Task Based Mechatronic System Design using Differential Evolution Strategies

Diseño de Sistemas Mecatrónicos Basado en Tareas usando Estrategias de Evolución Diferencial

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Abstract
A dynamic design approach for a mechatronic system called task based mechatronic system design approach (TBMSDA) is stated as a nonlinear dynamic optimization problem (NLDOP) and it is solved by using a differential evolution technique. The design of a parallel robot is carried out by this approach which integrates in a simultaneous way both the structure design parameters of a parallel robot and the PID controller gains in order to improve the position errors and the robot dexterity for executing a specific task. The TBMSDA considers the dynamic system as an equality constraint into the optimization problem. Through the TBMSDA, an optimal combination of the structure and PID controller gains for a planar five-bar parallel robot is obtained. Simulation results show the effectiveness of this design approach.
Keywords: Mechatronic design, Task based design, Evolutionary algorithms, System optimization, Parallel robot.

Resumen
Se establece un enfoque de diseño dinámico para un sistema mecatrónico denominado enfoque de diseño de sistemas mecatrónicos basado en tareas. Se plantea como un problema de optimización dinámica no lineal y su solución se basa en el uso de técnicas de evolución diferencial. Con este enfoque, se realiza el diseño de un robot paralelo al integrar en forma simultánea los parámetros del diseño de la estructura mecánica del robot paralelo y los parámetros del diseño de las ganancias del controlador PID con el propósito de optimizar el desempeño en los errores de posición y en la destreza del robot al ejecutar una tarea específica. En el problema de optimización se considera la dinámica del sistema como una restricción de igualdad. Con este enfoque de diseño, se obtiene una combinación óptima de la estructura y de las ganancias del controlador PID para un robot paralelo de cinco eslabones planar y para su controlador. Resultados de simulación muestran la efectividad de este enfoque de diseño mecatrónico.
Palabras clave: Diseño mecatrónico, Diseño basado en la tarea, Algoritmos evolutivos, Optimización de sistemas, robot paralelo.

1 Introduction

The mechatronic system design approach mainly involves the integrated design of the mechanical system and its embedded control system [Amerongen, 2003]. Hence, an improvement in this design is due to the compromise between the mechanical and controller performance indexes [Sudhendu and Asada, 1995; Ravichandran, et. al., 2006]. In order to appropriately choose both the mechanical and the control system parameters in an early mechatronic design stage, optimization techniques are used for simultaneously optimizing these parameters. Nevertheless, the task to be executed by the electro-mechanical system (task based design) is not considered by the mechatronic system design approach.

A special case of electro-mechanical systems is the robotic manipulator, which is usually used to execute one or a couple of tasks. Therefore a task based mechatronic system design approach becomes more suitable for the system performance. In [Kim, 2006], the link lengths of a parallel robot, a subclass of robotic manipulator, were optimized in order to perform a given task. A task based kinematic design approach for designing a parallel robot is developed. Therefore, in this design approach a static optimization problem is stated without considering the equality dynamic constraint of the dynamic system behavior. However, when the design is for a specific task, the dynamic system constraint must be included in order to provide states or data (as positions, velocities, applied torques, etc.) of the system more alike for the real manipulator behavior.

In this paper a mechatronic system design approach, which considers the task to be executed (TBMSDA), is formulated as a nonlinear dynamic optimization problem (NLDOP). This approach considers both equality and inequality nonlinear dynamic constraints in order to simultaneously find mechanical parameters and controller gains. In order to illustrate this approach, a five-bar parallel robot with its controller is designed. Given the dynamic model of the system with its controller, the set of all design variables (mechanical parameters and controller gains) must be chosen. In this case, the link lengths of the five-bar parallel robot and the proportional-integrative-derivative (PID) controller gains are selected. Usually, design variables are bounded in real applications or a special characteristic must be taken into account, hence they are included as constraints. The task trajectory to be executed by the robot must be transformed from the Cartesian space to independent generalized coordinate space of the dynamic system (as the joint space) in order to control the robot. The position error and a dexterity measure of the robot for following the task are the performance indexes to be optimized. A weighted sum approach for both indexes is implemented.

On the other hand, the TBMSDA will require a transcribing method for converting it from an infinite-dimensional problem to a finite-dimensional approximation in order to use nonlinear programming techniques [Betts, 2001] to solve it. These techniques require computing the gradient vector of the performance index with respect to the design variables and the sensitivity vector of the dynamic system states with respect to the same variables. When these techniques deal with nonlinear optimization problems, they can not guarantee convergence to the global optimum. On the other hand, Evolutionary algorithms (EAs) neither can guarantee the convergence to the global optimum. However, they are less prone to get trapped in local optima, even when dealing with fairly large and complex search spaces. So, EAs have been widely used to solve optimization problems in the last two decades. Additionally, it is precisely in nonlinear dynamic optimization problems when the use of EAs results particularly useful and highly competitive since they do not require the transcribing method, the gradient vector neither the sensitivity vector calculation. The EAs can be considered as a broad class of stochastic optimization techniques motivated by the natural evolution process found in biological organisms. This process can be explained as follows [Price, et. al., 2005]: Reproduction; a way to create new individuals from a current set of them. Mutation; small changes in the genetic codification of individuals which cause significant changes in their features. Competition; a mechanism to measure how fit is an individual to survive in a specific environment. Selection; leads to the maintenance or increase of population fitness, where fitness is defined as the ability to survive and reproduce in a specific environment.

The optimization algorithms used in a mechatronic system design approach involves the mathematical programming methods as the goal attainment [Portilla, et. al., 2007], the sequential quadratic programming method [Alvarez, et. al., 2005], and stochastic optimization methods as the (μ+λ) - evolutionary strategy [Ravichandran, et. al., 2006], the differential evolution (DE) strategy [Portilla, et. al., 2007].

Some of the DE strategy advantages are its simple structure, ease of use, speed and robustness. In this paper a DE strategy is used and a constraint handling approach must be included in this strategy in order to incorporate information about infeasibility into the fitness function for guiding the search. The constraint handling differential evolution strategy proposed by Mezura and Coello [Mezura and Coello, 2006] is used in order to handle constraints and objectives separately.

The rest of the paper is organized as follows; in Section II the TBMSDA is presented. In order to show the effectiveness of the approach, a practical example is stated in the same section. In order to solve the TBMSDA, the differential evolution algorithm with a simple constraint handling mechanism is presented in Section III. In Section IV a summary of the optimal results of the TBMSDA with different population sizes in the DE are shown and the simulation results of the closed-loop system with the optimal structure control parameters are presented. Finally, conclusions of this approach are summarized in Section V.

