

An Enhanced Hierarchical Hybrid Algorithm: In Galactic Swarm Optimization and Gravitational Search Algorithm

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Abstract. This paper presents a hierarchical hybrid metaheuristic combining Galactic Swarm Optimization (GSO) and the Gravitational Search Algorithm (GSA), using dynamic parameter adaptation strategies. The first strategy, FHGSO, applies fuzzy logic exclusively to the GSO component: it dynamically adapts the Level 2 cognitive and social coefficients of the inter-subpopulation interaction using the FGSO inference system. The second strategy, FHGSA, applies fuzzy logic exclusively to the GSA component: it dynamically adjusts the gravitational descent coefficient α and the Kbest selection ratio using the FGSA inference system. Both strategies use the normalized current iteration as the sole input, producing smooth transitions between exploration and exploitation. The theoretical foundation integrates classical Type-1 fuzzy set theory including formal definitions of fuzzy sets, membership functions, linguistic variables, and fuzzy logic operations with practical fuzzy inference system design using Mamdani methodology. This integration provides a mathematically rigorous approach to handling parameter uncertainty throughout algorithm execution. Three algorithm variants were systematically evaluated: (I) HGSO the base hybrid without fuzzy adaptation; (II) FHGSO the hybrid with FGSO adapting only the GSO Level 2 coefficients; and (III) FHGSA the hybrid with FGSA adapting only the GSA parameters α and Kbest. Experiments were conducted on seven classical benchmark functions at 10, 30, and 50 dimensions, and on the 12 IEEE CEC 2022 benchmark functions at 2, 10, and 20 dimensions. Results demonstrate that each fuzzy variant improves over the base HGSO on different problem types: FHGSO benefits functions sensitive to Level 2 diversity, while FHGSA excels on composite and high-dimensional problems where gravitational dynamics are the limiting factor.

Keywords. Gravitational search algorithm; galactic swarm optimization; fuzzy inference systems, dynamic parameter adaptation; hierarchical metaheuristics; benchmark optimization.

1 Introduction

Optimization represents a fundamental challenge in scientific and engineering domains. The task of locating optimal or near-optimal solutions within constrained feasible regions finds application across industrial design, financial modeling, machine learning, and numerous other areas.

Real world optimization problems typically exhibit characteristics non-convexity, multimodality, discontinuity, high dimensionality that render classical deterministic approaches ineffective.

Metaheuristic optimization algorithms have become indispensable tools for solving complex, non-linear, and high-dimensional optimization problems across engineering, data science, and applied mathematics. Among the most successful paradigms are physics-based algorithms and swarm intelligence methods, which model natural processes to navigate solution spaces efficiently.

The Gravitational Search Algorithm (GSA) [43] and Galactic Swarm Optimization (GSO) [32] are representative algorithms in their respective categories, each offering distinct strengths and complementary weaknesses.

GSA, introduced by Rashedi et al. in 2009, leverages Newtonian gravitational mechanics to drive agents toward better solutions. Its physics-based forces produce strong local exploitation, but the algorithm can suffer from premature convergence in complex multimodal landscapes [30]. GSO, by contrast, organizes its population into a hierarchical multi-level structure that mirrors cosmic organization from individual stars to galaxies, enabling simultaneous local and global exploration [3,32].

However, both algorithms are sensitive to their respective control parameters, which are typically set as static values. Fuzzy logic, originally formalized by Zadeh, provides a mathematical framework for managing uncertainty and vagueness. Type-1 fuzzy sets extend classical binary set membership through graded membership values, enabling representation of linguistic concepts with continuous transitions between membership states.

Dynamic parameter adaptation has emerged as a powerful strategy to improve metaheuristic performance across diverse optimization phases. Fuzzy logic-based adaptation, in particular, provides an interpretable and computationally lightweight mechanism for parameter control [2,10,12,38]. Previous works have demonstrated the effectiveness of fuzzy parameter tuning in GSA [28,46], GSO [3,7,29], and various other algorithms [11,12,18,30,34].

The primary motivation for this research is the observation that a hierarchical GSO-GSA hybrid algorithm involves parameters at two distinct algorithmic levels: (a) the Level 2 inter-subpopulation PSO coefficients c_3 and c_4 which govern the balance between personal best and global best influences among super-agents, and (b) the GSA-specific parameters α (gravitational decay rate) and K_{best} (number of influential agents) that control the gravitational search behavior. These two parameters set operate at conceptually different algorithmic levels, suggesting that separate, specialized fuzzy systems may be more effective than a single unified controller.

The main contributions of this work are:

- **Two Independent Type-1 Fuzzy Adaptation Strategies:** FHGSO uses the FGSO Type-1 Mamdani system exclusively to adapt GSO Level 2 coefficients (c_3 , c_4); FHGSA uses the FGSA Type-1 Mamdani system exclusively to adapt GSA parameters (α , K_{best}). Each variant isolates the contribution of one fuzzy controller to one algorithmic component.
- **Three-Variant Comparative Framework:** Systematic isolation of each fuzzy system's contribution through HGSO (base, no fuzzy), FHGSO (fuzzy GSO adaptation only), and

FHGSA (fuzzy GSA adaptation only), allowing direct attribution of performance gains to each fuzzy component.

- **Comprehensive CEC 2022 Evaluation:** Assessment on the 12 IEEE CEC 2022 single-objective benchmark functions at 2, 10, and 20 dimensions, providing standardized performance benchmarks.
- **Validated Exploration–Exploitation Balance:** Fuzzy rules encoding expert knowledge of how parameters should evolve over iterations, confirmed by comparative experimental results on classical and CEC benchmarks.

Paper organization Section 2 reviews the foundational algorithms and related work on Type-1 fuzzy dynamic parameter adaptation. Section 3 presents the HGSO base algorithm and its two Type-1 fuzzy variants (FHGSO and FHGSA) with full mathematical formulation of each inference system. Section 4 describes the experimental setup. Section 5 presents results and comparative analysis. Section 6 concludes the paper.

2 Background and Related Work

The development of effective metaheuristic algorithms requires a deep understanding of the theoretical frameworks that underpin their operation. This section establishes the necessary conceptual foundations for formulating the proposed algorithm, presenting a comprehensive review of three fundamental pillars: the mathematical theory of fuzzy sets and its implementation in inference systems, the formulation and characteristics of the Gravitational Search Algorithm (GSA) as a physics-based paradigm, and Galactic Swarm Optimization (GSO) as a hierarchical multi-population strategy.

Additionally, the state of the art in dynamic parameter adaptation using fuzzy logic is analyzed, identifying both significant advances in optimization with variable parameters and unexplored research opportunities in hybridizing these approaches.

