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Nonminimum Phase System and Control Design Applied to MEM Parallel-Plates

Karen Alicia Aguilar Cruz, Romeo Urbieta Parrazales, José de Jesús Medel Juárez

Instituto Politécnico Nacional,
Centro de Investigación en Computación,
Mexico

karen_ali320@hotmail.com, {rurbieta700, jjmedelj}@gmail.com

Abstract. The miniaturization or nanotechnology applied in control systems, redesigning various areas of science and technology wherein researches requires using nanoforms improving the local responses, and at the same time, working in parallel and sequential nanosystems. These studies, name MEMs' Control, requires knowing the control parameters and ranges operation. Commonly used in classical control applied now, to the nanostructures to be controlled. In our case, we present an example consisting in a modern analysis control not yet realized; observing the evolution process trajectory control, and simulation of nano-displacements in a specific case viewed throughout linear and nonlinear evolution concerning parallel plates described as a system to be controlled. Operating ranges considered the mechanical forces in transition and deformation plastic region. Therefore, we propose a nonminimum-phase control technique because it is efficient, precise, and high-performance technique. Under these considerations, the control parameters selected going to embedded in a nano control device that adjusting the digital nanodevice used in nanofluids precision.

Keywords. Parallel-plate characterization, phase advance control, nanotechnologies.

1 Introduction

The actuators and sensors technologies at the nano levels require nanotechnology considerations, although in reality developed as microtechnology also named as Nanotechnology [1].

As an example considered in this paper is the parallel-plates process operates through voltage source in a direct current; charging the plates, generating an electostatic field. Considering the Coulomb Law generating the internal force between the plates resulting is an actuator and in the same time, the internal reaction in the device is a capacitance, and is proportional to the voltage source and inverse to the charge applied. A mechanical system driven by an electrical source considering as an electromechanical actuator or transducer, used as a sensor. The mechanical movement between the plates loads energy and forces, the displacement generated is directly proportional to the square of the electric charge originated by the applied voltage, constituting a nonlinear process. While as a sensor measures, the capacitance originated between the two plates, approaching or moving away from the movable plate two stability regions. The capacitance constitutes a nonlinear process, directly proportional to the square root displacement [2, 3]

Works carrying out the displacement variable characterization determining an equilibrium point have led the two study areas: linear and nonlinear

The equilibrium point named as "pull in" describes the input voltage and relates the electric charge with the displacement ($Q_e(t)$, $X(t)$). The displacements inside the linear section are one-third of the squared electric charge, while the remaining two-thirds are the nonlinear zone [4].

Nonminimal-phase control applied to the $X(t)$ displacement allows better stability, precision, and reliability. However, there are few developed results in this area serving as a reference to others [5].

Then, this paper presents the control analysis, design, and simulation concerning parallel-plates, within the nonlinear area in a simple form [6].

The objective is to find the operating ranges and parameters control in each point of interest going

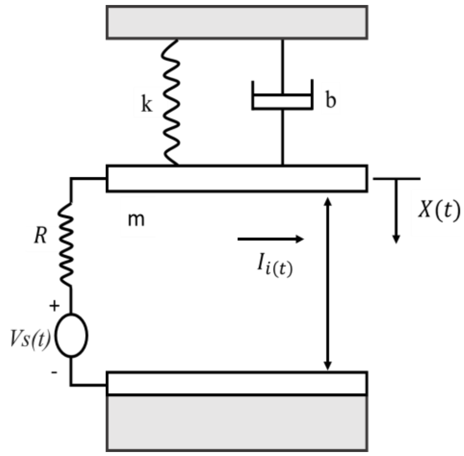

Fig. 1. Parallel-plates process

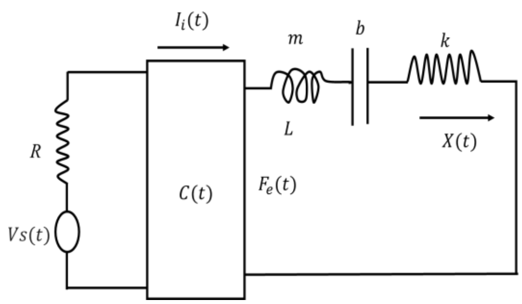

Fig. 2. Electromechanical model of parallel-plates

to used in the intelligent control for a displacement variable [7].

