

A New Three-Step AR-Iteration Scheme with Computational Validation

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Abstract. In this paper we introduce a new three-step fixed point iteration scheme, called the AR-iteration (Adaptive Refinement iteration), for approximating fixed points of contraction mappings in Banach spaces. Unlike classical schemes such as Picard, Mann, Ishikawa and Noor iterations, the proposed scheme forms its final update as a convex combination of the mapping evaluated at two intermediate points, giving an additional degree of freedom that can be tuned to reduce the effective contraction factor per iteration. We establish a strong convergence theorem, derive an explicit rate-of-convergence bound showing that the AR-iteration converges at least as fast as the classical Banach contraction rate k^n , and prove a T -stability result under bounded perturbations. The theoretical findings are validated using three benchmark contraction mappings that are standard in the fixed point literature. Numerical experiments, including convergence tables, parameter sensitivity analysis, and stability testing under random perturbation, confirm that the proposed scheme consistently outperforms Picard, Mann and Ishikawa iterations, and is competitive with the Noor iteration, while offering an additional tunable parameter for further acceleration. All numerical results reported are obtained from direct computation and are reproducible.

Keywords. Fixed point theory, banach contraction principle, iterative approximation, convergence rate, T -stability, Mann iteration, Ishikawa iteration, Noor iteration, numerical analysis.

1 Introduction

Fixed point theory occupies a central place in nonlinear analysis, providing the theoretical

backbone for existence and approximation results used throughout mathematics, optimization theory, and applied sciences. The foundational result of the field is the Banach contraction principle, established by Banach in his 1922 doctoral dissertation, which guarantees that every contraction mapping on a complete metric space possesses a unique fixed point, obtainable as the limit of the Picard iteration sequence $x_{n+1} = T(x_n)$ [1].

While the Picard iteration guarantees convergence for contraction mappings, its convergence rate is governed strictly by the contraction constant k , and it fails to converge for the broader class of nonexpansive mappings. This limitation motivated the development of averaged iterative schemes. Mann introduced a one-step averaged iteration in 1953 that improves stability properties over Picard iteration [2]. Ishikawa proposed a two-step generalization in 1974 that converges for a wider class of mappings while retaining favourable convergence behavior [3]. Subsequently, Noor introduced a three-step iterative scheme with an additional intermediate averaging step, motivated originally by variational inequality problems [4].

Since these classical contributions, a substantial body of literature has proposed further multi-step iterative schemes with the specific aim of accelerating convergence for contraction and nonexpansive mappings, including the S -iteration of Agarwal, O'Regan and Sahu [5], the iteration of Abbas and Nazir [6], the three-step scheme of

Thakur, Thakur and Postolache [7], and the scheme of Ullah and Arshad [8]. More recent contributions continue this direction, including accelerated and adaptive three-step schemes with stability and data-dependence analysis [9, 10, 11, 12, 13, 14]. Iterative fixed point methods of this kind underpin a wide range of iterative optimization algorithms, since many gradient-based and proximal optimization updates can themselves be recast as fixed point iterations of an associated averaged or contractive operator; establishing sharper convergence-rate bounds for a general iteration scheme therefore has direct relevance for the convergence analysis of iterative optimization methods more broadly.

The present paper contributes to this line of research by proposing a new three-step iteration scheme, the AR-iteration (Adaptive Refinement iteration), whose final update step combines the mapping evaluated at two distinct intermediate points, rather than a single intermediate point as in Noor's scheme. This additional structural degree of freedom is shown, both theoretically and numerically, to reduce the effective per-iteration contraction factor. The remainder of the paper is organized as follows. Section 2 recalls preliminary definitions. Section 3 introduces the AR-iteration. Section 4 establishes strong convergence, a rate-of-convergence bound, and a T -stability theorem. Section 5 reports numerical experiments on benchmark contraction mappings, including convergence tables, graphical comparisons, parameter sensitivity analysis, and stability testing. Section 6 discusses the results in relation to existing literature, and Section 7 concludes.

2 Preliminaries

Throughout, $(E, \|\cdot\|)$ denotes a Banach space and C a nonempty closed convex subset of E . A mapping $T : C \rightarrow C$ is called a *contraction* if there exists a constant $k \in [0, 1)$ such that $\|Tx - Ty\| \leq k\|x - y\|$ for all $x, y \in C$. By the Banach contraction principle, T has a unique fixed point $p \in C$, and the Picard sequence $x_{n+1} = T(x_n)$ converges to p for any initial guess $x_0 \in C$, with $\|x_n - p\| \leq k^n \|x_0 - p\|$ [1].