2.1 Type-1 Fuzzy Set Theory and Fuzzy Logic Systems

2.1.1 Type-1 Fuzzy Sets: Mathematical Foundations

Classical set theory employs a dichotomous membership model: elements either belong to a set (membership degree = 1) or do not (membership degree = 0). This sharp boundary proves inadequate for representing linguistic concepts with inherent gradations. Type-1 fuzzy sets generalize classical sets through membership functions $\mu_a: U \rightarrow [0,1]$, mapping universe elements to real-valued membership degrees within the unit interval.

Formally, a fuzzy set of type 1 A in the universe of discourse U is defined as shown in expression 1. The membership function $\mu_a(u)$ assigns to each element u a membership degree $\mu_a(u) \in [0,1]$. The support of A comprises all elements with non-zero membership as shown in equation 2. The kernel consists of elements with full membership as shown in equation 3 [48]:

$$A = \{(u, \mu_a(u)) \mid u \in U\}, \quad (1)$$

$$\text{supp}(A) = \{u \in U \mid \mu_a(u) > 0\}, \quad (2)$$

$$\text{core}(A) = \{u \in U \mid \mu_a(u) = 1\}. \quad (3)$$

2.1.2 Type-1 Fuzzy Sets: Mathematical Foundations

Membership functions define how membership levels vary within the universe of discourse. Common functional forms include triangular functions (the simplest and most computationally efficient), trapezoidal functions (extended plateaus), Gaussian functions (smooth, bell-shaped), and sigmoid functions (monotonic transitions). The graphical representation of a membership function is shown in Figure 1 below.

The choice of functional form depends on knowledge of the problem domain and computational limitations.

A linguistic variable represents a concept using linguistic terms (Low, Medium, High) with associated fuzzy sets. For example, the linguistic variable "Iteration Progress" could employ three

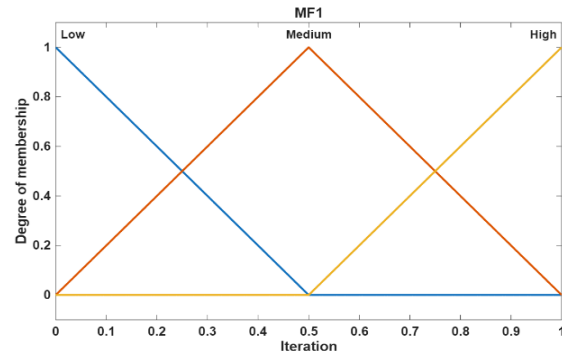


Fig. 1. Membership function

fuzzy sets: Initial Phase (low number of iterations), Intermediate Phase (intermediate progress), and Final Phase (near convergence). This linguistic representation allows for the intuitive formulation of rules and the interpretation of the problem [49].

2.1.3 Type-1 Fuzzy Sets: Mathematical Foundations

Fuzzy logic operations extend classical Boolean logic. The fundamental operations are: Fuzzy Union (OR), Fuzzy Intersection (AND), and Fuzzy Complement (NOT), as shown in the following equations. These operations preserve the intuitive meaning of logical connectives while extending them to continuous-valued membership degrees:

$$\mu_a \cup \mu_\beta(u) = \max(\mu_a(u), \mu_\beta(u)), \quad (4)$$

$$\mu_a \cap \mu_\beta(u) = \min(\mu_a(u), \mu_\beta(u)), \quad (5)$$

$$\mu_a'(u) = 1 - \mu_a(u). \quad (6)$$

The Mamdani inference engine, widely adopted in practical applications, operates in four stages: (1) Fuzzification converts precise inputs into membership degrees; (2) Rule evaluation applies fuzzy rules to fuzzy inputs; (3) Aggregation combines the outputs of multiple rules; (4) Defuzzification converts the aggregated fuzzy output into precise values. The Center of Gravity (COG) defuzzification method calculates as shown in the following equation [29]:

$$y^* = \frac{\sum \mu(y) \cdot y}{\sum \mu(y)}. \quad (7)$$

2.2 Gravitational Search Algorithm (GSA)

The Gravitational Search Algorithm (GSA), proposed by Rashedi, Nezamabadi-pour, and Saryazdi in 2009 [1], is a population metaheuristic inspired by Newton's law of universal gravitation. In classical mechanics, two bodies of masses M_1 and M_2 separated by a distance R attract each other with a force proportional to the product of their masses and inversely proportional to the square of the distance, as shown in equation 8, where G is the universal gravitational constant:

$$F = G \times M_1 \cdot M_2 / R^2. \quad (8)$$

The GSA translates this principle into an optimization framework by treating candidate solutions as objects with a gravitational mass proportional to their quality, so that the best solutions exert a greater attraction on their neighbors and direct the population toward the most promising regions of the search space.

2.2.1 Agent Representation and Mass Calculation

In GSA, each agent i represents a candidate solution described by its position $x_i \in \mathbb{R}^D$ in a D -dimensional search space. Each agent is associated with three types of mass: active gravitational mass M_{ai} (which exerts force on others), passive gravitational mass M_{pi} (which receives force from others), and inertial mass M_{ii} (which resists acceleration). In the standard formulation of GSA, all three are equal: $M_{ai} = M_{pi} = M_{ii} = M_i$. The mass of each agent is calculated from its fitness relative to the best and worst solutions in the current population, as shown in the following expressions:

$$m_i(t) = (fit_i(t) - worst(t)) / (best(t) - worst(t)), \quad (9)$$

$$M_i(t) = m_i(t) / \sum_j = 1N m_j(t), \quad (10)$$

where $fit_i(t)$ is the fitness of agent i at iteration t , and $best(t)$ and $worst(t)$ are the best and worst fitness values in the population at that iteration. This normalization ensures that masses sum to unity and that an agent with the best fitness receives the maximum mass value of 1, while an agent with the worst fitness receives mass 0.

Consequently, the population develops a natural hierarchy of attractiveness: agents occupying better regions of the fitness landscape become gravitationally dominant and pull the rest of the population toward them [30].

2.2.2 Gravitational Force and Acceleration

The gravitational force exerted on agent i by agent j along dimension d at iteration t is:

$$F_{ijd}(t) = G(t) \cdot M_i(t) \cdot M_j(t) / (R_{ij}(t) + \varepsilon) \cdot (x_{jd}(t) - x_{id}(t)), \quad (11)$$

where $R_{ij}(t) = \|x_i(t) - x_j(t)\|_2$ is the Euclidean distance between agents i and j , and ε is a small positive constant that prevents division by zero when two agents coincide in space. To avoid fully deterministic behavior and maintain stochasticity, the force from each neighbor is scaled by an independent uniform random number $rand_j \in [0, 1]$:

$$F^i_d(t) = \sum_{j \in K_{best}, j \neq i} rand_j \cdot F^{ij}_d(t). \quad (12)$$

The acceleration of agent i in dimension d is obtained by dividing the total force by its inertial mass, yielding $a_i^d(t) = F^i_d(t) / M_i(t)$. This formulation means that heavier agents (better solutions) experience less acceleration from gravitational forces relative to their mass an intended property that stabilizes the best solutions in the population while lighter agents (poorer solutions) are accelerated more strongly toward better regions [35].