2 Task based mechatronic system design as a NLDOP

The task based mechatronic system design problem consists in finding the structural design parameter vector $p_s \in R^{n_s}$ of the system and the controller design parameter vector $p_c \in R^{n_c}$ that maximize a weighted sum of n_j

dynamic performance indexes (J_i) subject to different constraints. These constraints involve the closed-loop nonlinear dynamic system (4) with the initial condition (5), and equality (2) and inequality (3) static/dynamic constraints which are imposed by the mechatronic system.

In the problem below, $f_{sc} : R^{n_v} \to R^{n_v}$ is a smooth, continuous, differentiable and nonlinear function, $x \in R^{n_v}$ is the state vector of the system, t is the time, x_0 is the chosen initial state vector and $\overline{q}(t)$, $\dot{\overline{q}}(t) \in R^n$ are the position and velocity vectors of the task which are transformed from the Cartesian space to the joint space in order to control the nonlinear dynamic system.

$$\underset{p_s,\, p_c}{Max}\left(J_1 + J_2 + \ldots + J_i\right)$$
$$J_i = \int_{t_0}^{t_f} L_i\left(x, p_s, p_c\right) dt, \qquad i = 1, 2, \ldots n_J \tag{1}$$

subject to

$$h\left(x, p_s, p_c, t\right) = 0 \tag{2}$$
$$g\left(x, p_s, p_c, t\right) \le 0 \tag{3}$$
$$\dot{x} = f_{sc}(x, \overline{q}(t), \dot{\overline{q}}(t), p_s, p_c, t) \tag{4}$$
$$x(t_0) = x_0 \tag{5}$$

2.1 Dynamic model of the parallel robot and its controller

Parallel manipulators are a special case of robotic manipulators which have a specific mechanical architecture. Their links form closed-loop chains which drive the end-effector (gripper) of the robot. These manipulators are widely used in airplane simulators, assembly of electronic boards, industrial fields such as pick and place robots, etc. Parallel manipulators have received considerable attention from the research community over the past two decades [Hunt, 1983]. Their studies are motivated by the fact that they can provide higher stiffness and accuracy than serial manipulators, especially for executing a specific task. However, the dynamics and kinematics of parallel manipulators are more complex than those arising in the analysis of serial ones.

The planar five-bar parallel robot is the manipulator to be designed and it is shown in Fig. 1. It has two degrees of freedom (d.o.f.), $n = 2$. The length, mass, inertia and mass center distance of the *i-th* link are represented by a_i, m_i, I_i, l_{ci}, respectively. $q = [q_1, q_2]^T \in R^2$, $\dot{q} = [\dot{q}_1, \dot{q}_2]^T \in R^2$ are the angular position vector and the angular velocity vector of the actuated links respectively, q'_3, q'_4 are the angular positions of the unactuated links, $u = [\tau_1, \tau_2]^T \in R^2$ is the applied generalized force vector and $p = [x_p, y_p]^T$ is the end-effector Cartesian coordinate vector of the planar five-bar parallel robot.

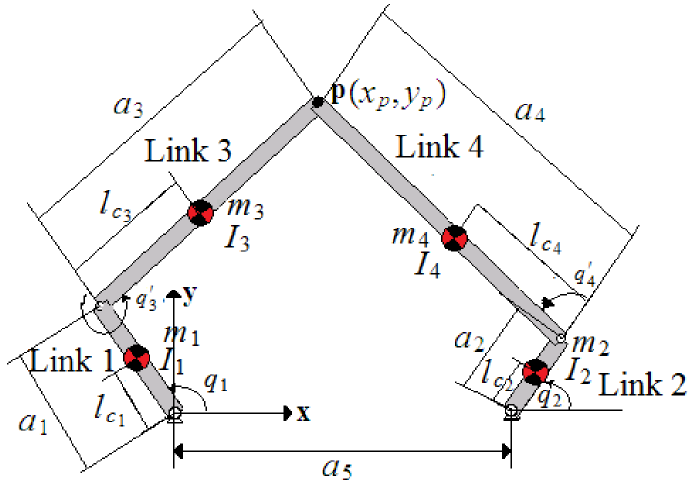


Fig. 1. The planar five-bar parallel robot schematic representation

The dynamic model is obtained by the reduced model method [Ghorbel, et. al., 2000]. This method generates n second-order nonlinear differential equations from the n d.o.f. of the closed chain mechanism, becoming then these equations similar to those of the dynamic equations of serial robots. For the planar five-bar parallel robot it is possible to get two explicit differential equations since the relationship between the dependent and independent generalized coordinates (closed-loop constraints) is an explicit equation.

The dynamic equations of the planar five-bar parallel robot are shown in (6), where $D(q') \in R^{2\times 2}$ is a symmetric positive definite inertia matrix, $C(q', \dot{q}') \in R^{2\times 2}$ is the Coriolis and centrifugal matrix, $g(q') \in R^2$ is the gravity vector, $u \in R^2$ is the applied generalized force vector. $q' \in R^4$ is a dependent generalized coordinate vector (the angular position of the free system q' in terms of the angular position of the constraint system q), where $\sigma(q)$ involves a parameterization of $q' \to q$. $\dot{q}'$ is the angular velocity of the free system in terms of the angular velocity of the constraint system $\dot{q}$.

$$\begin{aligned} \ddot{q} &= D(q')^{-1}\left(u - C(q', \dot{q}')\dot{q} - g(q')\right) \\ \dot{q}' &= \rho(q')\dot{q} \\ q' &= \sigma(q) \end{aligned} \tag{6}$$

where:

$$\begin{aligned} D(q') &= \rho(q')^T D'(q')\rho(q') \\ C(q', \dot{q}') &= \rho(q')^T C'(q', \dot{q}')\rho(q') + \rho(q')^T D'(q')\dot{\rho}(q') \\ g(q') &= \rho(q')^T g'(q') \\ \rho(q') &= [\tfrac{\partial\sigma(q)}{\partial q}, \tfrac{\partial\sigma(q)}{\partial q'}] \\ \dot{\rho}(q') &= \tfrac{d(\rho(q'))}{dt} \end{aligned} \tag{7}$$

The ordinary differential equation (6) describes the planar five-bar parallel robot dynamic behaviour. This can be written in state variable representation (8), where $x = [x_1, x_2, x_3, x_4]^T = [q_1,\ q_2,\ \dot{q}_1,\ \dot{q}_2]^T \in R^4$ is the state vector of the system, x_1, x_2 and x_3, x_4 are the positions and velocities of links 1 and 2 respectively, $u = [u_1, u_2]^T \in R^2$ is the control vector, and f_s is the right term of the planar five-bar parallel robot dynamics (6) represented in the state vector *x*.

