

# Heavy Tailed Network Delay: An Alpha-Stable *Model Retardo de Cola Pesada: Un Modelo Alfa-Estable*

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## Abstract

Adequate quality of IP services demands low transmission delays. However, packets traveling in a network are subject to a variety of delays that, in real-time applications, severely degrade the quality of service (QoS). This paper presents a general end-to-end delay model suitable for a multi-node path in the presence of heavy-tailed traffic. The proposed methodology is based on an alpha-stable random variable description. This allows us to define a network processing measure that relates the delay spread to the heavy tail characteristics of the traffic, the number of nodes in a route, and the processing speed at the nodes.

**Key words:** Network delay, Jitter, Alpha stable traffic, QoS.

## Resumen

Una calidad de servicio adecuada en redes IP demanda retardos de transmisión bajos. Sin embargo los paquetes que viajan en la red están sujetos a una variedad de retardos que, en el caso de servicios en tiempo real, degrada la calidad de servicio severamente. En este artículo se presenta un modelo general para el retardo de extremo a extremo en trayectorias multinodales y tráfico de cola pesada. La metodología propuesta se basa en una descripción alfa-estable. Esto permite definir una medida del procesamiento de la red que relaciona la variación del retardo con las características de cola pesada del tráfico, el número de nodos y la velocidad de procesamiento en los nodos.

**Palabras clave:** Retardo de red, Jitter, Tráfico alfa estable, Calidad de Servicio.

## 1 Introduction

Delay is recognized as an important impairment degrading communication performance. In particular, for real-time applications in a packet network, the service quality perception is very much dependent on the delay characteristics [1].

When packets travel through a network, they are subject to multiple-delay phenomena. For instance, in some real-time applications, packets are formed as the concatenation of active voice coded samples. When a packet is formed by samples originally separated by inactive periods, the first samples are said to experience a leftover delay. Also, in some protocols, corrupted or dropped-off packets are retransmitted. These can be perceived as out-of-sequence packets, and they can be characterized as exceedingly delayed packets. A packet in the network may also experience contention or queue-induced delays.

The fast growth of Internet traffic and the demand for real-time transmission services make the timing-error accumulation problem particularly acute. It is also recognized that current web networks are subject to a number of heavy tail characteristics so that, traditional Poisson/exponential models are no longer applicable [2], [3], and some design and analysis tools that consider the heavy-tailed nature of telecommunications need to be developed [4], [5].

Previous work on alpha-stable models presented in [6][7][8] by Gallardo et. al., Karasaridis et. al., and Harmantzis et. al., respectively, run through the queuing analysis of one node.

This proposal is based on the fact that Internet traffic delay requirements are related to *end-to-end network delay*, rather than to one node delay. In practice, the operation-processing speed may vary from node to node, and aggregated traffic presents a wide mixture of characteristics. For the sake of simplicity, in this paper, we assume that traffic characteristics are the same for all nodes along the paths in the network. The proposed model focuses on the heavy-tailed delay phenomenon from an alpha stable methodology perspective. Delay increases as the packet progresses in the network, and the delay is dependent on issues including the number of nodes in the path, the network traffic characteristics, and the processing speed of each node of the route. In this paper, we propose a network measure that relates these parameters to the delay performance.

## 2 The model

It has been observed that many network-related parameters (such as file lengths, required CPU times to complete a job, non-voice holding times, separation time between packets in a network, Ethernet packet count per time unit, etc.) exhibit heavy-tailed behavior [4], [5].

Heavy-tailed traffic characteristics, along with the number of stages in a route, impact the overall packet delay. For instance, a transmitted packet  $p(t)$ , as it progresses along a route, will experience [9] in each node  $i$  a delay  $\tau_i$  which is dependent on the number  $P_i$  of preexisting packets in the node and the service processing time  $T_i$  per packet in the node (this is  $\tau_i = T_i P_i$ ). As file sizes are heavy-tailed distributed, an equal packet size assumption supports the claim of a Paretian number of arrived packets per time unit. Thus, we assume that the number of packets arriving in a time window exhibit a heavy-tailed behavior which can conveniently be modelled by the use of a survivability function of the form

