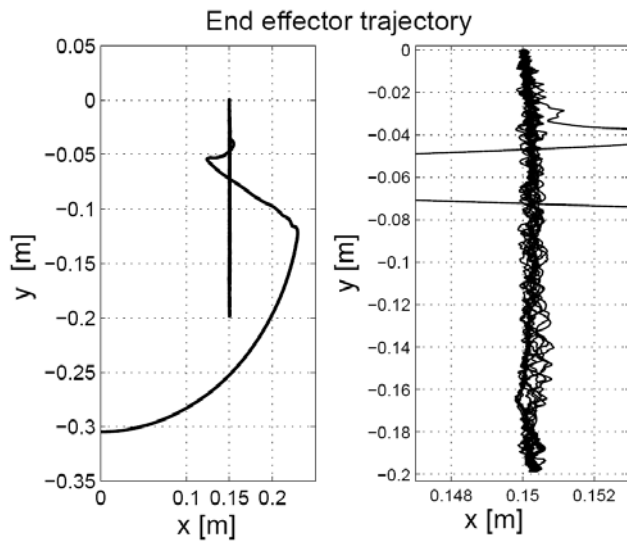
Fig. 5. End effector trajectory and its zoom for PID plus optimal control
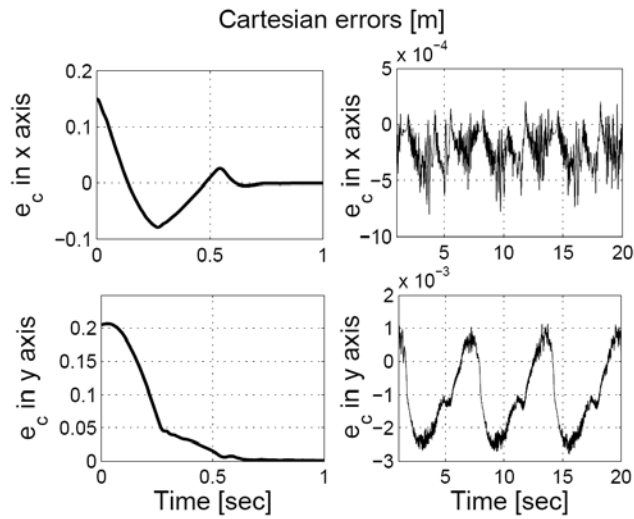
Cartesian errors [m]

Fig. 6. Cartesian errors in [m] and their zoom for optimization controller

Finally, the joint input torques, given by controller (5) are shown in Figure 7. Notice that the input control torques look very similar to the ones in Figure 5, however the controls are now a composition of the PID and the optimal part, as it is shown in Figures 8 and 9.
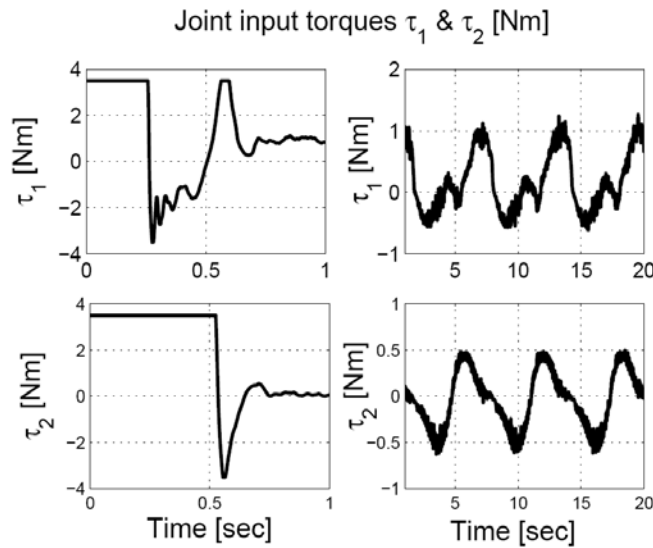
Joint input torques τ_1 & τ_2 [Nm]

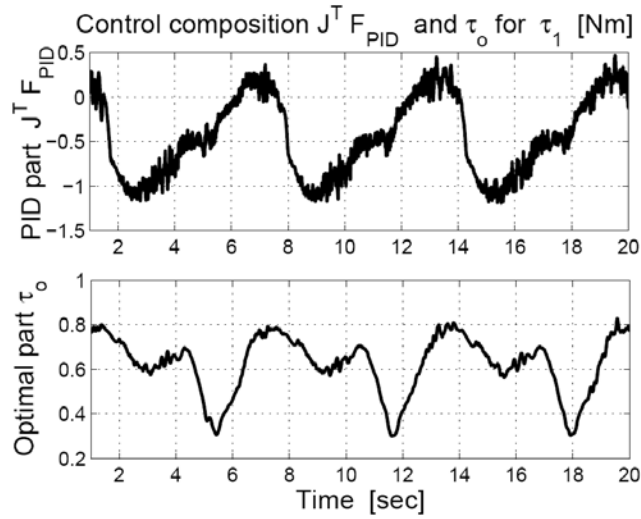**Fig. 7.** Joint input torques with optimization controller