$$\dot{x} = f_s\left(x, u, p_s\right) \tag{8}$$

In order to follow the desired task or desired trajectory by the end-effector, a control law must be considered. It is well known that the conventional PID controller does not guarantee the error asymptotic stability. Nevertheless, it is widely accepted by industry for automated machines. More than 90% of control loops implemented in industry use PID controllers [O'Dwyer, 2006]. With this in mind, two PID controllers are considered for the planar five-bar mechanism which can be represented by (9), where $e_i(t) = \overline{q}_i - x_i$, $\dot{e}_i(t) = \dot{\overline{q}}_i - x_{i+2}$ are the positioning and velocity error vector, $\overline{q},\ \dot{\overline{q}} \in R^2$ are the desired positioning and velocity vector, $k_p = [k_{p_{11}},\ k_{p_{22}}]$, $k_i = [k_{i_{11}},\ k_{i_{22}}]$, $k_d = [k_{d_{11}},\ k_{d_{22}}] \in R^{2\times 2}$ are diagonal positive definite matrices.

$$u_i = k_{p_{ii}} e_i(t) + k_{i_{ii}} \int_0^{t_f} e_i(t)dt + k_d \dot{e}_i(t), \quad for\ i = 1, 2 \tag{9}$$

2.2 Performance criteria

With the lack of the error asymptotic stability proof for the PID controller, the convergence of the position error to zero can not be guaranteed. Then, the positioning accuracy is stated as the performance index J_c to be minimized. This index is shown in (10), where $n = 2$ represents the parallel robot d.o.f.. $e_i(t) = \overline{q}_i - x_i$ is the position error between the angular position x_i and the desired position $\overline{q}_i$; $e_{i\,\max}$ represents the maximum error $e_i(t)$ in the time interval $[0, t_f]$ and t_f is the final time.

$$J_c = \sum_{i=1}^{n} \frac{\int_0^{t_f} e_i^2(t)dt}{\int_0^{t_f} e_{i\,\max}^2 dt} = \sum_{i=1}^{n} \frac{\int_0^{t_f} e_i^2(t)dt}{e_{i\,\max}^2 t_f} \tag{10}$$

Typically, the structural performance index is based on kinematic considerations [Stan, et. al., 2007]. This is not appropriate since the manipulator dynamic is not considered. Yoshikawa [Yoshikawa, 1985] introduced the concept of dynamic manipulability that depends on the Jacobian and the inertia matrix of the manipulator. This concept provides a geometric representation and a measure of the ability of a robot for performing end-effector accelerations along any task space direction for a set of joint torques. A measure of the dynamic manipulability is the volume of the dynamic manipulability ellipsoid [Yoshikawa, 1985]. The task based kinematic design proposed by Kim [Kim, 2006] was the first one, to the best of our knowledge, in using the dynamic manipulability measure as the performance index to be optimized. The link lengths of a parallelogram five-bar planar parallel robot were found by maximizing the dynamic manipulability measure. Nevertheless, the author considered a kinematic design optimization problem, where the constraint concerning the closed-loop nonlinear dynamic system behavior was not considered as an equality constraint.

In this paper the dynamic manipulability measure J_m (11) is used as other performance index to be optimized.

$$J_m = \frac{\int_0^{t_f} \frac{M}{\Psi} dt}{\int_0^{t_f} dt} = \frac{\int_0^{t_f} \frac{\sqrt[n_J]{\det(J(D^T D)^{-1} J^T)}}{\frac{tr(J(D^T D)^{-1} J^T)}{n_J}} dt}{t_f} = \frac{\int_0^{t_f} \frac{\sqrt[n_J]{\lambda_1 \lambda_2 \cdots \lambda_{n_J}}}{\frac{(\lambda_1 + \lambda_2 + \cdots + \lambda_{n_J})}{n_J}} dt}{t_f} \tag{11}$$

where t_0 and t_f are the initial and final time. $\det(\bullet)$ and $tr(\bullet)$ are the determinant and trace of the matrix $(\bullet)$ respectively, D is the inertia matrix. J is the Jacobian matrix (12) developed in [Liu, et. al., 2006], $n_J = 2$ represents the rank of J . λ_i represents an eigenvalue of $J(D^T D)^{-1} J^T$, M is the dynamic manipulability measure, Ψ represents the arithmetic mean average of the eigenvalues of $J(D^T D)^{-1} J^T$ and Ψ≥M is held.

$$J = J_q^{-1} J_p, \tag{12}$$

where:

$$J_p = \begin{bmatrix} (x_p - a_1 \cos q_1) & (y_p - a_1 \sin q_1) \\ (x_p - a_5 - a_2 \cos q_2) & (y_p - a_2 \sin q_2) \end{bmatrix}$$

$$J_q = \begin{bmatrix} J_{q_{11}} & 0 \\ 0 & J_{q_{22}} \end{bmatrix}$$

$$J_{q_{11}} = (y_p \cos q_1 - x_p \sin q_1) a_1$$

$$J_{q_{22}} = (y_p \cos q_2 + (a_5 - x_p) \sin q_2) a_2$$

It is important to note that J_m and J_c are dimensionless and their magnitudes are in the interval $[0,1]$.

2.3 Constraints

The constraints associated to the controller design consist of establishing a torque bound for each actuator in order to avoid actuator saturation. These constraints are shown in (13), where $u_j(x,\overline{q},\dot{\overline{q}},p_c,t)$ is the torque at the motor j , and $u_{j\max}$ is the maximum torque for the $j-th$ motor.

$$\left| u_j(x,\overline{q},\dot{\overline{q}},p_c,t) \right| \le u_{j\max} \quad \text{for } j = 1 \text{ to } n \tag{13}$$

The five-bar Grashof criteria [Ting, 1986] are the constraints associated to the structure and they are shown in (14). These criteria state that the planar five-bar parallel robot is a double-crank linkage if these criteria are fulfilled. Some kinds of inverse and forward kinematic singularities [Liu, et. al. 2006] are avoided by the fulfillment of Grashof criteria.

$$\begin{aligned} a_5 + a_1 + a_2 - a_3 - a_4 &\le 0 \\ a_1 + a_2 - a_3 &\le 0 \\ a_1 + a_2 - a_4 &\le 0 \\ a_3 - a_5 &\le 0 \\ a_4 - a_5 &\le 0 \\ a_5 - a_{5_{\max}} &\le 0 \\ -a_1 &< 0 \\ -a_2 &< 0 \end{aligned} \tag{14}$$

The link lengths of the planar five-bar parallel robot are represented by a_i for $i = 1,\ldots,5$, $a_{5_{\max}}$ is the maximum length of a_5. Other important constraint must include the temporary behavior of the closed-loop nonlinear dynamic system in all time interval $[0, t_f]$. This constraint can be expressed by differential equations as follows:

$$\dot{x} = f_{sc}(x, \overline{q}, \dot{\overline{q}}, p_c, p_s), \tag{15}$$

where f_{sc} is the nonlinear system (8) with the feedback controller (9).