$$\Pr\{P_i > P\} \sim \left(\frac{1}{P}\right)^\alpha . \quad (1)$$

This assumption implies, to ease formulation, the existence of very large buffers in the nodes and that files are instantly received. Thus, the accumulated delay  $\tau$  along a route can be considered to be the linear combination of multiple Pareto distributed variables, and after  $N$  routing stages, the received packet will be  $p(t-\tau)$  where

$$\tau = \sum_{i=1}^N P_i T_i . \quad (2)$$

Note that a proportion of the messages arrived to node  $i$  will have that node as a final destination while other packets will be diverted through several node outputs. Thus, only a proportion  $\eta$  of packets will travel to the next node of the concerned path, and  $P_{i+1}$  will be of the form  $P_{i+1} = \eta P_i + A_{i+1}$  where  $A_{i+1}$  will denote the packets proceeding from other uncorrelated sources. For the sake of simplicity, in our approximation it is assumed that the number of packets arriving to different nodes is independent. This approximation allows us to deal with an otherwise mathematically cumbersome problem [10],[11], and it has also been applied in other end-to-end delay modeling approaches [12].

It has been shown that for non-negative heavy-tailed random variables, the limit distribution of the sum can be alpha-stable distributed [5]. Also, in 1925, Lévy showed [13] that Pareto laws belong to the so-called stable-Paretian or stable non-Gaussian distributions, whose characteristic function  $C_{\alpha,\beta}^{\gamma,\mu}(\zeta) = E_\tau \{\exp(j\zeta\tau)\}$  is presented in the appendix.  $\alpha$ -stable random variables  $P$  are also conveniently denoted in the form  $P \sim S_\alpha(\gamma, \beta, \mu)$ .

### 2.1. Delay-Location and Delay Spread Criteria

Since delay has been expressed as  $\tau = \sum_{i=1}^N P_i T_i$ , it is convenient to examine some properties for the linear combination of  $\alpha$ -stable random variables. In particular, it can be noted that by using appendix A.3 formulation can easily be verified that if two independent random variables  $P_1$  and  $P_2$  are  $S_\alpha(\gamma, \beta, \mu)$ , then their linear combination  $T_1 P_1 + T_2 P_2$  is also  $\alpha$ -stable, [15], and it is a fact that

$$T_1 P_1 + T_2 P_2 \sim S_\alpha(\gamma(T_2^\alpha + T_1^\alpha)^{1/\alpha}, \beta, \mu(T_2 + T_1)). \quad (3)$$

This property enables total delay  $\tau = \sum_{i=1}^N P_i T_i$  to be modeled as an  $\alpha$ -stable variable<sup>1</sup>. This is  $\tau \sim S_\alpha(\gamma_T, \beta, \mu_T)$ , where dispersion and location parameters, respectively become

$$\gamma_T = \gamma \left[ \sum_{i=1}^N T_i^\alpha \right]^{1/\alpha} = \gamma |\bar{T}|_\alpha, \quad \text{and} \quad \mu_T = \mu \left[ \sum_{i=1}^N T_i \right] = \mu |\bar{T}|_1. \quad (4)$$

Since  $P_i \geq 0$ , then the distribution of the delay is totally skewed to the right when  $\beta = 1$ .

Note that for a given a vector  $\bar{T} = (T_1, T_2, \dots, T_N)$ ,  $T_i \geq 0$ ,  $|\bar{T}|_\sigma = \left[ \sum_{i=1}^N T_i^\sigma \right]^{1/\sigma}$  is referred to as the  $\sigma$ -norm or Minkowsky norm of  $\bar{T}$ . For  $\sigma=2$ ,  $|\bar{T}|_\sigma$  coincides with the Euclidean norm.