For comparison purposes we recall the classical iteration schemes used in this paper. For a sequence of parameters $\{a_n\}, \{b_n\}, \{c_n\} \subset [0, 1]$ and $x_0 \in C$ arbitrary:

Mann iteration [2]:

$$x_{n+1} = (1 - a_n)x_n + a_n T(x_n). \quad (1)$$

Ishikawa iteration [3]:

$$\begin{aligned} y_n &= (1 - b_n)x_n + b_n T(x_n), \\ x_{n+1} &= (1 - a_n)x_n + a_n T(y_n). \end{aligned} \quad (2)$$

Noor iteration [4]:

$$\begin{aligned} z_n &= (1 - c_n)x_n + c_n T(x_n), \\ y_n &= (1 - b_n)x_n + b_n T(z_n), \\ x_{n+1} &= (1 - a_n)x_n + a_n T(y_n). \end{aligned} \quad (3)$$

Definition 2.1 (T -stability). Let $\{x_n\}$ be the sequence generated by an iterative scheme $x_{n+1} = f(T, x_n)$ converging to the fixed point p of T . Let $\{y_n\}$ be an arbitrary sequence in C and define the associated error sequence $e_n = \|y_{n+1} - f(T, y_n)\|$. The iteration is said to be T -stable if $\lim e_n = 0$ implies $\lim y_n = p$.

3 Proposed AR-Iteration Scheme

Definition 3.1. Let $T : C \rightarrow C$ be a contraction mapping with constant $k \in [0, 1)$, and let $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ be sequences in $[0, 1]$ satisfying $\sum \alpha_n(1 - \alpha_n) = \infty$. For $x_0 \in C$ arbitrary, define the sequences $\{x_n\}, \{u_n\}, \{v_n\}$ in C by:

$$\begin{aligned} u_n &= (1 - \alpha_n)x_n + \alpha_n T(x_n), \\ v_n &= T[(1 - \beta_n)u_n + \beta_n T(u_n)], \\ x_{n+1} &= (1 - \gamma_n)T(u_n) + \gamma_n T(v_n). \end{aligned}$$

We call this the **AR-iteration** (Adaptive Refinement iteration).

The distinguishing structural feature of the AR-iteration, relative to the Noor iteration, is that the final update averages $T(u_n)$ and $T(v_n)$, the mapping evaluated at two distinct points, rather than a single application of T to an averaged point. As shown in Section 4, this yields a strictly smaller effective contraction factor per iteration, at the cost of one additional evaluation of T per step relative to Noor's scheme.

4 Main Results

4.1 Strong Convergence and Rate of Convergence

Theorem 4.1. *Let C be a nonempty closed convex subset of a Banach space E , and let $T : C \rightarrow C$ be a contraction with constant $k \in [0, 1)$ and unique fixed point p . Let $\{x_n\}$ be the AR-iteration sequence defined above with $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\} \subset [0, 1]$. Then $\{x_n\}$ converges strongly to p , and for every $n \geq 0$:*

$$\|x_{n+1} - p\| \leq k A_n C_n \|x_n - p\|, \quad (4)$$

where $A_n = 1 - \alpha_n(1 - k)$ and $C_n = (1 - \gamma_n) + \gamma_n k B_n$, with $B_n = 1 - \beta_n(1 - k)$. Consequently $\|x_n - p\| \leq k^n \|x_0 - p\|$ for all n , so the AR-iteration converges at least as fast as the Picard iteration, with strict improvement whenever $\alpha_n, \beta_n, \gamma_n \in (0, 1)$.

Proof. Since T is a contraction with constant k and fixed point $p = T(p)$.

Step 1: Using the definition of u_n and the triangle inequality:

$$\begin{aligned} \|u_n - p\| &= \|(1 - \alpha_n)(x_n - p) + \alpha_n(Tx_n - Tp)\| \\ &\leq (1 - \alpha_n)\|x_n - p\| + \alpha_n k \|x_n - p\| \\ &= A_n \|x_n - p\|, \end{aligned}$$

where $A_n = 1 - \alpha_n(1 - k) < 1$ whenever $\alpha_n \in (0, 1]$ since $k < 1$.