2.4 Design variables and development task

In the task based mechatronic system design approach is necessary to select both the structural design parameter vector $p_s \in R^{n_s}$ of the system and the controller design parameter vector $p_c \in R^{n_c}$. The structural parameters of the planar five-bar mechanism, which could affect the performance indexes (10) and (11), are the masses m_i, inertias I_i, mass center distances l_{ci} and the lengths a_i of the links. In this paper it is assumed that each link has a rectangular form with a width of $wd = 0.03(m)$. The structural design parameter vector $p_s = [a_1, a_2, a_3, a_4, a_5]^T \in R^5$ which includes the link lengths a_i is optimized. The masses m_i and mass center distances l_{ci} are considered constants. The p_s vector affects in a direct way the inertias I_i:

$$I_i = \tfrac{m_i(a_i^2 + wd^2)}{12}(Kgm^2) \qquad \text{for} \quad i = 1\ldots 4, \tag{16}$$

The PID controller gains are the controller design parameters to be optimized. These parameters are grouped in $p_c = [k_{p_{11}},\ k_{i_{11}},\ k_{d_{11}},\ k_{p_{22}},\ k_{i_{22}},\ k_{d_{22}}]^T \in R^6$.

A circle in the Cartesian space is given as the task specification to be followed by the end-effector of the planar five-bar parallel robot. This trajectory is shown in (17), where $c_x = 0.15(m)$, $c_y = 0.3(m)$ are the circle center coordinates in the Cartesian space, $r = 0.1(m)$ is the circle radius, t is the time and $f_{rec} = 0.25(Hz)$ is the frequency with a period of $4(s)$.

$$
\begin{aligned}
x_p &= c_x + r\cos(2\pi f_{rec} t) \\
\dot{x}_p &= -2\pi f_{rec} r \sin(2\pi f_{rec} t) \\
y_p &= c_y + r\sin(2\pi f_{rec} t) \\
\dot{y}_p &= 2\pi f_{rec} r \cos(2\pi f_{rec} t)
\end{aligned} \tag{17}
$$

The inverse kinematic model (IKM) is required to transform the trajectory from the Cartesian coordinates (desired task) to desired joint coordinates $(x_p, y_p) \Rightarrow \bar{q}$, $(\dot{x}_p, \dot{y}_p) \Rightarrow \dot{\bar{q}}$ in order to control the planar five-bar parallel robot. For this robot the IKM is developed in [Liu, et. al., 2006].

2.5 Optimization problem

The task based mechatronic design is stated as a nonlinear dynamic optimization problem in order to find both the optimal structure design parameters $p_s = [a_1, a_2, a_3, a_4, a_5]^T$ and the optimal controller design parameters $p_c = [k_{p_{11}}, k_{i_{11}}, k_{d_{11}}, k_{p_{22}}, k_{i_{22}}, k_{d_{22}}]^T$ of the planar five-bar parallel robot and its controller. Both optimal design parameters simultaneously minimize the positioning error (10) and maximize the dynamic manipulability measure (11) in order to execute a given task (17). This design is subject to the constraints at the motor torque (13), the Grashof criteria (14) and the dynamic behavior of the closed-loop nonlinear dynamic system (15) represented by differential equations with the initial condition vector $x(t_0) = x_{t_0}$.

A weighted-sum approach is used for handling both objectives into a single objective. Then the TBMSDA optimization problem can be formulated as follows:

$$\underset{p_s,\, p_c}{Max}\, (\mu_m J_m - \mu_c J_c) \tag{18}$$

subject to:

- The given task (17).
- Motor torque constraint (13).
- Grashof criteria (14).
- Closed-loop nonlinear dynamic system (15).
- Initial condition $x(t_0) = x_{t_0}$ of the differential equation (15).

where μ_c , μ_m are weights which assign relative importance among other performance indexes. As the performance indexes J_m and J_c are dimensionless and their magnitudes are in the interval $[0,1]$, the weights are in the interval (0,1]. It should be noticed that $\mu_c > \mu_m$ means that in the optimization process it is more important to minimize the position error than to maximize the dynamic manipulability measure. The other way around, $\mu_m > \mu_c$ means that that in the optimization process it is more important to maximize the dynamic manipulability measure than to minimize the position error and $\mu_c = \mu_m$ assign the same importance to both performances.

3 Differential Evolution

A differential evolution (DE) strategy which is an EA, is used in order to effectively solve the TBMSDA optimization problem. DE uses real coding of floating point numbers instead of the binary coding [Goldberg, 1989] for representing the design variables. DE is a population-based optimizer that solves the starting point problem by sampling the objective function at multiple, randomly chosen initial points. The key parameters of the DE strategy are: NP - the population size which is the set of individuals, CR - the crossover constant, F - the weight applied to random differential (scaling factor). It is often difficult to choose values for NP, F and CR since it depends on the specific problem. However, some general guidelines are available in [Price, et. al., 2005]. The DE strategy mutates and recombines the original population vectors called parents to produce other population of NP trial vectors called mutant vectors. Each parent is crossed by its respectively mutant vector to create one child. A deterministic mechanism with elitism is used for selecting between the parent and the child. If there are constraints, another selection mechanism which considers a constraint handling technique is required. There exist ten different DE variants proposed by Price and Storn [Price and Storn, 2005]: DE/best/1/exp, DE/rand/1/exp, DE/rand-to-best/1/exp, DE/best/2/exp, DE/rand/2/exp, DE/best/1/bin, DE/rand/1/bin, DE/rand-to-best/1/bin, DE/best/2/bin, DE/rand/2/bin. Depending of the problem at hand, different DE variants is implemented. In [Mezura, et. al., 2006] the performance of a set of DE variants when solving a common benchmark was studied. They identify which variant is more suitable to solve both unimodal - separable/nonseparable and multimodal - separable/nonseparable problems. The results obtained in that paper showed that DE/rand/1/bin and DE/rand/2/bin were the algorithms which provided the best performance in the search of the global optimum for multimodal – separable/nonseparable. With these in mind and as the dynamic optimization problem (18) is multimodal due to the highly nonlinear constraint in the system dynamics (15), the DE/rand/1/bin is implemented in this paper.