Since quality perception depends on delay, it is desirable to maintain the accumulated delay figures within specified limits. Thus, delay location and dispersion ( $\mu_T$  and  $\gamma_T$ ) can respectively be upper bounded by quality criteria  $\mu_T \leq \mu_T^*$  and  $\gamma_T \leq \gamma_T^*$ . This leads, using (4), to design constraints of the form

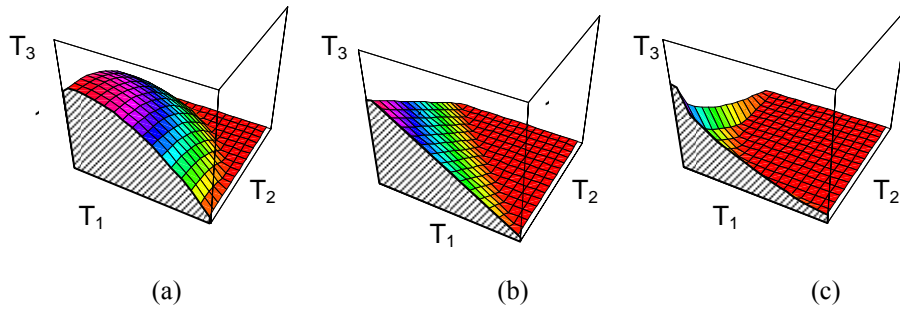
$$|\bar{T}|_1 = \left[ \sum_{i=1}^N T_i \right] \leq \frac{\mu_T^*}{\mu}, \quad (5-a)$$

and

$$|\bar{T}|_\alpha = \left[ \sum_{i=1}^N T_i^\alpha \right]^{1/\alpha} \leq \frac{\gamma_T^*}{\gamma}. \quad (5-b)$$

which, for a given delay allowance ( $\mu_T^*$  or  $\gamma_T^*$ ), limit the number of nodes  $N$  in a path and/or the processing times  $\{T_i\}$ . Since  $T_i \geq 0$ , equations (5-a) and (5-b) define, for  $\{T_i\}$ , feasibility regions contained in an  $N$ -dimensional  $\sigma$ -sphere that contracts as the stability index  $\alpha$  diminishes. Figure 1 illustrates normalized feasibility regions in a 3-D space  $(T_1, T_2, T_3)$  for different stability indexes.

<sup>1</sup> Evidence that overall delay exhibits a heavy-tailed behavior has been reported in [13]. Reported data were examined, and it was found that they can be adequately matched by an alpha stable distribution.



**Fig. 1.** Contraction of feasibility region as a function of the stability index. Feasible region; (a)  $N=3, \alpha=2$ ; (b)  $N=3, \alpha=1$ ; (c)  $N=3, \alpha=1/2$

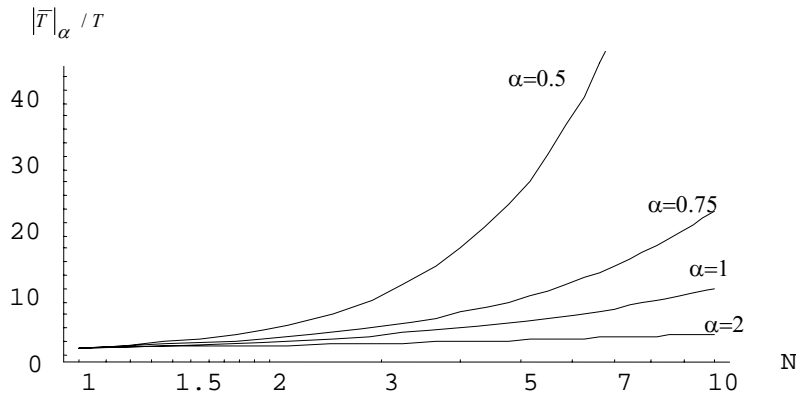
In the case of a homogeneous scenario ( $T_i = T$ ), the number of allowable nodes in the route is bounded by

$$N \leq \min \left\{ \left\lceil \frac{\mu_T^*}{\mu} \frac{1}{T} \right\rceil, \left\lceil \frac{\gamma_T^*}{\gamma} \frac{1}{T} \right\rceil^\alpha \right\}. \tag{6}$$

Although,  $\mu_T^*$  and  $\gamma_T^*$  can both be used as a network-design criteria, in real-time applications, quality perception degradation is often more sensitive to delay dispersion. Therefore, in this paper, we will emphasize the spread behavior  $\alpha$ .