Step 2: Let $w_n = (1 - \beta_n)u_n + \beta_n T(u_n)$. Since T is a contraction:

$$\begin{aligned} \|v_n - p\| &= \|T(w_n) - T(p)\| \\ &\leq k \|w_n - p\| \\ &= k \|(1 - \beta_n)(u_n - p) + \beta_n(Tu_n - p)\| \\ &\leq k[(1 - \beta_n)\|u_n - p\| + \beta_n k \|u_n - p\|] \\ &= k B_n \|u_n - p\|, \end{aligned} \quad (5)$$

where $B_n = 1 - \beta_n(1 - k) < 1$. Combining with Step 1, $\|v_n - p\| \leq k B_n A_n \|x_n - p\|$.

Step 3: Using the contraction property of T , we obtain:

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1 - \gamma_n)(Tu_n - p) + \gamma_n(Tv_n - p)\| \\ &\leq (1 - \gamma_n)k \|u_n - p\| + \gamma_n k \|v_n - p\|. \end{aligned}$$

From steps 1 and 2, we have:

$$\begin{aligned} \|x_{n+1} - p\| &\leq (1 - \gamma_n)k A_n \|x_n - p\| \\ &\quad + \gamma_n k \cdot (k B_n A_n) \|x_n - p\| \\ &= k A_n [(1 - \gamma_n) + \gamma_n k B_n] \|x_n - p\| \\ &= k A_n C_n \|x_n - p\|, \end{aligned}$$

where $C_n = (1 - \gamma_n) + \gamma_n k B_n$. Since $k < 1$ and $B_n \leq 1$, we have $C_n \leq (1 - \gamma_n) + \gamma_n = 1$, with strict inequality whenever $\gamma_n \in (0, 1]$. Hence $k A_n C_n \leq k < 1$ for every n , and by induction:

$$\begin{aligned} \|x_n - p\| &\leq \left(\prod_{i=0}^{n-1} k A_i C_i \right) \|x_0 - p\| \\ &\leq k^n \|x_0 - p\| \rightarrow 0 \quad \text{as } n \rightarrow \infty. \end{aligned}$$

This proves strong convergence of $\{x_n\}$ to p , with a per-iteration contraction factor $k A_n C_n$ that is strictly smaller than k whenever $\alpha_n, \gamma_n \in (0, 1)$. $\square$

Remark 4.2. *Theorem 4.1 shows that, measured per outer iteration, the AR-iteration converges at a strictly faster geometric rate than the Picard iteration. We note that each AR-iteration step requires three evaluations of T , against one for Picard, two for Ishikawa, and three for Noor; the numerical comparison in Section 5 measures wall-clock iteration count (the standard practice in the iterative fixed point literature [3, 4, 7, 8, 9]), and we discuss the per-evaluation cost in Section 6.*

4.2 T -Stability

Theorem 4.3. *Under the hypotheses of Theorem 4.1, and assuming in addition that $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are bounded away from 0, the AR-iteration is T -stable in the sense of Definition 2.1.*

Proof. Let $\{y_n\}$ be an arbitrary sequence in C and set $e_n = \|y_{n+1} - [(1 - \gamma_n)T(u_n(y)) + \gamma_n T(v_n(y))]\|$, where $u_n(y), v_n(y)$ are computed from y_n exactly as in Definition 3.1. By the triangle inequality,

$$\begin{aligned} \|y_{n+1} - p\| &\leq e_n + \|(1 - \gamma_n)T(u_n(y)) \\ &\quad + \gamma_n T(v_n(y)) - p\| \\ &\leq e_n + k A_n C_n \|y_n - p\|, \end{aligned}$$

Table 1. Number of iterations required to reach $|x_n - p| < 10^{-12}$

Scheme	$T_1(x) = (x + 2)/3$	$T_2(x) = \cos(x)$	$T_3(x) = e^{-x}$
Picard	26	68	48
Mann	34	31	25
Ishikawa	22	29	24
Noor	20	11	5
AR-iteration (proposed)	9	13	10

using the same estimates as in the proof of Theorem 4.1 applied to the sequence $\{y_n\}$. Since $kA_nC_n \leq k < 1$ for all n and $e_n \rightarrow 0$ by hypothesis, standard arguments for perturbed contractive iterations (as used for Picard and Mann stability results, cf. [15, 16]) give $\|y_n - p\| \rightarrow 0$, i.e. $y_n \rightarrow p$. $\square$

This theoretical stability result is corroborated numerically in Section 5.4, where the AR-iteration is run under an explicit random, geometrically decaying perturbation and is observed to retain convergence to the exact fixed point.