In order to handle constraints, it is necessary to incorporate the simple constraint handling mechanism of Mezura [Mezura, 2005] in the DE/rand/1/bin. This mechanism guides the population to the feasible region which is based on three criteria:

I. Between two feasible individuals, the one with a higher fitness wins.
II. A feasible individual wins over an infeasible one.
III. Between two infeasible individuals, the one with the lowest amount of constraint violation wins, and its performance index is established as zero.

The pseudo code of the differential evolution algorithm with the constraint handling mechanism is given in Appendix 1.

4 TBMSDA Optimization Results

The TBMSDA optimization results are found by using the constraint handling differential evolution (CHDE) algorithm shown in Appendix 1. The scaling factor F takes a different value at each generation in the interval $0.3 \le F \le 0.9$ and the crossover constant CR is in the interval $0.8 \le CR \le 1$ at each optimization process (see Appendix 1). These values are obtained via a uniformly distributed pseudo random number function. In this integrated design, it is assumed that each link has a rectangular form with a width of $wd = 0.03(m)$. Other assumption is that masses and mass center distances of the links have the following values: $m_1 = 0.2168(kg)$, $m_2 = 0.1445(kg)$, $m_3 = 0.5059(kg)$, $m_4 = 0.5782(kg)$, $l_{c1} = 0.075(m)$, $l_{c2} = 0.05(m)$, $l_{c3} = 0.175(m)$, $l_{c4} = 0.2(m)$. The inertia moments are computed as (16) for $i = 1,\ldots,4$. These inertia moments will be different when the link lengths a_i are changing in the optimization process. The final time, which the performance index and the dynamic constraints are evaluated, is $t_f = 5(s)$. The initial condition vector of the dynamic system (15) for the planar five-bar parallel robot is $x(t_0) = [3.4906, 0, 0, 0]^T$.

As stated in Section 2, the trajectory tracking of a circle is the given task to be executed by the parallel robot. On the other hand, the constant parameters associated to the constraints (13) and (14) are selected as: $a_{5_{\max}} = 0.5(m)$, $u_{1\max} = u_{2\max} = 5(Nm)$. The weights in (18) are chosen as $\mu_c = 1$ and $\mu_m = 1$ in order to give equal importance in achieving the TBMSDA objectives. By selecting these weights, the CHDE algorithm searches for optimal parameters p_c^* and p_s^*.

Seven independent optimization processes or runs with different population size NP are performed. The termination criterion of the CHDE algorithm is when all individuals of the population converge to a unique solution with an error of 1×10^{-6}. This termination criterion is shown in the algorithm of Appendix 1. A comparative result of the computing time, the population size, the maximum number of generations G_{max} and the number of violated constraints among the runs is shown in Table 1. This computational time was obtained with a PC with a 3.4 GHz Pentium IV processor and 1.5GB of RAM. It is observed, that when the population size is increased, more computing time is spent. The worst case is when NP=5 because three constraints in (13) are violated.

Table 1. Comparative results of the convergence of the CHDE algorithm with different population sizes

Run	Computing time (h)	NP	G_{max}	Violated constraints
0	1.10	5	14997	3
1	1.70	10	11613	0
2	8.86	25	9273	0
3	10.07	50	3690	0
4	20.53	100	2797	0
5	155.88	300	4610	0
6	163.30	500	2627	0

In Table 2, the optimal control structure parameters and the optimal performance index are presented for each run. It is important to note that in all runs, the Grashof criteria (14) are satisfied. The maximum performance index is found with 50, 100, 300 and 500 individuals. With these individuals, the CHDE algorithm converges to the same optimal control structure parameters and the same performance function value. As a result, the algorithm can explore the entire space of feasible solutions when the population size is chosen larger or equal to 50 individuals. On the other hand, the algorithm can not explore the entire space of feasible solutions when the population size is chosen less than 50 individuals because the population converges prematurely to another feasible solution.

The convergence graphics of the performance function mean of the run 1 to 6 are shown in Fig. 2. In the CHDE algorithm (see Appendix 1 or the third criterion of the constraint handling mechanism), when the solution is not feasible, that is, one or some constraints are violated; the performance index is established as zero. Then, it is observed in Fig. 2, when the population size *NP* is small, there are more generations (iterations) which the performance indexes are zero. That means that all individuals of the population at these generations do not converge to a feasible region. So, the best trade off between the computing time and the convergence of the optimal results is when the population size is of 50 individuals (NP=50).

Table 2. Optimal control structure parameters and optimal performance index for each run

Run	Optimal structure vector $p_s^* = [a_1, a_2, a_3, a_4, a_5]^T$	Optimal control vector $p_c^* = [k_{p_{11}}, k_{i_{11}}, k_{d_{11}}, k_{p_{22}}, k_{i_{22}}, k_{d_{22}}]^T$	Performance Index ($\mu_m J_m$-$\mu_c J_c$)
0	$[0.1881, 0.1289, 0.3422, 0.3574, 0.3791]^T$	$[31.8718, 65.3404, 24.8893, 40.7563, 7.4507, 1.3525]^T$	0.8614
1	$[0.1536, 0.1483, 0.3666, 0.4723, 0.4990]^T$	$[14.9732, 9.1253, 46.1561, 91.7972, 0.6129, 0.3275]^T$	0.9718
2	$[0.1524, 0.1430, 0.3498, 0.4676, 0.4854]^T$	$[22.9173, 27.7205, 82.8989, 99.9980, 0.9015, 0.5975]^T$	0.9766
3	$[0.1546, 0.1424, 0.3448, 0.4789, 0.5]^T$	$[26.5116, 27.8242, 99.999, 99.9999, 0.9745, 0.5836]^T$	0.9768
4	$[0.1546, 0.1424, 0.3448, 0.4789, 0.4999]^T$	$[26.5116, 27.8242, 99.999, 99.9999, 0.9745, 0.5836]^T$	0.9768
5	$[0.1546, 0.1424, 0.3448, 0.4789, 0.4999]^T$	$[26.5116, 27.8242, 99.999, 99.9999, 0.9746, 0.5836]^T$	0.9768
6	$[0.1545, 0.1424, 0.3449, 0.4789, 0.5]^T$	$[26.4814, 27.7952, 99.9407, 99.9977, 0.9769, 0.5845]^T$	0.9768

The desired trajectory and the end-effector behavior of the planar five-bar parallel robot with the optimum control structure parameters of the run 3 is shown in Fig. 3a. A error between them is observed in Fig. 3b. The trajectory tracking of the end-effector-effector presents a good performance with a Cartesian position error smaller than $0.0001(m)$. As the performance index of the TBMSDA optimization is formulated as a weighted-sum approach, the CHDE algorithm finds an unique solution that depends on the selection of the weights μ_c and μ_m. Then, the Cartesian position error could be diminished by decreasing the magnitude of the weight μ_m.