Since  $\left[ \sum_{i=1}^N T_i^\alpha \right]^{1/\alpha} = |\bar{T}|_\alpha$  relates the traffic stability index  $\alpha$  and the number of nodes  $N$  in a path, as well as the processing speed in the nodes  $T_i$ , we name  $|\bar{T}|_\alpha$  the Network Processing Factor. We draw attention to the fact that, consistently, a similar network processing factor can be obtained using other methodology such as extreme value theory; see [12]. Note that in the case of all the nodes having the same processing speed, the norm  $|\bar{T}|_\alpha$  changes with  $1/\alpha$  power of the number of nodes. This implies, for strong heavy-tail traffic (i.e., for  $\alpha < 1$ ), a strong impact of the number of nodes on the network-processing factor. Figure 2 shows the variation of  $|\bar{T}|_\alpha / T$  with  $N$  for different  $\alpha$

Note that delay constraints in equation (5) impact on routing strategies requirements because packets should travel along paths contained within a feasibility region of the form  $|\bar{T}|_\alpha \leq Q$ , where  $Q$  can be specified according to a specified dispersion  $c$



**Fig. 2.** Normalized network-processing factor as a function of nodes and stability index.

## 2.2 Bound-Delay Criterion

In other network specification approaches, it is required that the delays should not exceed a threshold value  $\tau_{\max}$  for at least a proportion  $p$  of time, this is,

$$\Pr\{\tau \leq \tau_{\max}\} > p. \quad (7)$$

In order to guarantee an adequate quality of service, delay variations should be kept within specified constraints. For instance, perceived quality for voice services is considered excellent if delay does not exceed  $\tau_{\max}=150$  msec. The service is considered good for  $\tau_{\max}=300$  msec, while for delays beyond 300 msec the service is considered to be poor, and delays above 450 msec are considered unacceptable. Similarly, delay constraints for on-line video services have been suggested [1], [16], [17].

Network delay has been shown to be  $\tau \sim S_{\alpha}(\gamma|T|_{\alpha}, 1, \mu|T|_1)$ . Probability density functions for  $\alpha$ -stable variables do not have a closed form, except in a few particular cases. Nevertheless, the probability density function (PDF) and the characteristic function are a Fourier pair, and inequality (7) can be expressed as

$$F_{\tau}(\tau_{\max}) = \int_{-\infty}^{\tau_{\max}} \int_{-\infty}^{\infty} C_{\alpha,1}^{\gamma|\bar{T}|_{\alpha}, \mu|\bar{T}|_1}(\zeta) e^{-j\zeta x} d\zeta dx > p. \quad (8)$$

It is often advantageous to describe a distribution in terms of a standard representation that includes unitary dispersion ( $\gamma=1$ ) and a null-location parameter ( $\mu=0$ ).

Representation of the form  $x \sim S_{\alpha}(1, \beta, 0)$  can be achieved by recalling that alpha-stable variables have scaling and shifting properties. That is, for  $x \sim S_{\alpha}(\gamma, \beta, \mu)$ , and for any positive constant  $w$ ,

$$wx \sim S_{\alpha}(w\gamma, \beta, w\mu), \quad (9-a)$$

and

$$\text{if } y=x-\mu, \quad y \sim S_{\alpha}(\gamma, \beta, 0). \quad (9-b)$$

Considering that  $\beta=1$  (because delays are non-negative) and since  $\tau \sim S_{\alpha}(\gamma|T|_{\alpha}, 1, \mu|T|_1)$ , using (9), we can define  $\tau^* = \frac{\tau}{\gamma|\bar{T}|_{\alpha}} - \frac{|\bar{T}|_1}{|\bar{T}|_{\alpha}} \frac{\mu}{\gamma}$ , to obtain  $\tau^* \sim S_{\alpha}(1, 1, 0)$ , then condition (7) can be expressed as

$$\Pr\{\tau \leq \tau_{\max}\} = \Pr\left\{\tau^* \leq \frac{\tau_{\max}}{\gamma|\bar{T}|_{\alpha}} - \frac{|\bar{T}|_1}{|\bar{T}|_{\alpha}} \frac{\mu}{\gamma}\right\} = \int_{-\infty}^{\tau_{\max}^*} \int_{-\infty}^{\infty} C_{\alpha,1}^{1,0}(\zeta) e^{-j\zeta x} d\zeta dx > p. \quad (10)$$

where  $\tau_{\max}^* = \frac{\tau_{\max}}{\gamma|\bar{T}|_{\alpha}} - \frac{|\bar{T}|_1}{|\bar{T}|_{\alpha}} \frac{\mu}{\gamma}$ . Note that in the case of a homogeneous scenario ( $T_i=T$  for all  $i$ ),  $|\bar{T}|_{\alpha} = N^{1/\alpha}T$  and