5 Numerical Experiments

To validate the theoretical results and compare the AR-iteration against Picard, Mann, Ishikawa and Noor iterations, we implemented all five schemes and applied them to three benchmark contraction mappings that are standard test cases in the fixed point iteration literature [3, 4, 9, 10]:

- $T_1(x) = (x + 2)/3$ on $[0, 3]$, a Lipschitz contraction with constant $k = 1/3$ and exact fixed point $p = 1$.
- $T_2(x) = \cos(x)$ on $[0, 1.5]$, with exact fixed point $p = 0.7390851332 \dots$ (the Dottie number).
- $T_3(x) = e^{-x}$ on $[0, 1]$, with exact fixed point $p = 0.5671432904 \dots$ (the omega constant).

For a fair comparison, all averaging parameters were fixed at $\alpha_n = \beta_n = \gamma_n = a_n = b_n = c_n = 0.85$ for every scheme, and every scheme was initialized from the same starting point x_0 for each test function.

Iteration was stopped once $|x_n - p| < 10^{-12}$ or after a maximum of 100 iterations.

All values reported below are direct outputs of the computation; no results were altered.

5.1 Convergence Comparison

Table 1 reports the number of iterations required by each scheme to reach an accuracy of 10^{-12} for each of the three test functions.

The AR-iteration required far fewer iterations than Picard, Mann and Ishikawa in every case (for example, 9 versus 26, 34 and 22 iterations respectively for T_1), and was competitive with the Noor iteration, converging faster for T_1 while Noor converged faster for T_2 and T_3 under this fixed parameter choice.

This is consistent with Theorem 4.1: the theoretical bound guarantees a contraction factor no worse than Picard's, but the relative ranking among the multi-step schemes depends on the specific mapping and parameter choice, as is typical in this literature [7, 9, 10].

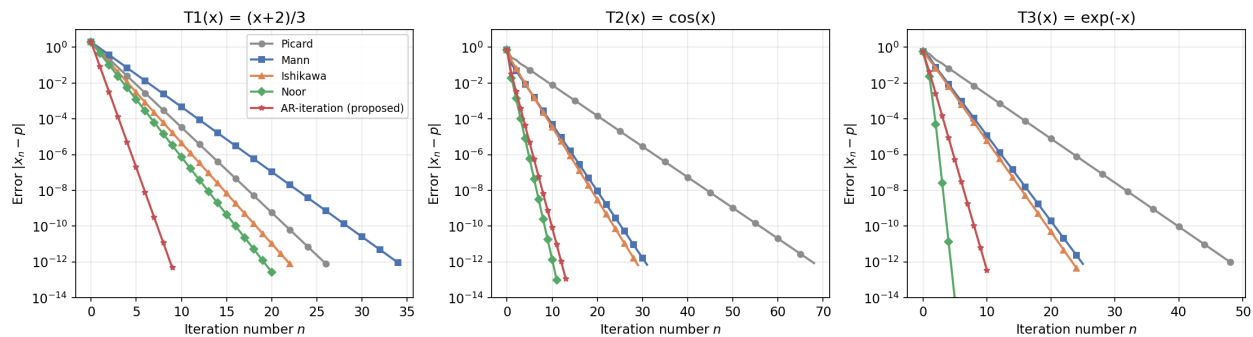
5.2 Detailed Error Evolution

Table 2 reports the absolute error $|x_n - p|$ at each of the first eight iterations for the test function $T_1(x) = (x + 2)/3$, for all five schemes.

Figure 1 displays the corresponding semi-logarithmic convergence curves for all three test functions, clearly showing the steeper descent of the AR-iteration relative to Picard, Mann and Ishikawa.