This work includes, in the first, a description of the displacement process; in second, the transfer function obtained, after that, the control and finished with specific results, shown in tables and graphics based on MATLAB® and Simulink® modules.

2 Parallel Plates Process Models

2.1 Process of Parallel-Plates

The parallel plate's process represented in Figure 1 is an nanoelectromechanical process. In one hand, the electrical process is a direct voltage source $V_s(t)$ that feeds two plates where the positive pole connects the moving plate and the negative, to the nonmoving [8].

The mechanical process constituted by two metal plates separated a nanodistance $X(t)$, one fixed in the low, embedded in a no viscous medium serving as a dielectric media; and another mobile plate, which supported by a nanospring with elastic coefficient k inside a medium with nanoviscosity coefficient b, making the movement smoother and more delicate.

2.1 Electromechanical Model

The electromechanical model is a quadrupole compound by an nanoelectrical capacitance $C(t)$, expressed in nanofarads as seen in Figure 2, leading the two electrical meshes, represented by a directly input voltage source $V_s(t)$ and by an electrical output grid using electromechanical equivalences, in nanovolts.

The output voltage of the capacitor represents the nanoelectrostatic force $F_e(t)\ [nN]$, the nanomass m of the moving plate is the nanoelectric inductance $L\ [nH]$, the volume of the viscous medium is regarded as the nanoelectrical capacitance $C(t)\ [nF]$, and the nanodisplacement $X(t)\ [nm]$ of the spring as the nanoelectrical resistance viewed as k.

The input mesh is feed with a direct nanovoltage source $V_s(t)$ conducting an nanoelectric input current $I_i(t)$ loading the plates with a charge $Q(t)$ and generating an nanoutput voltage $V_c(t)$ and current $I_c(t)$.

The nanoelectric accumulated charge $Q(t)$ in the plate surface A, the distance $d = g_0 - X(t)$ between plates considering g_0 as the initial distance between them and the nanodielectric permittivity factor $\in$, leads the nanoelectrical capacitance $C(t)$ depending on the plate's geometry.

The Second Kirchhoff Law allows characterizing the mesh described through the Equations (1, 2):

$$V_s(t) \ = R * \frac{dQ(t)}{dt} + V_c(t), \tag{1}$$

$$R\frac{dQ(t)}{dt} + \frac{\left(g_o - X(t)\right)}{\in A}Q(t) = V_s(t). \tag{2}$$

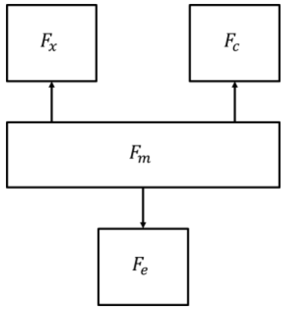

Fig. 3. Parallel-plates equilibrium model

Table 1. Pull-in variables and parameters

Variables	Parameters
$v_{pi} = \sqrt{\frac{8 * k * g_0^2}{27 * C_0}}$	$A = \frac{g_0 * C_0}{\in}$
$q_{pi} = \frac{3}{2} * C_o * v_{pi}$	$R = \frac{r}{w_o * C_0}$
$V_s(t) = v_{pi} * v_{i(t)}$	$t = \frac{\tau}{w_o}$
$V_c = v_{pi} * v_i(t)$	
$X(t) = g_o * x(t)$ $Q(t) = q(t) * q_{pi}$	

The output mesh represents the nanomechanical force source causing the displacement $X(t)$ through the mechanical components m, b and k, being analogous to the nanoelectric circuit including the nanoelectrostatic force $F_e(t)$ based on the nanocapacitor voltage $V_c(t)$ generated between the plates [9].