$\tau_{\max}^* = \frac{\tau_{\max}}{\gamma N^{1/\alpha}T} - \frac{\mu}{\gamma} N^{\frac{\alpha-1}{\alpha}}$ . Since distributions are monotonic non-decreasing functions, it is seen that the higher the

processing time  $T$ , the more difficult it will be to satisfy the  $\Pr\{\tau^* \leq \tau_{\max}^*\} > p$  requirement. Note that the overall delay performance is dependent not only on the traffic characteristic parameters (such as  $\alpha, \gamma, \mu$  for an individual node) but also on the number of nodes and their processing speed through the defined network-processing factor.

It has already been pointed out that closed general expressions for integrals (8) and (10) do not exist, and distributions should be obtained numerically. Nevertheless, it has been shown [14], [18] that convenient reparametrization (out-

lined below) allows numerical calculation of  $\square$ -stable distributions. In particular, it has been shown that for a stability index  $\alpha \neq 1$ , the characteristic function of a random variable  $x \sim S_\alpha(1, \beta, 0)$  can be expressed as

$$C_{\alpha,\delta}^\lambda(\zeta) = \exp\left\{-\lambda|\zeta|^\alpha \exp\left\{-i\frac{\alpha\delta\pi}{2}\text{sign}(\zeta)\right\}\right\}. \tag{11}$$

where the parametric representations (A.1) and (11) are related as  $\lambda = \gamma/\cos(\Phi(\alpha))$ ;  $\Phi(\alpha)=[\alpha-1-\text{sign}(\alpha-1)]\pi/2$  and for  $\beta=1$ ,  $\delta=2\Phi(\alpha)/\pi$ . The notation  $x \sim S_\alpha^*(\lambda, \delta)$  can be used to indicate that the characteristic function of the distribution of  $x$  is of the form (11), while  $f^*(x|\alpha|\square|\square)$  and  $F^*(x|\alpha|\square|\square)$  will denote the corresponding pdf and CDF. This leads to the following equivalences [14]

$$f(x;\alpha, 1, 0,1) \stackrel{D}{=} f^*(x;\alpha, 2\Phi(\alpha)/\pi, 1/\cos(\Phi(\alpha))), \tag{12-a}$$

and

$$F(x;\alpha, 1, 0,1) \stackrel{D}{=} F^*(x;\alpha, 2\Phi(\alpha)/\pi, 1/\cos(\Phi(\alpha))). \tag{12-b}$$

Also considering the properties of alpha-stable density functions (A.7) and (A.8) we obtain

$$f^*(x; \alpha, 2\Phi(\alpha)/\pi, 1/\cos(\Phi(\alpha))) \stackrel{D}{=} (1/\cos(\Phi(\alpha)))^{-1/\alpha} \tag{13-a}$$

$$f^*(x/(\cos(\Phi(\alpha)))^{-1/\alpha}; \alpha, 2\Phi(\alpha)/\pi, 1),$$

and

$$F^*(x; \alpha, 2\Phi(\alpha)/\pi, 1/\cos(\Phi(\alpha))) \stackrel{D}{=} F^*(x/(\cos(\Phi(\alpha)))^{-1/\alpha}; \alpha, 2\Phi(\alpha)/\pi, 1). \tag{13-b}$$