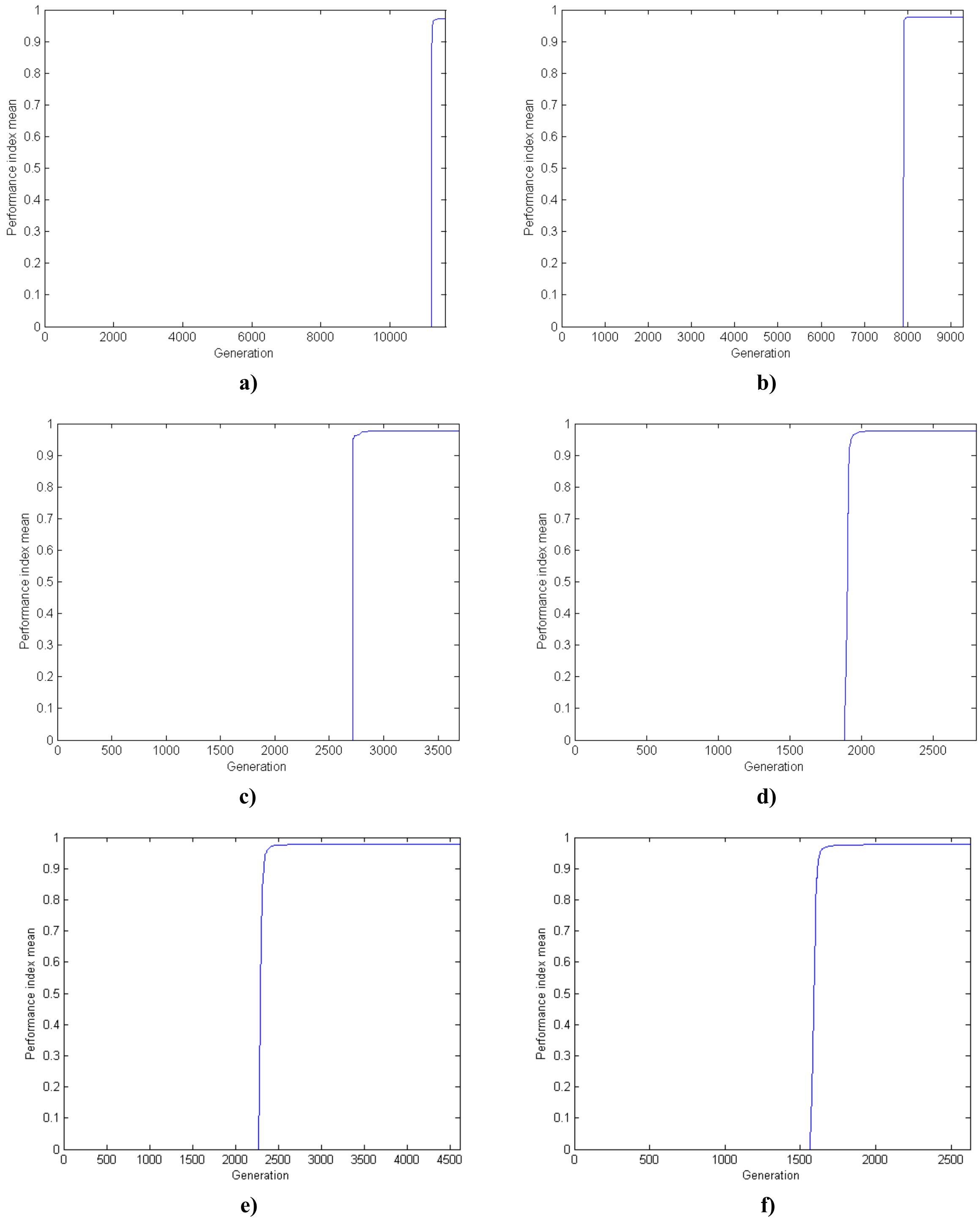


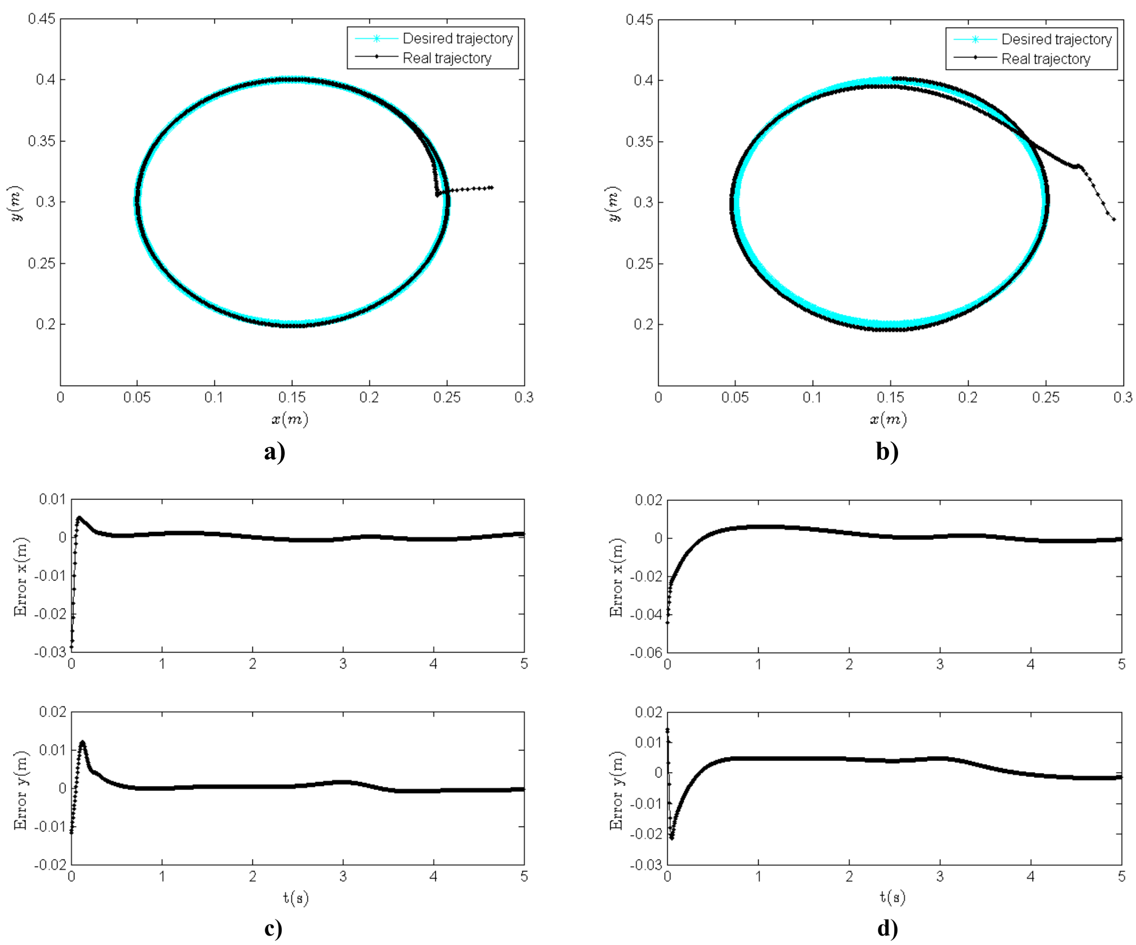
Fig. 2. Performance index mean of the population per generation. **a)** NP=10, **b)** NP=25, **c)** NP=50, **d)** NP=100, **e)** NP=300 and **f)** NP=500