2.2.3 Gravitational Force and Acceleration

The velocity and position of each agent are updated at each iteration as follows:

$$v_i^d(t+1) = rand_i \cdot v_i^d(t) + a_i^d(t), \quad (13)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1). \quad (14)$$

The random factor $rand_i \in [0, 1]$ applied to the velocity serves as an inertial weight, introducing randomness into the motion and preventing agents from following overly deterministic trajectories [41]. In the hybrid HGSO formulation adopted in this work, this basic GSA velocity update is extended with a PSO component and an explicit inertia weight w , as detailed in Section 3.

2.2.4 Gravitational Constant and Kbest Mechanism

Two key control mechanisms regulate the balance between exploration and exploitation in the GSA. First, the gravitational constant $G(t)$ decays exponentially with each iteration, as shown in equation 15:

$$G(t) = G_0 \cdot \exp(-\alpha \cdot t / T), \quad (15)$$

where G_0 is the initial gravitational constant, α is the decay coefficient, and T is the total number of iterations. As $G(t)$ approaches zero, the gravitational forces between agents weaken, effectively freezing the population and allowing it to converge to the solutions found during the exploration phase. Choosing a larger α accelerates this decay, biasing the algorithm toward exploitation at the cost of exploration time. Second, the Kbest strategy restricts which agents contribute to the gravitational force on agent i . At each iteration t , only the Kbest(t) highest-quality agents are considered, as shown below:

$$Kbest(t) = \text{round}((N - (N - 1) \cdot (t / T))). \quad (16)$$

This linear decay scheme starts with all N agents contributing to the force at $t = 0$ (maximum exploration, maximum diversity of attractive sources) and ends with only the single best agent at $t = T$ (full exploitation, pure elitist attraction). The combination of decaying G and decaying Kbest creates a two-mechanism shift from broad stochastic exploration to focused exploitation, which is fundamental to GSA's ability to balance global and local search [43].

2.2.5 Strengths and Limitations of GSA

GSA's physics-inspired mechanism provides several advantages: (I) the mass-based selection of attractive agents is self-adaptive, requiring no explicit archive or external ranking structure; (II) the Kbest mechanism provides a principled, parameter-free mechanism for transitioning from exploration to exploitation; (III) the algorithm exhibits strong exploitation capability in unimodal and low-dimensional problems.

However, GSA also presents notable limitations. The exponential decay of G is sensitive to the choice of α : too aggressive a decay causes premature stagnation before the global optimum is

identified, while too slow a decay extends the exploration phase unnecessarily. Furthermore, as $G \rightarrow 0$ all agents effectively stop moving, making it difficult to escape local optima once the population has been prematurely captured. These limitations motivate the integration of GSA into a hierarchical framework that provides structural diversity management, as proposed in this work.

2.3 Galactic Swarm Optimization (GSO)

Galactic Swarm Optimization, proposed by Muthiah-Nakarajan and Noel in 2016 [4], is a hierarchical multi-population metaheuristic that draws its organizational structure from cosmological observations. In the universe, matter is organized across multiple scales: individual stars orbit within star clusters, star clusters are gravitationally bound within galaxies, and galaxies themselves interact within galactic superclusters.

GSO mirrors this hierarchy by organizing candidate solutions into nested populations that evolve on different timescales and spatial scales of the search space. The fundamental insight motivating GSO is that exploration and exploitation can be decoupled across hierarchical levels: lower levels conduct intensive local search within subregions, while upper levels coordinate global information exchange.

In its original formulation, GSO employs hierarchical levels: particle level in which individual agents within each subswarm update their positions using PSO with personal best and local swarm best as attractors; swarm level in which the best agent of each subswarm acts as a representative that interacts with representatives of other subswarms, guided by the personal best of the representative and the global galaxy best; and galaxy level in which the best representatives from swarm level interact at the highest scale, guided by the overall best position found across the entire population.

The velocity update at Level 1 for particle p in subswarm k is:

$$v_p(t+1) = w \cdot v_p(t) + c_1 \cdot r_1 \cdot (pbest_p - x_p(t)) + c_2 \cdot r_2 \cdot (lbest_k - x_p(t)), \quad (17)$$

where $pbest_p$ is the personal best position of particle p , $lbest_k$ is the best position found within subswarm k , and c_1 , c_2 are cognitive and social

coefficients respectively. At Level 2, the representative of subswarm k updates its position using a similar PSO rule but with the galaxy best ($gbest$) as the social attractor instead of the local subswarm best:

$$v_k(t+1) = w \cdot v_k(t) + c_3 \cdot r_3 \cdot (pbest_k - x_k(t)) + c_4 \cdot r_4 \cdot (gbest - x_k(t)). \quad (18)$$

2.3.1 Hierarchical Diversity Preservation

The principal advantage of GSO's hierarchical structure is its natural mechanism for maintaining population diversity. Because subpopulations at Level 1 evolve independently for multiple iterations before sharing information through Level 2, each subswarm has the opportunity to explore a distinct region of the search space.

The best solution from each subpopulation is then collected and exposed to gravitational influence from the galaxy best at Level 2, creating an information channel that propagates globally useful information downward while allowing each subpopulation to retain locally discovered solutions. This contrasts with flat single-population algorithms, where all agents are immediately influenced by the global best, leading to rapid convergence that sacrifices diversity.

The epoch-based organization further reinforces this diversity property. An epoch consists of a full cycle of Level 1 iterations across all subpopulations followed by Level 2 iterations. Multiple epochs allow the algorithm to repeatedly refine solutions at both levels, with Level 2 results potentially redirecting Level 1 subpopulations toward more promising regions in subsequent epochs.

In this work, a simplified two-level hierarchy is adopted: Level 1 corresponds to independent subpopulation search (galaxies), and Level 2 corresponds to the interaction of super-agents formed from the best solution of each galaxy, guided by the global galaxy best. This simplification preserves the essential exploration-exploitation decoupling of GSO while reducing the computational overhead of the scheme.

2.3.2 Parameter Sensitivity of GSO

The performance of GSO is sensitive to several parameters. The coefficients c_3 and c_4 at Level 2 control the balance between the representative's

personal attraction toward its own best experience and its social attraction toward the galaxy best. When c_3 dominates, representatives retain memory of locally discovered optima and are less influenced by the global best, preserving diversity but potentially slowing convergence. When c_4 dominates, all representatives are strongly attracted to the current galaxy best, accelerating convergence but risking premature loss of diverse solutions.