Of particular interest is the change of variable  $z=x/(\cos(\Phi(\alpha)))^{-1/\alpha}$  as  $z \sim S_\alpha^*(1, \alpha)$  and it has been reported [14], [18] that the distribution of  $S_\alpha^*(1, \alpha)$  has convergent series representations. Thus, the changes of variables

$$\tau^{**} = \frac{\tau^*}{\cos^{-1/\alpha}(\Phi(\alpha))} \quad \text{and} \quad \tau_{\max}^{**} = \frac{\tau_{\max}^*}{\cos^{-1/\alpha}(\Phi(\alpha))}$$

enable us to express distribution (13) as a convergent series and, in fact, the distribution of  $\tau_{\max}^{**}$  becomes

$$F_{\tau^{**}}(\tau_{\max}^{**}) = 1 - \frac{1}{\pi\alpha} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\Gamma(\alpha n + 1)}{n!n} \sin(n\pi\alpha) \tau_{\max}^{**\alpha n}, \quad \text{for } \alpha < 1 \text{ and } \tau_{\max}^{**} > 0, \tag{14}$$

and

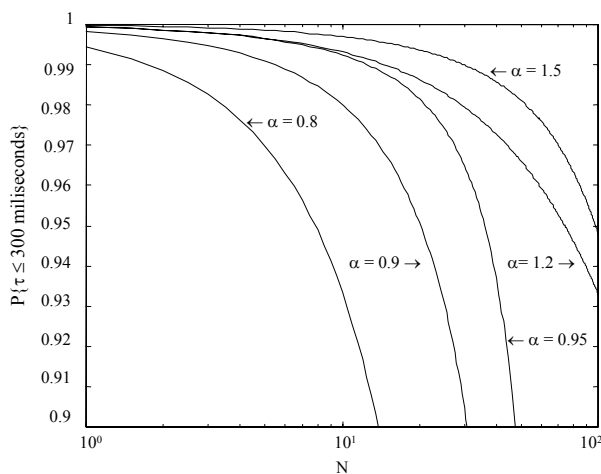
$$F_{\tau^{**}}(\tau_{\max}^{**}) = \begin{cases} \frac{1}{2} - \frac{\Phi(\alpha)}{\alpha\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\Gamma(\frac{n}{\alpha} + 1)}{n!n} \sin(\frac{n\rho\pi}{\alpha}) \tau_{\max}^{**n}, & \text{for } \alpha > 1 \text{ and } \tau_{\max}^{**} > 0, \\ 1 - \frac{1}{2} - \frac{\Phi(\alpha)}{\alpha\pi} - \frac{1}{\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\Gamma(\frac{n}{\alpha} + 1)}{n!n} \sin(\frac{n\rho'\pi}{\alpha}) \tau_{\max}^{**n}, & \text{for } \alpha > 1 \text{ and } \tau_{\max}^{**} < 0, \end{cases} \tag{15}$$

where  $\rho = \frac{\alpha + \delta}{2}$  and  $\rho' = \frac{\alpha - \delta}{2}$ .

### 3 Results

Under a bound-delay criterion, with expressions (14) and (15), we studied the impact of the path length (the number of nodes) on the accumulated delay as well as on its expected improvement due to the use of faster processing nodes. To illustrate these relationships, a homogeneous scenario with an even processing time per packet at each node was assumed ( $T_i$ ). It was assumed that nodes can process an average of some 1100 packets per second, with peaks of up to 3000 pkt/sec. We recall that variance does not exist for  $\alpha$ -stable distributions if  $\alpha < 2$ . Therefore, the dispersion parameter was set as the difference between the maximum and minimum expected values. Mean accumulated delays are consistent with experimentally reported figures, which also support the heavy-tailed nature of path delay, [1].

Figure 3 presents as a function of the number of nodes the probability of satisfying the criterion  $\tau \leq 300msec$  for a set of stability indexes of overall packet delay. Results show that the greater the number of nodes, the harder to satisfy the quality criterion. The figure also shows the adverse effect of the tail weight on the delay requirement. In particular, it can be seen that for very heavy tails ( $\alpha < 1$ ),  $\Pr\{\tau \leq \tau_{max}\}$  degrades very fast with the number of nodes. A homogeneous scenario with servers processing time  $T=10^{-6}sec/request$  was assumed.