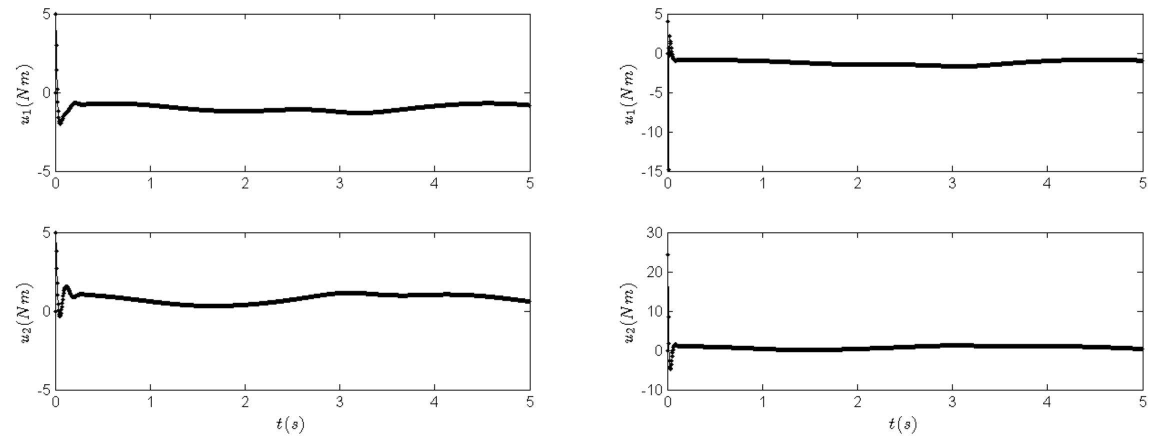
Note that at $t = t_0$ (at the beginning of the real trajectory), the end-effector of the parallel robot with optimum link lengths is near to the desired task (circle trajectory). Then it requires a minimum motor torque to reach the desired trajectory.

The applied torques at the actuated joints with the optimum control structure parameters of the run 3 are shown in Fig. 3c. These torques hit the motor torque constraints (13) at the first milliseconds. Note that (13) are fulfilled at all time.

On the other hand, in Fig. 3d – 3f, the planar five-bar parallel robot behavior with the design parameters of the worst case (run 0), are shown. With the design parameters of run 0, the Cartesian position error is larger (see Fig. 3e) and the end-effector of the parallel robot at $t = t_0$ is further away of the desired task. So, as the desired task is further away, the actuated joints require more power in order to reach the desired task. This produce that the motor torque constraints (13) are not satisfied (see Fig. 3f).

In short, the construction of the prototype, a redesign process of the structure and a calibration process of the controller system are not required because in this approach the optimal mechanical design parameters and the optimal controller design parameters are simultaneously obtained with the task based mechatronic system design approach. As a result, the TBMSDA approach dramatically reduces the design time of a mechatronic system.





e) **f)**

Fig. 3. Desired trajectory and end-effector behavior of the planar five-bar parallel robot with the optimal control structure parameters **a)** of the run 3, **b)** of the run 0. Cartesian position error between the desired task and the end-effector behaviour **c)** of the run 3, **d)** of the run 0. The input torques for the trajectory tracking of a circle with optimal control structure parameters **e)** of the run 3, **f)** of the run 0

5 Conclusion

Parallel robot design is a huge challenge due to the complexity of the dynamic model and the presence of many singularities. Consequently, it is not an easy task to find the mechanical and the control parameters in such a way as to optimize the system performance. Hence, in this paper, a task based mechatronic system design approach for the optimal design of a planar five-bar parallel robot and its controller is developed. A nonlinear dynamic optimization problem is formulated in order to find the optimal PID controller gains and the optimal link lengths of the parallel robot. Both design parameters simultaneously maximize the dynamic manipulability (a dexterity measure) and minimize the position error for the end-effector trajectory tracking of a desired task. Static and dynamic constraints are considered, such as, the motor torque constraints, the Grashof criteria and the nonlinear dynamic system behavior.

The CHDE algorithm is used in order to solve the TBMSD problem. This algorithm finds an optimum selection of the link lengths and the PID controller gains for following a desired task. In the particular case of the planar five-bar parallel robot, the performance on the positioning errors, dexterity and applied torque for a given task are improved in a considerable time (10h) with the proposed approach.

One of the major difficulties in applying this approach lies in finding a dynamic model of the parallel robot that represents the real system. On the other hand with the TBMSDA approach dramatically reduces the design time of a mechatronic system because the construction of the system, a redesign process of the system structure and a calibration process of the controller system are not required.

Several runs of the algorithm with different population size NP are compared. For the particular case of the planar five-bar parallel robot, the CHDE algorithm can not converge to the best solution with a population size less than 50 individuals. As a conclusion, the algorithm can not explore the entire space of feasible solutions with this population size. With 5 individuals in the population, the Grashoft criteria are fulfilled but some of the motor torque constraints are violated. This gives the worst case of the runs. When the population size is small, there are more generations which the population does not converge to feasible solutions. With a population greater than or equal to 50 individuals the algorithm converges to the same optimum parameters. So, the best trade off between the computing time and the convergence of the optimal results is when NP=50.

The CHDE algorithm converges to the same optimum parameters with a population greater than or equal to 50 individuals. This solution depends on the given weights for each performance index.

Future work will include multicriteria optimization [Osyczka, 1984] in order to provide all design solutions for which a single objective can not be improved without compromising some other objectives (Pareto optimal set). Hence, the Pareto optimal set will help to the decision maker. In addition, the experimental results of the TBMSDA will be carried out. However, it will be necessary to implement another optimization process in order to adjust from the optimal structure parameters to geometry of the links, as m_i and l_{ci} will remain constant as considered above.