The appropriate balance shifts as optimization progresses: early iterations benefit from high c_3 (explore locally discovered bests), while later iterations benefit from high c_4 (converge toward global best). This observation motivates the FHGSO variant proposed in this work, in which c_3 and c_4 are dynamically controlled by a Type-1 fuzzy system [7].

2.4 Hybridization: Combining GSA and GSO

The complementary strengths of GSA and GSO create a natural motivation for hybridization. GSA provides a powerful physics-based local search mechanism through gravitational forces, ensuring that agents within a subpopulation converge efficiently toward locally superior positions. However, GSA lacks a structural mechanism for maintaining multi-scale diversity across the population.

GSO, conversely, excels at managing population diversity through its hierarchical multi-population organization, but its PSO-based update at each level does not provide the same intensity of local search as GSA's gravitational dynamics. A hybrid algorithm that replaces PSO at Level 1 with GSA retains GSO's hierarchical diversity management while substituting a more powerful local search engine within each subpopulation.

The proposed hybrid velocity update at Level 1 combines gravitational acceleration from GSA with a PSO cognitive-social component, weighted equally with a coefficient of 0.5:

$$v_p(t+1) = w(t) \cdot v_p(t) + 0.5 \cdot a_{grav,p(t)} + 0.5 \cdot v_{psp,p(t)}, \quad (19)$$

$$v_{psp,p(t)} = c_1 \cdot r_1 \cdot (pbest_p - x_p(t)) + c^2 \cdot r^2 \cdot (lbest_k - x_p(t)), \quad (20)$$

$$w(t) = 1 - t / (Iteration_{max} + 1), \quad (21)$$

where $a_{grav,p}(t) = G(t) \times F_p(t)$ is the gravitational acceleration computed from Kbest interactions as described in Section 2.1, and $w(t)$ is a linearly decreasing inertia weight that transitions from 1 (full velocity memory at $t = 0$) toward 0 (no velocity memory at $t = Iteration_{max}$). The coefficients c_1 and c_2 are randomly sampled from $U(0, 2.05)$ once per trial and remain constant throughout that trial, introducing inter-trial variability while maintaining consistency within a trial.

2.5 Type-1 Fuzzy Dynamic Parameter Adaptation

Dynamic parameter adaptation using Type-1 fuzzy logic has proven highly effective across multiple metaheuristic frameworks. The core principle is to replace fixed parameter values with fuzzy inference rules that map observable algorithm state variables (such as the normalized current iteration) to parameter outputs, producing smooth and interpretable transitions across the optimization timeline.

Bernal et al. [3] introduced fuzzy logic parameter adaptation in GSO, demonstrating that dynamically adjusting the PSO coefficients at Level 1 based on iteration progress significantly improves solution quality on benchmark functions. Subsequent work by the same group [5,6,7] extended this approach to multiple GSO parameter groups and to hybrid GSO formulations, consistently showing that fuzzy-driven parameter transitions outperform static configurations.

Olivas et al. [35] applied a comparable Type-1 Mamdani system to adapt α and Kbest in GSA, demonstrating measurable improvements on unimodal and multimodal benchmarks; a later extension [37] substituted interval Type-2 fuzzy logic, providing additional robustness at the cost of higher computational complexity. Caraveo et al. [8,9], Peraza et al. [39,40], and Amador-Angulo et al. [1] further validated the fuzzy adaptation paradigm across bee colony, harmony search, and other algorithms.

The common design pattern across this body of work is a single fuzzy input (normalized iteration $t/T \in [0,1]$) and Mamdani inference with centroid defuzzification. The membership functions encode

the qualitative rule that parameters promoting exploration should be high when iteration is low and low when iteration is high, with the opposite pattern for parameters promoting exploitation. This design requires only three or five triangular membership functions per variable and three or five rules, making it computationally negligible relative to the cost of fitness evaluations.

2.6 Research Gap and Motivation

Despite the extensive individual literature on GSA adaptation [35,37], GSO adaptation [3,5,6,7], and general fuzzy metaheuristic parameter control [8,9,18,34,40], no prior work has constructed a GSO-GSA hybrid algorithm and systematically evaluated the effect of independent Type-1 fuzzy adaptation applied separately to the GSO component (c_3, c_4) and to the GSA component ($\alpha, Kbest$).

The present work addresses this gap by proposing a single base hybrid algorithm (HGSO) and deriving two independent fuzzy variants FHGSO (fuzzy-adapted GSO coefficients only) and FHGSA (fuzzy-adapted GSA parameters only) that allow the contribution of each fuzzy system to be assessed independently and compared directly.

3 Proposed Algorithm: HGSO and Fuzzy Adaptation Variants

Based on established theoretical foundations, the HGSO (Hierarchical Gravitational Search Optimization) algorithm is presented as a two-level hybrid architecture that synthesizes diversity management through hierarchical structures characteristic of GSO with the local search intensity provided by the gravitational dynamics of GSA. The proposed algorithm is complemented by two independent adaptation variants using fuzzy logic.

The detailed mathematical formulation presented below specifies the population structure, the velocity and position update mechanisms at each level, the modified fitness functions, and the complete specification of the two Mamdani fuzzy inference systems that control the transition between exploration and exploitation phases. This modular architecture facilitates the isolation and

independent evaluation of each adaptation strategy's contribution to the overall performance of the algorithm.

3.1 Algorithm Overview and Design Rationale

The base algorithm, HGSO (Hierarchical Gravitational Search Optimization), implements a two-level hierarchical optimization framework that combines the diversity management architecture of GSO with the physics-based local search of GSA. Level 1 handles intra-subpopulation search: each of the subpop galaxies evolves independently for Iteration1 iterations using a hybrid GSA-PSO velocity update, enabling intensive local search within each subpopulation's current region. Level 2 handles inter-subpopulation interaction: the best solution from each galaxy is designated as a super-agent, and these super-agents interact for Iteration2 iterations using a hybrid gravitational-PSO update guided by the galaxy-wide best solution, enabling global information exchange and redirection of subpopulations.

Two Type-1 fuzzy adaptation variants are derived from HGSO by replacing specific static parameters with the outputs of a Mamdani fuzzy inference system. FHGSO activates the FGSO system which maps normalized iteration to c_3 and c_4 values exclusively at Level 2, leaving all GSA parameters static. FHGSA activates the FGSA system which maps normalized iteration to α and a Kbest value exclusively at Level 2, leaving all GSO coefficients static.

In both variants, Level 1 operates with the same static parameters as the base HGSO, and all non-adapted parameters retain their fixed values. This controlled design allows each experiment to isolate the performance contribution of one fuzzy controller.