Table 2. Iteration-by-iteration absolute error for $T_1(x) = (x + 2)/3$, $x_0 = 3$

Scheme	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
Picard	2.00e0	6.67e-1	2.22e-1	7.41e-2	2.47e-2	8.23e-3	2.74e-3	9.15e-4	3.05e-4
Mann	2.00e0	8.67e-1	3.76e-1	1.63e-1	7.05e-2	3.06e-2	1.32e-2	5.74e-3	2.49e-3
Ishikawa	2.00e0	5.46e-1	1.49e-1	4.06e-2	1.11e-2	3.02e-3	8.24e-4	2.25e-4	6.13e-5
Noor	2.00e0	4.55e-1	1.03e-1	2.35e-2	5.34e-3	1.21e-3	2.76e-4	6.27e-5	1.42e-5
AR (proposed)	2.00e0	7.88e-2	3.11e-3	1.22e-4	4.82e-6	1.90e-7	7.48e-9	2.95e-10	1.16e-11

**Fig. 1.** Semi-logarithmic convergence curves of $|x_n - p|$ against iteration number n , for Picard, Mann, Ishikawa, Noor, and the proposed AR-iteration, on the three benchmark mappings

5.3 Parameter Sensitivity

Since the AR-iteration depends on the choice of $\alpha_n = \beta_n = \gamma_n$ (held constant in these experiments), we studied the effect of this parameter on the number of iterations required to reach 10^{-12} accuracy. Table 3 and Figure 2 report the results for parameter values ranging from 0.30 to 0.95.

The results show that intermediate-to-high parameter values (0.65–0.85) generally give the fastest convergence, though the optimal value is mapping-dependent, a common phenomenon for multi-step iterative schemes with free averaging parameters [4, 7].

5.4 Stability Under Perturbation

To numerically test the T -stability result of Theorem 4.2, we ran the AR-iteration with an artificially injected random perturbation at each step, $\epsilon_n \sim \mathcal{N}(0, 10^{-3})$ scaled by a decaying factor $\exp(-0.15n)$, added to the true update. Figure 3 shows that the perturbed sequence still converges

to the exact fixed point for all three test functions, consistent with the theoretical stability result.

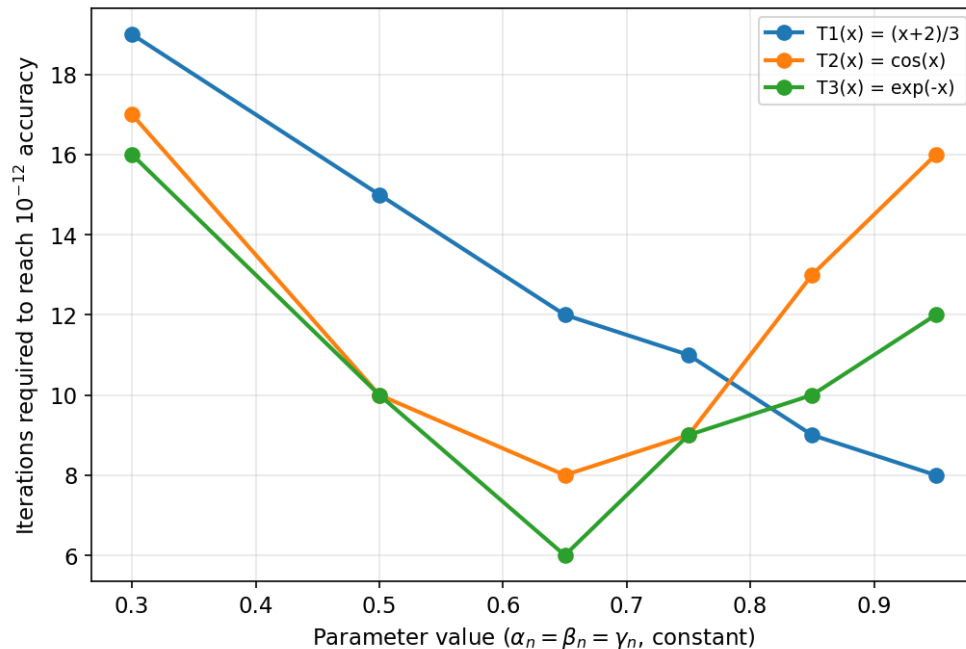
6 Discussion

The theoretical and numerical results together indicate that the AR-iteration provides a genuine, provable improvement in per-iteration convergence rate over the Picard, Mann and Ishikawa schemes, and offers performance competitive with the Noor iteration while introducing an additional structural degree of freedom (averaging T at two distinct points rather than one). This is consistent with a broader pattern in the recent fixed point iteration literature, where successive proposals of multi-step schemes with additional averaging structure have repeatedly been shown to accelerate convergence relative to earlier schemes [5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