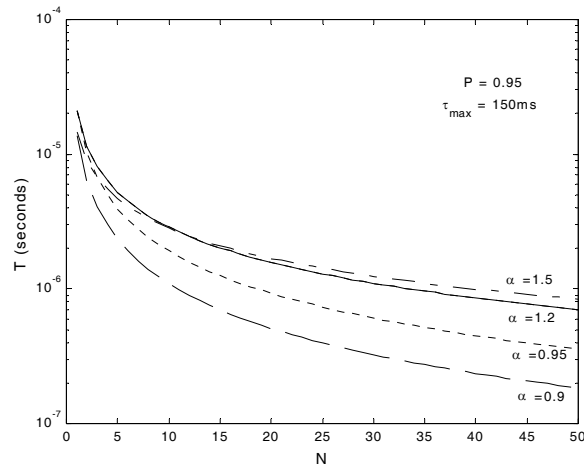
**Fig. 3.** Probability of keeping delay within specified constraint as a function of the number of nodes and stability index; where  $T=10^{-6}sec/request$

While the network-processing factor constraint (5) defines a feasibility region and establishes a relationship between the processing speed ( $1/T_i$ ) and the number of nodes  $N$ , distribution analysis (eq 14 and 15) allows studying the trade-off between processing speed and the path length for a  $\Pr\{\tau \leq \tau_{max}\} > p$  requirement. Thus, the figure 4-a shows, for different stability indexes and the  $\Pr\{\tau \leq 150msec\} = 0.95$  requirement, that faster processing speed allows larger number of nodes in a path giving more flexibility to the routing algorithms. However, this allowance diminishes as the stability index decreases. This penalty is higher for smaller stability indexes.

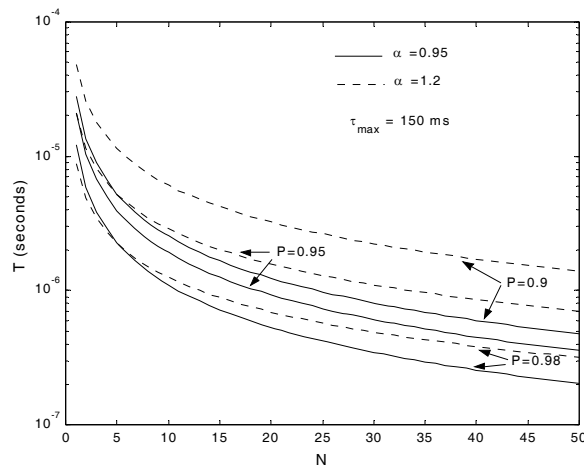
For different stability indexes and different probabilities  $\Pr\{\tau \leq 150msec\} = p$ , figure 4-b depicts the relationship between the processing time and the number of nodes. It can be seen how a decrease on the proportion of packets required to satisfy the  $\tau_{max}$  criterion diminishes the processing time needed. Also note that for a given processing time and stability index, a higher probability ( $p$ ) of packets satisfying the delay criterion will be achieved reducing the number of nodes.

### 4 Conclusions

Approximate delay analysis in the presence of heavy-tailed jitter has been presented. Delay performance is shown to depend (through a defined network-processing factor) on the traffic stability characteristics as well as on the number and speed of nodes along the transmission path. The heavier the traffic tail is (smaller  $\alpha$ ), the poorer the delay performance is. Therefore, routing solutions must be placed within a feasibility region defined by the adopted quality criterion and the heavy-tail characteristic. Proposed methodology based on alpha-stable analysis can be applied both to homogeneous or heterogeneous scenarios.



(a)



(b)

Fig. 4. Processing speed ( $1/T$ ) – Path length trade-off

a) impact of stability index b) impact of delay distribution  $p$ .



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## 5 Appendix. Mathematical Concepts

### Survival function

A survival function describes the probability that a variable  $X$  takes on a value greater than a number  $x$ . The survival function  $S(x)$  and the distribution function  $D(x)$  are related by

$$D(x) + S(x) = P(X \leq x) + P(X > x) = 1 \quad . \quad (\text{A.1})$$

**Heavy-tailed distributions**

A random variable  $X$  is a heavy-tailed distribution if its survival function decays as a

$$P[X > x] \sim x^{-\xi} \quad \text{as } x \rightarrow \infty, \quad 0 < \xi < 2. \tag{A.2}$$

Heavy-tailed distributions have the following properties: If  $\xi < 2$ , then the distribution has infinite variance. If  $\xi < 1$ , then the distribution has infinite mean. Thus, as  $\xi$  decreases, a probability mass increases at the tail of the distribution.