3.2 Mathematical Formulation

3.2.1 Population Structure and Initialization

The total population consists of subpop galaxies: Population = {Gal₁, Gal₂, ..., Gal_{subpop}}. Each galaxy Gal_k contains PopSize agents. For each galaxy k and agent p, the initialization is:

$$x_{k,p}(0) = x_{min} + (x_{max} - x_{min}) \cdot rand(D, 1), \quad (22)$$

$$v_{k,p}(0) = v_{min} + (v_{max} - v_{min}) \cdot rand(D, 1) \text{ where } v_{min} = x_{min}, v_{max} = x_{max}. \quad (23)$$

The personal best of each agent is initialized to its starting position. The local best of galaxy k is the agent with the lowest fitness value at initialization. The global galaxy best is the minimum fitness agent across all galaxies.

Per trial randomization: before initializing the populations for each independent trial, four coefficients c_1, c_2, c_3, c_4 are independently drawn from the uniform distribution $U(0, 2.05)$. These coefficients remain constant throughout the trial, introducing inter-trial variability that contributes to the robustness of the reported statistics.

In the FHGSO variant, $c_3(r)$ and $c_4(r)$ are overridden at each iteration of Level 2 by the FGSO fuzzy output; the initial random draw is therefore inactive for those two parameters.

3.2.2 Intra-Subpopulation Gravitational Search

At Level 1, each galaxy evolves independently. For galaxy k at iteration $r \in [0, \text{Iteration1}]$:

Step 1 Gravitational constant: $G(r) = G_0 \cdot \exp(-\alpha \times r / \text{Iteration1})$, with $G_0 = 100$ and $\alpha = 20$ (static, identical for all three variants at Level 1).

Step 2 Mass normalization: For each agent p in galaxy k:

$$\begin{aligned} m_{k,p}(r) &= (fit_{k,p}(r) - worst_k(r)) / (best_k(r) - worst_k(r)); \quad M_{k,p}(r) \\ &= m_{k,p}(r) / \sum_p m_{k,p}(r). \end{aligned} \quad (24)$$

Step 3 Kbest selection: $Kbest_1(r) = \max(1, \text{round}(\text{PopSize} - (\text{PopSize} - 1) \cdot r / \text{Iteration1}))$. Agents are ranked by fitness and only the top $Kbest_1$ contribute to gravitational forces. This linear decay applies identically in all three variants at Level 1.

Step 4 Force and acceleration:

$$F_{k,p}(r) = \sum_{j \in Kbest_1, j \neq p} rand \cdot M_{k,j}(r) \cdot (x_{k,j}(r) - x_{k,p}(r)) / (\|x_{k,j} - x_{k,p}\| + \varepsilon), \quad (25)$$

$$a_{grav,k,p}(r) = G(r) \cdot F_{k,p}(r). \quad (26)$$

Step 5 Hybrid velocity update:

$$v_{pso,k,p}(r) = c_1 \cdot r_1 \cdot (pbest_{k,p} - x_{k,p}(r)) + c_2 \cdot r_2 \cdot (lbest_k - x_{k,p}(r)), \quad (27)$$

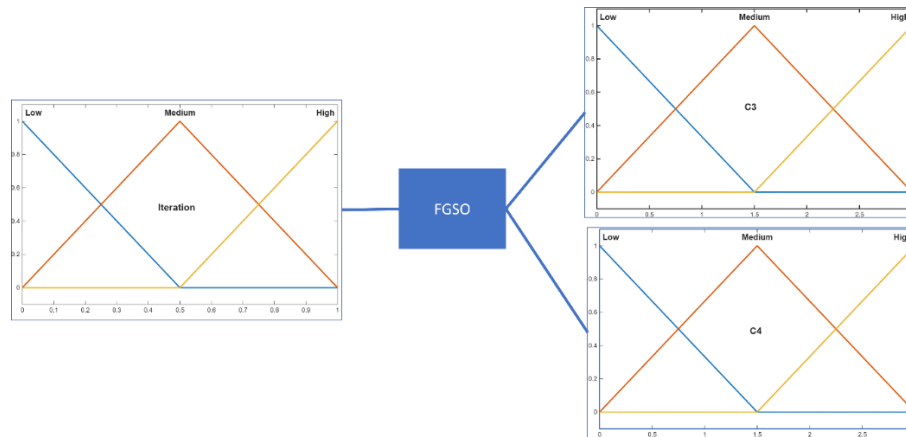


Fig. 2. Graphical structure of the FGSO

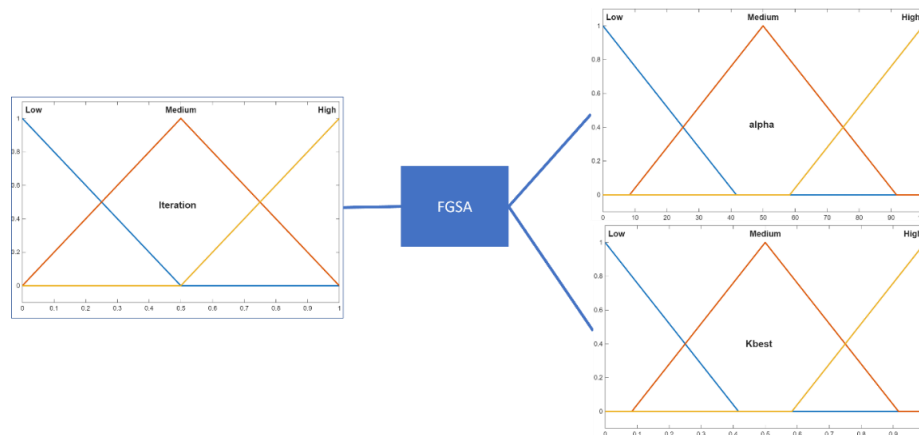


Fig. 3. Graphical structure of the FGSA

$$w(r) = 1 - r / (Iteration_1 + 1), \quad (28)$$

$$v_{k,p}(r+1) = w(r) \cdot v_{k,p}(r) + 0.5 \cdot a_{grav,k,p}(r) + 0.5 \cdot v_{ps0,k,p}(r). \quad (29)$$

Velocity is clamped to $[v_{min}, v_{max}]$ and positions to $[x_{min}, x_{max}]$ after each update.

3.2.3 Inter-Subpopulation Interaction via Super-Agents

At the end of each epoch's Level 1 phase, the best solution from each galaxy k is extracted to form super-agent k : $super_k.x = lbest_k$, $super_k.pbest = lbest_k$, $super_k.v = 0$ (velocity reset to zero). Super-agents then interact for $Iteration_2$ iterations.

At each iteration r of Level 2, the fuzzy system is evaluated first (for FHGSO or FHGSA), and the

resulting parameter values are used for that iteration's update. The input to the fuzzy system is the normalized iteration $t_{norm} = r / Iteration_2 \in [0, 1]$.