Figure 3 shows an equilibrium diagram representing the principal forces involved in the parallel plates system, accomplishing to the Nano-Newton, Pascal, and Hook laws. Where F_m is the nanomechanical force from the accelerated plate nanomass, F_c, the nanoforce from the damping material viscosity and the nanomoving plate velocity, F_x, the force due to the spring and the plate displacement, F_e the electrostatic force, which is the stored electrostatic energy change $U_e(t)$ in the moving plate [10].

To obtain $F_e(t)$ from the variation of $U_e(t)$, it is considered that the nanoenergy in t is defined as the integral of the nanoelectric potential P_s stored in the moving plate in a differential time t , as seen in Equation (3):

$$U_e(t) \cong \int_0^t P_s(t)dt = \int_0^t V_s(t)I_s(t)dt\,. \tag{3}$$

This expression depends on the nanodistance $X(t)$, the nanoelectric charge $Q(t)$ and the nanoplate geometric characteristics, as presented in Equation (4):

$$U_e(t) = \frac{1}{c}\int_0^t Q(t)dQ = \frac{x(t)}{2\epsilon A}\,Q(t)^2. \tag{4}$$

From Equation (4) and considering the changes of $U_e(t)$ respect the nanodisplacements $X(t)$ we obtain Equation (5):

$$F_e(t) \cong \left(\frac{dU_e(t)}{dX(t)}\right) = \frac{Q(t)^2}{2\epsilon A}. \tag{5}$$

Substituting the corresponding equations viewed in Figure 3 diagram, and also we have the equilibrium Equation (6), which is a second order differential equation:

$$\frac{Q(t)^2}{2\in A} = m\frac{d^2X(t)}{dt^2} + b\frac{dX(t)}{dt} + kX(t). \tag{6}$$

3 Parallel-Plate Transfer Function

The nanoelectromechanical process model is described by two differential equations of the first and second order, obtaining the system transference function. It is necessary to normalize the differential equations within the equilibrium point; then, a linearization should be performed, followed by a description in state variables and after all, a description in the Laplace domain [11].

3.1 Variables and Parameters Pull-in

In this initial stage, the first transformation is carried out using the equilibrium nanovoltage v_{pi} named as "pull-in".

By this transformation, the normalized variables and parameters mentioned in Table 1 are obtained. Here ζ and ω_0 are the damping and oscillation frequency [12].

Variables and parameters from Table 1 are now applied to Equation (2) to obtain Equation (7):

$$\ddot{q}(t) + \frac{1}{r}[1 - x(t)]q(t) = \left(\frac{2}{3}\right)\left(\frac{1}{r}\right)v_i(t). \quad (7)$$

Moreover, then, according to Equation (6) we are obtaining (8):

$$\ddot{x}(t) + 2\zeta\,\dot{x}(t) + x(t) = \frac{q^2(t)}{3}. \quad (8)$$

3.2 State Variable Transformation

The state variable transformation obtained is necessary to describe the Equations (7, 8) in first order equations. The variables of interest are the nanodisplacement $x(t)$, the nanoelectric charges $q(t)$ in the plates, and the nanospeed plates $p(t)$ around the equilibrium point (X_{eq}, Q_{eq}, P_{eq}). From this equilibrium point, a transformation in t has the corresponding deviations $\phi x(t)$, $\phi q(t)$, and $\phi p(t)$, that leads to the state variables in Equations (9, 10, 11) [13].

$$\frac{dx_1(t)}{dt} = x_2(t) = m_1(t), \quad (9)$$

$$\frac{dx_2(t)}{dt} = -x_1(t) - 2\zeta x_2(t) + \frac{1}{3}x_3^2(t) = m_2(t), \quad (10)$$

$$\frac{dx_3(t)}{dt} = -\frac{1}{r}\big(1 - x_1(t)\big)x_3(t) + \frac{2}{3r}v_i(t) = m_3(t). \quad (11)$$