**Alpha-stable distribution**

The alpha-stable PDF and the cumulative distribution function (CDF) are not analytically expressible (a few exceptions are the Gaussian, Cauchy and Levy distributions). However, this family of distributions is represented by their characteristic function, as in the following equation:

$$C_{\alpha,\beta}^{\gamma,\mu}(\zeta) = \exp\left\{j\mu\zeta - \gamma^\alpha |\zeta|^\alpha [1 - j\beta \text{sign}(\zeta)\omega(\zeta, \alpha)]\right\}. \tag{A.3}$$

Where 
$$\omega(\zeta, \alpha) = \begin{cases} \tan \frac{\alpha\pi}{2}, & \text{if } \alpha \neq 1, \text{ and} \\ \frac{2}{\pi} \log|\zeta|, & \text{if } \alpha = 1, \end{cases} \quad \text{sign}(\zeta) = \begin{cases} 1 & \text{for } \zeta > 0 \\ 0 & \text{if } \zeta = 0 \\ -1 & \text{for } \zeta < 0 \end{cases}$$

$\mu \in (-\infty, \infty)$  is known as the location parameter;  $\gamma > 0$  is the scale or dispersion parameter;  $\beta \in [-1, 1]$  is the symmetry or skewness parameter defined as  $\beta = \lim_{x \rightarrow \infty} \frac{1 - F(x) - F(-x)}{1 - F(x) + F(-x)}$ , where  $F(x)$  is the corresponding distribution; and  $\alpha$  is known as the stability index.

**Properties of  $\alpha$ -stable random variables**

1.- Let  $X_1$  and  $X_2$  be independent random variables, then  $X_1 + X_2 \sim S_{\mathcal{E}}(\gamma, \beta, \mu)$  with

$$\gamma = \left(\gamma_1^\alpha + \gamma_2^\alpha\right)^{1/\alpha}, \quad \beta = \frac{\beta_1\gamma_1^\alpha + \beta_2\gamma_2^\alpha}{\gamma_1^\alpha + \gamma_2^\alpha}, \quad \mu = \mu_1 + \mu_2. \tag{A.4}$$

2.-For any  $0 < \alpha < 2$ ,

$$X \sim S_{\mathcal{E}}(\gamma, \beta, 0) \Leftrightarrow -X \sim S_{\mathcal{E}}(\gamma, -\beta, \mu). \tag{A.5}$$

3.- Let  $X \sim S_{\mathcal{E}}(\gamma, \beta, \mu)$  and let  $a$  be a real constant. Then

$$X_1 + a \sim S_{\mathcal{E}}(\gamma, \beta, \mu + a). \tag{A.6}$$

4.- Let  $X \sim S_{\mathcal{E}}(\gamma, \beta, \mu)$  and let  $a$  be a real constant. Then

$$aX \sim S_\alpha(|\alpha|^\gamma, \text{sign}(\alpha)\beta, \alpha\mu) \quad \text{if } \alpha \neq 1. \tag{A.6}$$

The associated PDF and CDF are respectively denoted as  $f(x; \alpha, \gamma, \beta, \mu)$  and  $F(x; \alpha, \gamma, \beta, \mu)$ . The PDF has correspondingly the following scaling and shifting properties:

$$f(x; \alpha, \gamma, \beta, \mu) \stackrel{D}{=} \gamma^{-1/\alpha} f(\gamma^{-1/\alpha}(x-\mu); \alpha, 1, \beta, 0). \tag{A.7}$$

$$f(x; \alpha, \gamma, \beta, \mu) \stackrel{D}{=} \gamma^{-1/\alpha} f(\gamma^{-1/\alpha}(x-\mu); \alpha, 1, \beta, 0). \tag{A.8}$$

For more details of alpha-stable distributions, see [15].



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