For FHGSA: $[\alpha(r), Kbest_2(r)] \leftarrow FGSA(t_{norm})$. The α output determines the gravitational constant: $G(r) = G_0 \times \exp(-\alpha(r) \times r / Iteration_2)$. The $Kbest_2(r)$ output is used directly as the number of super-agents contributing to gravitational forces. In HGSO and FHGSO, $\alpha = 20$ (static) and $Kbest_2$ retains the value computed during the last Level 1 iteration.

For FHGSO: $[c_3(r), c_4(r)] \leftarrow FGSO(t_{norm})$. In HGSO and FHGSA, c_3 and c_4 retain the values randomly drawn at the start of the trial.

The Level 2 super-agent velocity update mirrors the Level 1 structure:

$$v_{ps0,k}(r) = c_3(r) \cdot r_3 \times (pbest_k - x_k(r)) + c_4(r) \cdot r_4 \cdot (galaxy_x - x_k(r)), \quad (30)$$

$$v_k(r+1) = w(r) \cdot v_k(r) + 0.5 \cdot a_{grav,k}(r) + 0.5 \cdot v_{ps0,k}(r); \quad w(r) = 1 - r/(Iteration_2 + 1). \quad (31)$$

3.3 Fuzzy Dynamic Parameter Adaptation Strategies

Two independent Type-1 Mamdani fuzzy inference systems are designed, each targeting a distinct parameter group of the Level 2 super-agent update. Both systems share the same structural design: one input variable (normalized iteration $t_{norm} \in [0, 1]$), two output variables, three triangular membership functions per variable, and three rules. Centroid defuzzification is used.

Each system is applied independently in a separate algorithm variant; they are never activated simultaneously.

3.3.1 FGSO: Fuzzy System for c_3 and c_4 (FHGSO Variant)

The FGSO system governs the Level 2 PSO social coefficients c_3 and c_4 of the super-agent update. The design rationale is grounded in the dynamics of inter-subpopulation coordination: in early iterations (low t_{norm}), super-agents should be strongly influenced by their personal best (high c_3) to preserve the locally discovered optima of their respective galaxies and avoid premature convergence toward a possibly non-optimal galaxy best.

As iterations progress (high t_{norm}), the balance shifts toward the galaxy best (high c_4) to accelerate final convergence. The antithetic rule structure c_3 decreasing while c_4 increases ensures that the total PSO influence magnitude remains bounded across all iterations. Figure 2 shows the graphical structure of the FGSO fuzzy system.

3.3.2 FGSA: Fuzzy System for α and Kbest (FHGSA Variant)

The FGSA system governs the gravitational parameters α and Kbest2 used during Level 2 super-agent interaction. The design rationale mirrors the well-established understanding of GSA dynamics: a low α in early iterations allows $G(r)$ to remain large for longer, sustaining strong

gravitational forces and broad search; a high Kbest2 in early iterations ensures that super-agents are attracted by many peers, maintaining diversity at the inter-subpopulation level.

As optimization progresses, increasing α accelerates the decay of $G(r)$ toward zero, effectively halting motion and crystallizing the solution, while decreasing Kbest2 focuses gravitational forces to the single best super-agent, implementing pure elitist attraction. The inverse relationship between α and Kbest2 across the three rules thus encodes an exploration to exploitation transition in gravitational terms, complementary to the PSO-based transition implemented by FGSO. Figure 3 shows the graphical structure of the FGSA fuzzy system.

4 Experimental Setup

Rigorous validation of the proposed algorithm requires a carefully designed experimental protocol that allows for both robustness assessment in problems of known complexity and performance evaluation in challenging optimization scenarios representative of the state of the art.

This section describes in detail the algorithm's operating parameters, adjusted according to the problem's dimensionality, and the characteristics of the test functions used in two distinct experimental phases.

The first experimental phase uses well-characterized classical mathematical benchmark functions in varying dimensionalities, while the second phase incorporates the 12 functions of the IEEE CEC 2022 boundary-constrained optimization standard, which introduce realistic features such as optimum displacements, rotated landscapes, and hybrid and composite structures.

4.1 Algorithm Parameters

The parameter settings for the HGSO, FHGSO, and FHGSA algorithms were adjusted according to the problem's dimensionality, ensuring a balance between complexity and exploration capacity. At lower dimensions, fewer iterations and a smaller number of subpopulations were used, while at higher dimensions, the number of iterations and

Table 1. Parameters of HGSO, FHGSO and HGSA

Dimension	Epochs	Iteratiton1	Iteratiton1	PopSize	Subpops
10	5	100	300	5	10
30	5	280	600	5	20
50	9	250	700	5	20

Table 2. Classical benchmark functions

Function	Formula (summary)	Domain	Optimum	Type
F1 Rosenbrock	$\sum [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-5, 10]$	0 at (1,...,1)	Unimodal
F2 Sum Squares	$\sum i \cdot x_i^2$	$[-10, 10]$	0 at origin	Unimodal
F3 Zakharov	$\sum x_i^2 + (\sum 0.5i \cdot x_i)^2 + (\sum 0.5i \cdot x_i)^4$	$[-5, 10]$	0 at origin	Unimodal
F4 Shubert	$\prod_i [\sum_{j=1}^5 j \cdot \cos((j+1)x_i + j)]$	$[-10, 10]$	≈ -186.731	Multimodal
F5 Beale	Sum of squared residuals	$[-4.5, 4.5]$	0 at (3, 0.5)	Multimodal
F6 Booth	$(x+2y-7)^2 + (2x+y-5)^2$	$[-10, 10]$	0 at (1, 3)	Unimodal
F7 Dixon-Price	$(x_1 - 1)^2 + \sum i(2x_i^2 - x_{i-1})^2$	$[-10, 10]$	0 (analytical)	Unimodal

subpopulations was significantly increased to promote diversity and convergence.

The values of $G_0 = 100$ and $\alpha = 20$ were kept constant at the basic level. Similarly, the c_1 , c_2 , c_3 and c_4 were randomly generated in each trial under a uniform distribution in $[0, 2.05]$, with the particularity that at the second level, c_3 and c_4 , α , and k_{best} were redefined by FGSO and FGSA, respectively.

Finally, statistical robustness was ensured by running 30 independent tests per configuration, allowing for the evaluation of the stability and consistency of the results. The base algorithm, HGSO (Hierarchical Gravitational Search Optimization), implements a two-level hierarchical optimization framework that combines the diversity management architecture of GSO with the physics-based local search of GSA.

The parameters used to perform the experiment are shown in Table 1.

4.2 Benchmark Mathematical Functions

The first experimental phase evaluated all three variants on seven established benchmark functions. These functions span diverse landscape

characteristics: unimodal, multimodal, separable, and non-separable structures.

The second experimental phase evaluated the three variants on the 12 functions of the IEEE CEC 2022 Single-Objective Bound Constrained Optimization benchmark. This suite is specifically designed to challenge modern optimization algorithms with features including shifted optima, rotated landscapes, hybrid compositions, and composite function structures. Functions F1–F5 represent basic transformed functions, F6–F8 are hybrid functions, and F9–F12 are composite functions [23].