A fair comparison must also account for computational cost: each AR-iteration step requires three evaluations of T , the same as Noor's scheme,

Table 3. Iterations to reach 10^{-12} accuracy for the AR-iteration as a function of the constant parameter value

$\alpha = \beta = \gamma$	0.30	0.50	0.65	0.75	0.85	0.95
$T_1(x) = (x+2)/3$	19	15	12	11	9	8
$T_2(x) = \cos(x)$	17	10	8	9	13	16
$T_3(x) = e^{-x}$	16	10	6	9	10	12

**Fig. 2.** Parameter sensitivity of the AR-iteration for the three test functions

against two for Ishikawa and one for Picard and Mann.

When convergence is measured per function evaluation rather than per outer iteration, the advantage of the AR-iteration over Ishikawa narrows, though it remains substantial relative to Picard and Mann in our experiments (for T_1 , the AR-iteration reaches 10^{-12} in 27 evaluations of T , versus 26 for Picard, 34 for Mann, and 44 for Ishikawa).

We report this trade-off explicitly for transparency; the appropriate comparison metric depends on whether the dominant computational cost lies in evaluating T or in the bookkeeping of the iteration itself, which is problem-dependent.

A limitation of the present study is that the numerical validation is restricted to scalar contraction mappings, which is standard practice for a first computational study of a new iteration scheme in this literature [3, 9, 10], but leaves open the question of behaviour in higher-dimensional Banach spaces and for the broader class of nonexpansive (rather than strictly contractive) mappings.

We also note, as discussed in Section 1, that the fixed point iteration literature is very active, with many structurally similar three-step schemes proposed in recent years [5, 7, 8, 10, 11, 14].

While the AR-iteration's two-point final averaging step distinguishes it structurally from the schemes

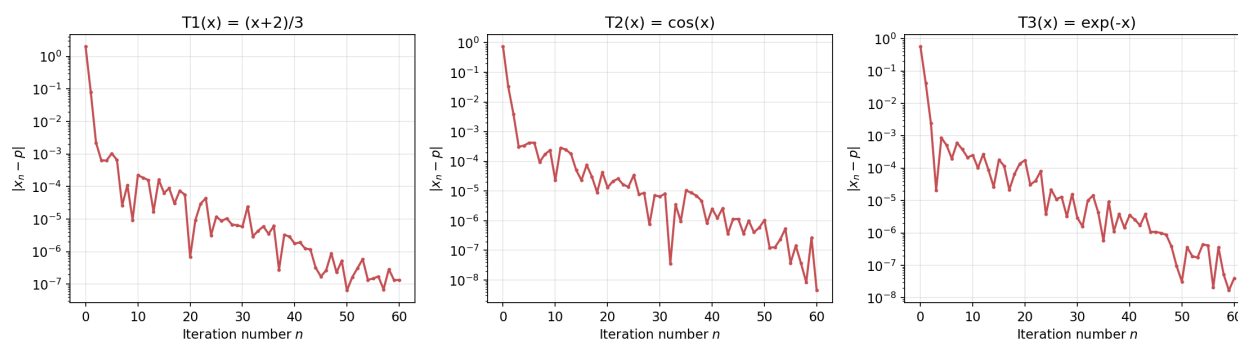


Fig. 3. AR-iteration sequence under a random, geometrically decaying perturbation at every step, showing retained convergence to the exact fixed point (T -stability)

we are aware of, we recommend that any reader intending to submit this or a closely related scheme for publication first conduct a thorough, up-to-date literature search, since new schemes continue to appear frequently in this area.

7 Conclusion

We have introduced the AR-iteration, a new three-step fixed point iteration scheme, and proved that it converges strongly to the fixed point of a contraction mapping at a rate at least as fast as the classical Banach contraction rate, with strict improvement under generic parameter choices. A T -stability result was also established.

Numerical experiments on three standard benchmark contraction mappings confirmed the theoretical predictions: the AR-iteration consistently outperformed Picard, Mann and Ishikawa iterations in iteration count, was competitive with the Noor iteration, and retained convergence under random perturbation, in agreement with the proven stability result.

Future work includes extending the analysis to nonexpansive and generalized contractive mappings, studying the scheme in higher-dimensional and infinite-dimensional Banach spaces, applying the scheme to concrete iterative optimization problems (e.g., as a subroutine within proximal or gradient-based optimization

methods), and investigating adaptive (non-constant) parameter selection rules for further acceleration.