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Appendix 1: Pseudo code of the differential evolution algorithm [Mezura, et. al., 2006]

Choosing the population number NP and the parameter number D to be optimized.
Pseudo randomly initializing CR in an interval [0.8, 1].
Create a random initial population $\vec{x}^i_{G=1}$ for $i = 1, \ldots, NP$.
Do
  Pseudo randomly initializing F in an interval [0.3, 0.9].
  For $i = 1$ **to** NP
    Selecting three different individuals r_1, r_2, r_3 of the population G_i in a pseudo random way.
    Selecting an integer number in a pseudo random way between [1, D], which is stored in the *jrand* variable.
    For $j = 1$ **to** D
      Uniform crossover:
      If ((rand(0,1) $\leq$ CR) **or** (j = jrand))
        Mutation:
        $\vec{u}^i_j = \vec{x}^{r_3}_{j,G} + F\left(\vec{x}^{r_1}_{j,G} - \vec{x}^{r_2}_{j,G}\right)$
      Else
        $\vec{u}^i_j = \vec{x}^i_{j,G}$
      End If
    End For
    Constraint handling mechanism:
    If ((constraints evaluated at $\vec{x}^i_{j,G+1}$ = 0) and (constraints evaluated at $\vec{u}^i_j$ = 0))
      If (performance function evaluated at $\vec{u}^i_j$ is better than performance function evaluated at $\vec{x}^i_{j,G+1}$)
        $\vec{x}^i_{j,G+1} = \vec{u}^i_j$
      Else
        $\vec{x}^i_{j,G+1} = \vec{x}^i_{j,G}$
      End If
    Else
      If ((constraints evaluated at $\vec{x}^i_{j,G+1}$ =0) and (constraints evaluated at $\vec{u}^i_j$ $\neq$ 0))
        $\vec{x}^i_{j,G+1} = \vec{x}^i_{j,G}$
      Else
        If ((constraints evaluated at $\vec{x}^i_{j,G+1}$ $\neq$ 0) and (constraints evaluated at $\vec{u}^i_j$ =0))
          $\vec{x}^i_{j,G+1} = \vec{u}^i_j$
        Else

If (constraints evaluated at $\vec{x}^{i}_{j,G+1}$ $\leq$ constraints evaluated at $\vec{u}^{i}_{j}$)

$\vec{x}^{i}_{j,G+1} = \vec{x}^{i}_{j,G}$

Else

$\vec{x}^{i}_{j,G+1} = \vec{u}^{i}_{j}$

End If

End If

End If

Performance function at $\vec{x}^{i}_{j,G+1}$ is zero, that is, $J(\vec{x}^{i}_{j,G+1}) = 0$

End If

End For

while $\left(\max\left(J(\vec{x}_{G+1})\right) - \min\left(J(\vec{x}_{G+1})\right)\right) > 1\times10^{-6}$

References

1. **Alvarez, J., C. A. Cruz** and **E. Portilla**, "Parametric optimal design of a pinion-rack based continuously variable transmission", IEEE/ASME International Conference on Advance Intelligent Mechatronics, 2005, pp.24-28.
2. **Amerongen, J.,** "Mechatronic design", *Mechatronics*, 13(10): 1045-1066 (2003).
3. **Betts, J. T.,** *Practical methods for optimal control using nonlinear programming*. SIAM, Philadelphia; 2001.
4. **Ghorbel, F. H., O. Chételat, R. Gunawardana** and **R. Longchamp**, "Modeling and set point control of closed-chain mechanisms: theory and experiment", *IEEE Transactions on Control Systems Technology*, 8(5): 801-815 (2000).
5. **Goldberg, DE.,** *Genetic algorithms in search, optimization and machine learning*, Addison-Wesley: Reading, MA, U.S.A., 1989.
6. **Hunt, K. H.,** "Structural kinematics of in-parallel-actuated robot arms". *ASME Journal of Mechanisms, Transmissions and Automation in Design*, 105(4): 705-712 (1983).
7. **Kim, J. Y.,** "Task based kinematic design of a two DOF manipulator with a parallelogram five-bar link mechanism", *Mechatronics*, 16(6): 323-329 (2006).
8. **Liu, X-J., J. Wang** and **G. Pritschow**, "Kinematics, singularity and workspace of planar 5R symmetrical parallel mechanisms", *Mechanism and Machine Theory*, 41(2): 145-169 (2006).
9. **Ma, Ou** and **J. Angeles**, "The concept of dynamic isotropy and its applications to inverse kinematics and trajectory planning", *IEEE International Conference on Robotics and Automation*, 1990, 481-486.
10. **Mezura, E.** and **C. A. Coello**, "A simple multimembered evolution strategy to solve constrained optimization problems", *IEEE Transactions on Evolutionary Computation* , 9(1), 2005, 1-17.
11. **Mezura, E., J. Velázquez** and **C. A. Coello**, "A comparative study of differential evolution variants for global optimization", *Proceedings of the 8th annual conference on Genetic and evolutionary computation*, 2006, 485-492.
12. **Osyczka, A.,** *Multicriterion optimization in engineering*, John Wiley & Sons, New York, 1984.
13. **O'Dwyer, A.,** *Handbook of PI and PID controller tunning rules,* Imperial College Press, London, 2006.
14. **Portilla, E. A., E. Mezura, J. Alvarez, C. A. Coello** and **C. A. Cruz**, "Integration of structure and control using an evolutionary approach: An application to the optimal concurrent design of a CVT", *International Journal for Numerical Methods in Engineering*, 71(8): 883-901 (2007).
15. **Price, K. V., R. M. Storn** and **J. A. Lampinen**, *Differential evolution approach to global optimization*, Springer - Verlag Berlin Heidelberg, Germany, 2005.
16. **Ravichandran, Th., D. Wang** and **G. Heppler**, "Simultaneous plant-controller design optimization of a two-link planar manipulator". *Mechatronics*, 16(3-4): 233-242 (2006).
17. **Spong, M. W.** and **M. Vidyasagar**, *Robot dynamics and control*, John Wiley and Sons, New York; 1989.

18. **Stan, S.-D., V. Mătieş** and **R. Bălan**, "Optimization of 2 dof micro parallel robots using genetic algorithms", *Proceedings of International Conference on Mechatronics*, 2007, pp. 1-6.
19. **Sudhendu, R.** and **H. Asada**, "Integrated structure/control design of high speed flexible robots based on time optimal control". *Journal of dynamic systems, measurement, and control*, 117(4), 503-512 (1995).
20. **Ting, K. L.,** "Five-bar Grashof criteria". *Journal of Mechanism, Transmissions and Automation in Design*, 108, 533-537 (1986).
21. **Yoshikawa, T.,** "Dynamic manipulability of robotic mechanism", *J. Robotic Systems*, 2(1), 113-124 (1985).

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