Fig. 8. Joint input torques composition of optimization controller τ_{o1}
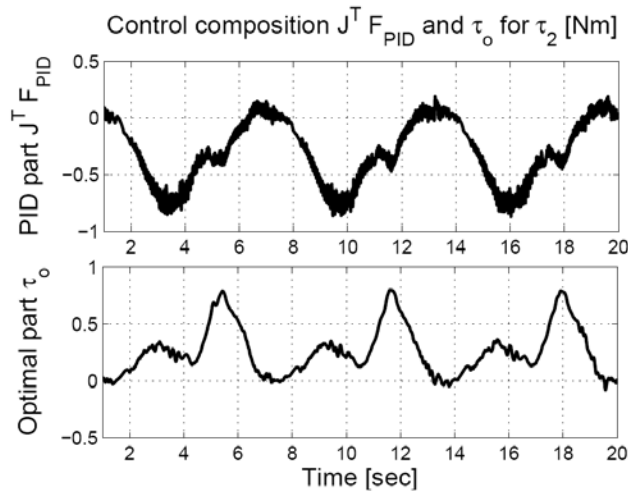

Fig. 9. Joint input torques composition of optimization controller τ_{o2}

2) Discussion of experimental results: From comparison of Figures 2, 5, 3 and 6 it is easy to conclude that the joint optimization control decreases the Cartesian tracking errors, thus improving the performance of the closed loop system, with respect to the PID Cartesian control. Furthermore, the inclusion of the optimal control τ_o in the input torques (5) has implications on the general behavior of the closed loop system, particularly on transient period.

On the one hand, from Figures 4, 7, 8 and 9 combined with figures 2, 5, it can be concluded that at transient the PID Cartesian part dominates the behavior of the closed loop system, that is why the approaching curve from the initial condition to the desired vertical plane are alike. On the other hand, once the Cartesian errors are small, the optimal part comes into play to improve the performance of the system, this behavior is shown at the composition of the input torques of figures 8 and 9. All the above agrees with the design of the controller, where the PID Cartesian part was introduced for stabilization purposes and the optimal part for performance improvement.

5 Conclusions

A joint/Cartesian optimal control for robot manipulators has been introduced, it combines a PID Cartesian controller for stabilization purposes and an on-line optimal control for performance improvement purposes. The controller is intended for tracking, nonetheless it does not require the inverse kinematics of the robot. The optimization part of the controller is based on the gradient flow approach, such that sensitivities are required.

Experimental results show better performance of the closed loop with the optimization controller than with the PID Cartesian control alone, which implies that both controllers complement themselves for decreasing the Cartesian tracking errors. The tuning of the control gains has been done by trail and error; however it seems plausible that Lyapunov stability techniques can be used for selection of the control gains. The stability proof and tuning gain rules are the work of further extensions. Also as further work it is considered the application of the optimal controller to a three dof redundant robot, where higher performance of the controller is expected due to the multiple solutions of the kinematics location problem. Robustness of the proposed controller to disturbance will be addressed as further works as well.

6 Acknowledgment

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Alejandro Rodríguez Ángeles *Ingeniero en Comunicaciones y Electrónica en el Instituto Politécnico Nacional, 1996. Maestro en Ciencias en Ingeniería Eléctrica en el Centro de Investigación y de Estudios Avanzados del I.P.N., 1997. Doctor en Ciencias en Ingeniería Mecánica en la Universidad Tecnológica de Eindhoven, 2002. Miembro del Sistema Nacional de Investigadores, nivel 1, desde enero del 2004 a la fecha. Investigador Titular de la Sección de Mecatrónica del Departamento de Ingeniería Eléctrica del CINVESTAV-IPN, desde diciembre del 2005.*

Carlos Alberto Cruz Villar *Ingeniero en Comunicaciones y Electrónica por la ESIME-IPN, 1996. Doctor en Ciencias por el CINVESTAV-IPN, 2001. Profesor Asociado y Jefe del Departamento de Ciencias Básicas en la UPIITA-IPN de 1999 al 2000. Investigador Adjunto, en la Sección de Mecatrónica del Departamento de Ingeniería Eléctrica del CINVESTAV-IPN, desde agosto del 2001.*

David Muro Maldonado *Maestro en Ingeniería Eléctrica del Centro de Investigación y Estudios Avanzados del I.P.N., 2007. Ingeniero en Mecatrónica en la Unidad Profesional Interdisciplinaria de Ingeniería y Tecnologías Avanzadas del I.P.N., 2005. Actualmente es Ingeniero de diseño en el Departamento de Hidráulica de Bosch Rexroth, S. A. de C. V., en su oficina matriz de México D.F.*