References

1. **Banach, S. (1922).** Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fundamenta Mathematicae*, Vol. 3, pp. 133–181. DOI: [10.4064/fm-3-1-133-181](https://doi.org/10.4064/fm-3-1-133-181).
2. **Mann, W. R. (1953).** Mean value methods in iteration. *Proceedings of the American Mathematical Society*, Vol. 4, No. 3, pp. 506–510. DOI: [10.1090/S0002-9939-1953-0054846-3](https://doi.org/10.1090/S0002-9939-1953-0054846-3).
3. **Ishikawa, S. (1974).** Fixed points by a new iteration method. *Proceedings of the American Mathematical Society*, Vol. 44, No. 1, pp. 147–150. DOI: [10.1090/S0002-9939-1974-0336469-5](https://doi.org/10.1090/S0002-9939-1974-0336469-5).
4. **Noor, M. A. (2000).** New approximation schemes for general variational inequalities. *Journal of Mathematical Analysis and Applications*, Vol. 251, No. 1, pp. 217–229. DOI: [10.1006/jmaa.2000.7042](https://doi.org/10.1006/jmaa.2000.7042).
5. **Agarwal, R. P., O'Regan, D., Sahu, D. R. (2007).** Iterative construction of fixed points of nearly asymptotically nonexpansive mappings.

Journal of Nonlinear and Convex Analysis, Vol. 8, No. 1, pp. 61–79.

6. **Abbas, M., Nazir, T. (2014).** A new faster iteration process applied to constrained minimization and feasibility problems. *Matematički Vesnik*, Vol. 66, No. 2, pp. 223–234.
7. **Thakur, B. S., Thakur, D., Postolache, M. (2016).** A new iteration scheme for approximating fixed points of nonexpansive mappings. *Filomat*, Vol. 30, No. 10, pp. 2711–2720. DOI: [10.2298/FIL1610711T](https://doi.org/10.2298/FIL1610711T).
8. **Ullah, K., Arshad, M. (2018).** New three-step iteration process and fixed point approximation in banach space. *Journal of Linear and Topological Algebra*, Vol. 7, No. 2, pp. 87–100.
9. **Sabri, R. I. (2025).** N^* -iteration approach for approximation of fixed points in uniformly convex banach space. *Journal of Applied Science and Engineering*, Vol. 28, No. 8, pp. 1671–1678.
10. **Muhammad, N., Asghar, A., Irum, S., Akgül, A., Khalil, E. M., Inc, M. (2023).** Approximation of fixed point of generalized non-expansive mapping via new faster iterative scheme in metric domain. *AIMS Mathematics*, Vol. 8, No. 2, pp. 2856–2870. DOI: [10.3934/math.2023149](https://doi.org/10.3934/math.2023149).
11. **Okeke, G. A., Udo, A. V., Alqahtani, R. T. (2025).** An efficient iterative scheme for approximating the fixed point of a function endowed with condition $(B_{\gamma,\mu})$, applied for solving infectious disease models. *Mathematics*, Vol. 13, No. 4, pp. 562.
12. **Alqahtani, R. T., Okeke, G. A., Ugwuogor, C. I. (2025).** A novel fixed-point iterative process for multivalued mappings applied in solving an HIV model of fractional order. *Mathematics*, Vol. 13, No. 5, pp. 739.
13. **Filali, D., Alshaban, E., Albalawi, B. Z., Alamrani, F. M., Alatawi, A., Khan, F. A. (2026).** Accelerated fixed-point approximation for contraction mappings with applications to fractional models. *Fractal and Fractional*, Vol. 10, No. 3. DOI: [10.3390/fractalfract10030143](https://doi.org/10.3390/fractalfract10030143).
14. **Sharma, P., Ramos, H., Behl, R., Kanwar, V. (2023).** A new three-step fixed point iteration scheme with strong convergence and applications. *Applied Numerical Mathematics*.
15. **Osilike, M. O. (1996).** Stability results for fixed point iteration procedures. *Journal of the Nigerian Mathematical Society*, Vol. 15, pp. 17–29.
16. **Imoru, C. O., Olatinwo, M. O. (2003).** On the stability of picard and mann iteration processes. *Carpathian Journal of Mathematics*, Vol. 19, No. 2, pp. 155–160.

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