Table 2 shows the classic benchmark functions used in the Phase 1 experiments. The tests were performed with 10, 30, and 50 dimensions in 30 independent trials.

Table 3 shows the functions according to IEEE CEC 2022 (2, 10 and 20 dimensions were tested).

For the classical benchmark suite, mean fitness values across 30 independent trials were reported alongside the identification of the best-performing variant per function and dimension. For CEC 2022, mean fitness values are compared against the theoretical optimal value of each function, with the best-performing variant highlighted. Additional

Table 3. IEEE-CEC 2022 functions

f	Description	Domain	Optimal Value	Type
F1	Shifted Zakharov Function	[−100,100]	300	Basic
F2	Shifted Rosenbrock's Function	[−100,100]	400	Basic
F3	Shifted Expanded Scaffer's F6	[−100,100]	600	Basic
F4	Shifted Non-Continuous Rastrigin's	[−100,100]	800	Basic
F5	Shifted Levy Function	[−100,100]	900	Basic
F6	Hybrid Function 1 (N=3)	[−100,100]	1800	Hybrid
F7	Hybrid Function 2 (N=4)	[−100,100]	2000	Hybrid
F8	Hybrid Function 3 (N=4)	[−100,100]	2200	Hybrid
F9	Composite Function 1 (N=5)	[−100,100]	2300	Composite
F10	Composite Function 2 (N=4)	[−100,100]	2400	Composite
F11	Composite Function 3 (N=5)	[−100,100]	2600	Composite
F12	Composite Function 4 (N=6)	[−100,100]	2700	Composite

metrics (standard deviation, best, worst) are provided in the discussion.

5 Experimental Setup

This section presents the results obtained from the proposed method and its different variations.

Seven classical mathematical functions were used, evaluated in 10, 20, and 30-dimensional spaces, as well as the IEEE-CEC 2022 reference functions described previously.

The experiments show consistent behavior in the method's ability to adapt to increasing dimensionality, revealing both strengths and limitations in more complex scenarios.

Comparing the classical functions with those from IEEE-CEC 2022 reveals significant differences: while the former offers a controlled framework for validating the algorithm's stability, the latter introduce challenges closer to real-world optimization problems, where robustness and generalizability become critical. In this regard, the results suggest that the proposed method maintains competitive performance against demanding benchmarks.

However, in this case, areas for improvement are identified in highly multimodal and high-dimensional functions.

5.1 Classical Benchmark Functions

At 10D, the most challenging function is Rosenbrock. FHGSA achieves $3.546e-5$, versus FHGSO ($3.999e-4$) and HGSO (0.00175424), approx. 98.0% improvement over the base. GSA gravitational dynamics are the bottleneck for Rosenbrock. Functions with zero optima (Sum Squares, Zakharov, Beale, Booth) are solved to machine precision by all variants. The Shubert function reaches its global optimum in all cases.

Table 5. Mean fitness values on classical benchmarks 30D. Bold = best variant. Values shown are means over 30 runs.

At 30D, the most challenging function is Rosenbrock. FHGSA achieves $2.783e-8$, versus FHGSO ($4.743e-5$) and HGSO ($3.261e-4$), approx. 100.0% improvement over the base. GSA gravitational dynamics are the bottleneck for Rosenbrock. Functions with zero optima (Sum Squares, Zakharov, Beale, Booth) are solved to machine precision by all variants. The Shubert function reaches its global optimum in all cases.

Table 6. Mean fitness values on classical benchmarks 50D. Bold = best variant. Values shown are means over 30 runs.

At 50D, the most challenging function is Rosenbrock. FHGSA achieves $2.513e-12$, versus FHGSO ($3.177e-5$) and HGSO ($3.440e-5$), approx. 100.0% improvement over the base. GSA

Table 4. Experimental results 10 D

Function	HGSO	FHGSO	FHGSA	Best
Rosenbrock	0.00175424	3.999e-4	3.546e-5	FHGSA
Sumsquare	0	0	0	HGSO
Zakharov	0	0	0	HGSO
Shubert	-186.731	-186.731	-186.731	HGSO
Beale	1.797e-26	5.476e-28	9.440e-22	FHGSO
Booth	9.977e-25	1.580e-29	2.102e-24	FHGSO
Dixon-Price	0	0	0.182481	HGSO

Table 5. Experimental results 30 D.

Function	HGSO	FHGSO	FHGSA	Best
Rosenbrock	3.261e-4	4.743e-5	2.783e-8	FHGSA
Sumsquare	0	0	0	HGSO
Zakharov	0	0	0	HGSO
Shubert	-186.731	-186.731	-186.731	HGSO
Beale	0	0	0	HGSO
Booth	0	0	0	HGSO
Dixon-Price	0	0	0	HGSO

gravitational dynamics are the bottleneck for Rosenbrock. Functions with zero optima (Sum Squares, Zakharov, Beale, Booth) are solved to machine precision by all variants.

The Shubert function reaches its global optimum in all cases.

5.2 CEC 2022 Benchmark Functions

At 2 dimensions, all three variants achieve the theoretical optimum across all 12 functions, confirming that the hierarchical architecture provides sufficient search power for low-dimensional CEC problems. No differentiation between fuzzy and non-fuzzy variants is observed at this dimensionality (see Table 7).

At 10 dimensions, the two fuzzy variants show complementary advantages. FHGSO achieves the exact optimum on F3 (Expanded Scaffer's F6) while HGSO deviates slightly (600.46), demonstrating that Level 2 coefficient diversity benefits this function. FHGSA achieves the exact optimum on F9 (Composite Function 1, value 2300) while HGSO reaches 2529.3 and FHGSO

reaches 2524.9, demonstrating that gravitational dynamics are the binding constraint for composite functions. Basic functions F1–F2, F5, F10, F12 are solved optimally by all variants (see Table 8).

At 20 dimensions, the complementary nature of the two fuzzy strategies is most evident. FHGSO achieves the optimum on F3 and records 1967.1 on F6 (vs. 2046.4 for HGSO), a 3.9% improvement attributable to enhanced Level 2 diversity. FHGSA maintains the exact optimal value on F9 (2300) as the only variant to do so, while HGSO reaches 2480.8 and FHGSO reaches 2480.8. FHGSA also matches the optimum on F11 (2600) where FHGSO degrades to 2708.6. These patterns confirm that composite functions (F9, F11) benefit most from adaptive gravitational dynamics, while hybrid functions (F3, F6) are more sensitive to Level 2 PSO coefficient adaptation (see Table 9).