3.3 Taylor's Linearization Method

The state variables viewed in Equations (9, 10, 11) represent a matrix equation containing the three nanostates $m_1(t)$, $m_2(t)$ and $m_3(t)$, which are linearized using the Taylor method through partial derivatives, obtaining Equation (12) [14]:

$$\left.\begin{array}{l} \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2\zeta & \frac{2}{3}x_3(t) \\ \frac{1}{r}x_1(t) & 0 & \frac{1}{r}(x_1(t) - 1) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \\ \quad + \begin{bmatrix} 0 \\ 0 \\ \frac{2}{3r} \end{bmatrix} \left[\frac{\partial v_i(t)}{\partial x_i(t)} \frac{dx_i(t)}{dt}\right], \\ \frac{dy(t)}{dt} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} \end{array}\right\}. \quad (12)$$

3.4 Laplace Description

Equation (12) is a linear state equation containing differential terms with which is no possible to operate. A normalization process and the application of the Laplace transformation, to factorize the differential terms, lead to an algebraic equation that allows obtaining the transfer function $T(s)$ of the parallel plates process in Equation (13):

$$T(s) = \frac{X(s)}{V_i(s)} = \frac{4Q_{eq}}{9\begin{pmatrix} rs^3 + \\ (1 - X_{eq} + 2\zeta r)\,s^2 + \\ (r + 2\zeta(1 - X_{eq})s + \\ (1 - 3X_{eq}) \end{pmatrix}}, \quad (13)$$

where r is the normalized source resistance, ζ, the spring nanodamping constant and X_{eq}, the moving plate displacement from the equilibrium point. Having $M = 4Q_{eq}$, $N = r$, $P = 1 - X_{eq} + 2\zeta r$, $Q = r + 2\zeta(1 - X_{eq})$ and $R = 1 - 3X_{eq}$, Equation (14) can be expressed as in Equation (14):

$$T(s) = \frac{M}{9(Ns^3 + P\,s^2 + Qs + R)}, \quad (14)$$

for this project, the considered parameter values are $r = 0.8$, $\zeta = 1.8$, which are obtained experimentally, and $X_{eq} = [0.4, 0.9]$, defined from the moving plate displacement range above the equilibrium value or nonlinear zone [15]

4 Nonminimum Phase Control

The transfer function (14) describes a non-linear process called nonminimum phase process.

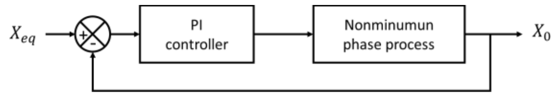

Fig. 4. PI controller application to nonminimum phase process

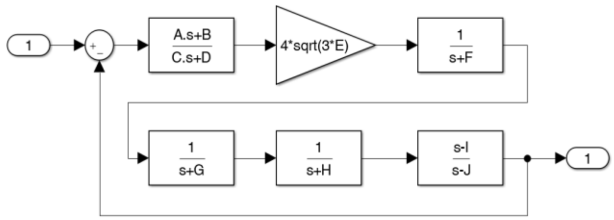

Fig. 5. Parallel-plate process viewed as a block diagram including the nonminimal phase control

Table 2. Nonminimum phase PI control parameters

X_{eq_i}	*A_i*	*B_i*	*C_i*	*D_i*
0.4	4.0	4.0	4	0.010
0.5	3.5	3.5	4	0.009
0.6	3.0	3.0	4	0.007
0.7	2.7	2.7	4	0.005
0.8	2.5	2.5	4	0.004
0.9	2.3	2.3	4	0.003

Table 3. Nonminimum phase parallel plates process parameters

X_{eq_i}	E_i	F_i	G_i	H_i	I_i
0.4	1.0376	2	3.6495	0.0556	0.0556
0.5	1.0351	2	3.6313	0.1400	0.0400
0.6	1.0329	2	3.5972	0.2256	0.2256
0.7	1.0309	2	3.6138	0.3122	0.3122
0.8	1.0291	2	3.5812	0.3999	0.3999
0.9	1.0276	2	3.5661	0.4883	0.4883

Due to the parallel plate displacements make the system being out of the equilibrium point, it is necessary a control block stabilize the system considering the equilibrium point as a reference.

A proposal for this problem is a classical control approach with a PI controller feedback applied to the process for each X_{eq} input as indicated in Figure 4, adjusting both parameters of control [16].

The complete system represented in the block diagram shown in Figure 5 is applying the PI controller. The controller parameters A_i, B_i, C_i, and D_i, as well as the parallel plates process parameters, E_i, F_i, G_i, H_i, and I_i, described for each input $\{X_{eq_i}\}$ in Tables 1 and 2, respectively, remembering that a PI structure requires a different adjustment for each input. For Table 2, $E_i = X_{eq_i}$.

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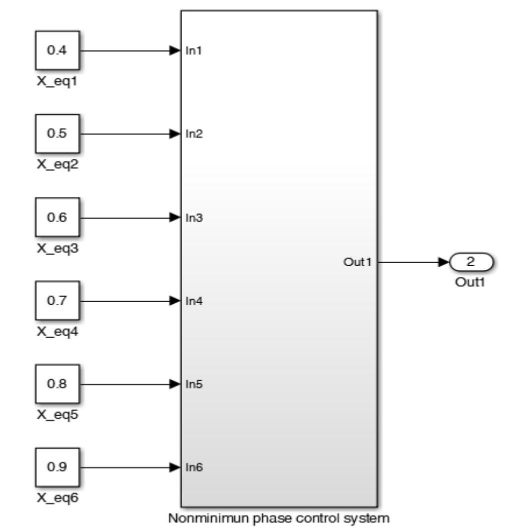

Fig. 6. Parallel plate process control simulation viewed into block arrangement

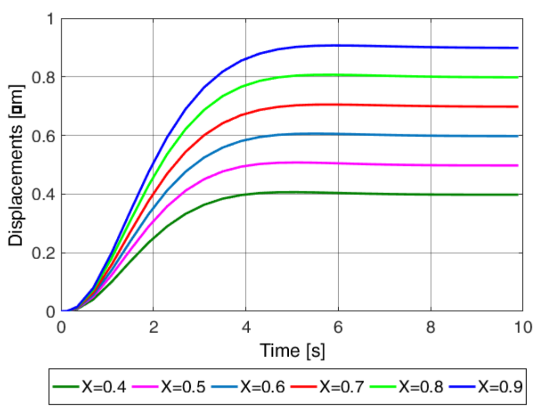

Fig. 7. Controlled displacements for different inputs for the nonminimum phase process

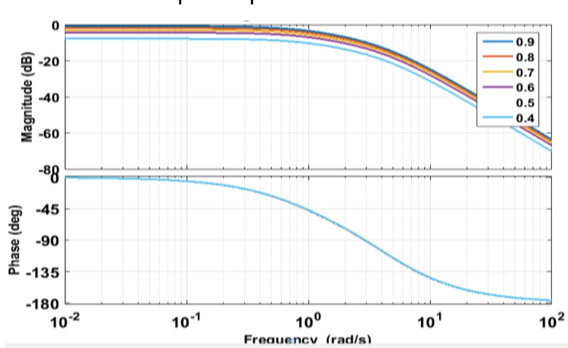

Fig. 8. Bode diagram of the controlled nonminimum phase process

5 Nonminimal Phase Process Control Simulation and Results

A simulation to analyze the control performance is carried out considering the block diagram of Figure 5 as a submodule, having six in total, one for each displacement of interest. All of them are encapsulated in a system with six inputs and one output, as seen in Figure 6.

Figure 7 shows the simulation results in where each value of the set $\{X_{eq_i}\}, i = 1,2,3,$ viewed as the reference is possible to appreciate the controller effects when approximate to the reference in time.

Figure 8 presents the results in the bode diagram considering the frequency domain.

6 Conclusions

It was possible to characterize the nonlinear variable displacement of the parallel plate's process movements through a transfer function using Laplace transformation concerning (13, 14).

The nonminimum phase pole of the system was annulated using classical control techniques, allowing the adjustment the displacement variable within the linear region, having no overshoots in the results, as seen in Figure 7.

With this proposal now is possible to take advantage of a bigger range of movement for the moving plate, using the non-linear region of the process functioning. Besides, we are defining performance intervals that allows constructing the fuzzy application control for future works.

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Corresponding author is José de Jesús Medel Juárez.