5.3 Comparative Analysis of Fuzzy Adaptation Contributions

The comparison across HGSO, FHGSO, and FHGSA reveals the distinct and complementary

Table 6. Experimental results 30 D

Function	HGSO	FHGSO	FHGSA	Best
Rosenbrock	3.440e-5	3.177e-5	2.513e-12	FHGSA
Sumsquare	0	0	0	HGSO
Zakharov	0	0	0	HGSO
Shubert	-186.731	-186.731	-186.731	HGSO
Beale	0	0	0	HGSO
Booth	0	0	0	HGSO
Dixon-Price	0	0	0	HGSO

Table 7. Results of CEC 2022 for 2 D

f	Optimal	HGSO	FHGSO	FHGSA	Best
F1	300	300	300	300	HGSO
F2	400	400	400	400	HGSO
F3	600	600	600	600	HGSO
F4	800	800	800	800	HGSO
F5	900	900	900	900	HGSO
F6	1800	0.0000	0.0000	0.0000	HGSO
F7	2000	0.0000	0.0000	0.0000	HGSO
F8	2200	0.0000	0.0000	0.0000	HGSO
F9	2300	2300	2300	2300	HGSO
F10	2400	2400	2400	2400	HGSO
F11	2600	2600	2600	2600	HGSO
F12	2700	2700	2700	2700	HGSO

roles of each fuzzy system. FHGSO which adapts the Level 2 GSO coefficients $c3$ and $c4$ primarily improves functions where the inter-subpopulation interaction is the bottleneck. This is most evident on CEC 2022 functions F3 and F6 at 10 and 20 dimensions, where Level 2 diversity controls the algorithm's ability to sample distinct basins.

FHGSA which adapts the GSA parameters α and K_{best} primarily benefits functions where the intensity and selectivity of gravitational search are decisive, as evidenced by improvements on CEC composite functions F9 and F11 and the classical Rosenbrock function across all dimensions.

Notably, neither FHGSO nor FHGSA dominates the other across all test cases. FHGSO (Type-1 fuzzy on GSO coefficients only) outperforms FHGSA on F3 and F6 at higher CEC

dimensions, while FHGSA (Type-1 fuzzy on GSA parameters only) outperforms FHGSO on composite CEC functions F9 and F11 and on the Rosenbrock classical function.

This functional specialization confirms that the two parameter groups control genuinely different behavioral aspects of the hybrid algorithm and that their relative importance is problem-dependent. A future variant combining both FGSO and FGSA Type-1 systems together (FHGSO+FHGSA) would determine whether additive or synergistic improvements arise.

A consistent pattern across both benchmark suites is that the performance gap between HGSO and each fuzzy variant widens with increasing dimensionality. At 2D (CEC), all variants are equivalent. At 10D, advantages are marginal.

Table 8. Results CEC 2022 for 10 D

f	Optimal	HGSO	FHGSO	FHGSA	Best
F1	300	300	300	300.0000	HGSO
F2	400	400	400	400	HGSO
F3	600	600.4571	600	600	FHGSO
F4	800	800	804.2736	800	HGSO
F5	900	900	900	900	HGSO
F6	1800	1829.2413	1824.2914	1827.0005	FHGSO
F7	2000	2009.8863	2007.0873	2009.8166	FHGSO
F8	2200	2203.9682	2204.3409	2204.4534	HGSO
F9	2300	2529.2844	2524.8649	2300	FHGSA
F10	2400	2400	2400	2400	HGSO
F11	2600	2600	2600.0058	2600	HGSO
F12	2700	2700	2700	2700	HGSO

Table 9. Results CEC 2022 for 20 D

f	Optimal	HGSO	FHGSO	FHGSA	Best
F1	300	301.2444	300.7353	301.2808	FHGSO
F2	400	400	400	400	HGSO
F3	600	605.5090	600	600	FHGSO
F4	800	800	817.8197	800	HGSO
F5	900	900	900	900	HGSO
F6	1800	2046.3922	1967.1333	2097.8697	FHGSO
F7	2000	2035.9618	2038.1940	2038.9498	HGSO
F8	2200	2225.4187	2224.5096	2225.1818	FHGSO
F9	2300	2480.8075	2480.7879	2300	FHGSA
F10	2400	2400	2400	2400	HGSO
F11	2600	2600	2708.5903	2600	HGSO
F12	2700	2700	2700	2700	HGSO

At 20D and 50D, FHGSA achieves up to several orders of magnitude improvement on the Rosenbrock function over HGSO, while FHGSO achieves the most significant gains on hybrid CEC functions.

This scalability profile is consistent with the theoretical motivation: as dimensionality increases, fixed parameters become progressively suboptimal because the appropriate balance between exploration and exploitation shifts more dramatically across the optimization timeline, making dynamic adaptation increasingly valuable.

6 Conclusions and Future Work

This paper introduced two independent fuzzy dynamic parameter adaptation strategies for a hierarchical GSO-GSA hybrid algorithm (HGSO).

The FHGSO variant applies the FGSO inference system exclusively to the GSO Level 2 PSO coefficients c_3 and c_4 , dynamically controlling the balance between personal and global best attraction in inter-subpopulation interactions.

The FHGSA variant applies the FGSA inference system exclusively to the GSA

parameters α (gravitational decay) and Kbest (agent selection ratio), dynamically controlling the intensity and selectivity of the gravitational search.

Both systems use a single normalized input (current iteration) and Mamdani inference with centroid defuzzification.

Experimental results on seven classical benchmark functions (10, 30, 50 dimensions) and 12 IEEE CEC 2022 functions (2, 10, 20 dimensions) demonstrate that: (1) the base HGSO already solves the majority of functions to machine precision, confirming the power of the hierarchical framework; (2) FHGSO provides improvements on functions sensitive to Level 2 diversity notably CEC F3 and F6 at 10D and 20D through adaptive c_3/c_4 control; (3) FHGSA provides improvements on composite functions and the Rosenbrock function notably CEC F9 and F11 through adaptive α and Kbest control; and (4) neither fuzzy variant universally dominates the other, confirming that the two parameter groups address distinct aspects of the optimization dynamics.

Future Research Directions include the following possibilities:

- **Interval Type-2 Fuzzy Extension:** Extend both FGSO and FGSA from Type-1 to interval Type-2 fuzzy inference systems [28,32,33,34] to handle uncertainty in the input-output mapping, potentially improving robustness on noisy or rotated benchmark landscapes.
- **Adaptive Rule Base:** Investigate self-adaptive or learned fuzzy rules, potentially using evolutionary algorithms or reinforcement learning to optimize membership function parameters for specific problem classes.
- **Combined FHGSO+FHGSA Variant:** Investigate whether simultaneously activating both the FGSO and FGSA systems yields additive or synergistic improvements, and identify the conditions under which the combined variant is most beneficial.
- **Real-World Applications:** Apply the fuzzy variants to engineering optimization such as neural network hyperparameter tuning, fuzzy controller design [13,17], and power system optimization [22].

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