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Development of Generalized Type-2 Fuzzy Systems for Integrating Recurrent Neural Network Ensembles in Bitcoin Time Series Prediction Using Particle Swarm and Genetic Algorithm

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Abstract. This article presents a model for time series prediction. This is achieved by optimizing an ensemble recurrent neural network, and its integration is performed using fuzzy systems. These are type-1, generalized interval type-2, and Sugeno and Mamdani logic. The algorithms used for optimization were genetic algorithm (GA) and particle swarm optimization (PSO). The data used correspond to the Bitcoin time series.The main goal is to improve prediction accuracy by integrating the outputs of the neural network with those of the fuzzy systems. Each system consists of one output variable and five input variables, with two membership functions per input, resulting in a total of 32 rules. The results demonstrate that both Type-2 and Type-1 fuzzy systems provide accurate predictions of market behavior, which is further supported by statistical validation.

Keywords. Type-2 generalized fuzzy system, time series prediction, ensemble neural networks, Mamdani model, Sugeno model.

1 Introduction

Time series analysis is a statistical method applied to examine data gathered at consistent time intervals [1-3]. Its main objective is to identify patterns, trends and temporal relationships in data to make predictions or make informed decisions. [4-6]. Time series are found in various disciplines, such as financial, data mining and climate [7-11]. Common examples include stock prices, daily temperatures, a company's monthly sales, and air pollution levels [12-14].

The use of fuzzy logic has expanded in multiple areas, including: Process Control: It is used in air conditioning systems, speed control in automobiles, and regulation of electric motors, allowing smoother and more efficient adjustments [15-17]. Diagnostic and Decision-Making Systems: In medicine, it is used to diagnose diseases by handling imprecise or subjective data from symptoms and medical tests [18-24]. Pattern Recognition: It is applied in artificial intelligence for image, voice, and writing recognition, improving accuracy in data classification [25-29]. Optimization and Automation: Found in route planning, traffic management, and financial analysis systems, allowing more adaptive decisions in a complex environment [29-30]. Smart Appliances: Many modern devices, such as air conditioners, cameras, and washing machines, use fuzzy logic to improve their performance based on changing conditions [31-33].

This work mainly contributes to the design of Generalized Fuzzy Type Systems to handle uncertainty in ENN decisions [34-39]. This paper offers a robust, flexible, and adaptive approach for forecasting highly volatile time series, such as the cryptocurrency market. Its value lies in the synergistic combination of deep learning techniques, advanced fuzzy logic, and evolutionary algorithms to address the challenges of uncertainty and nonlinearity in financial data.

In the field of artificial intelligence and evolutionary computation, several methods derived from natural phenomena are used to address complex optimization problems. These approaches are used due to their ability to find efficient solutions in large, nonlinear search spaces. [40-43]. Genetic algorithms refer to the foundations of evolutionary theory, incorporating

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mechanisms such as mutation, natural selection, reproduction, and genetic crossover. In a GA, a population of potential solutions (called chromosomes) evolves iteratively.

The evaluations are based on a fitness function; the algorithm selects and combines the best individuals, promoting continuous improvement of the population. In contrast, particle swarm optimization (PSO) draws inspiration from the collective behavior of natural swarms, including schools of fish or flocks of birds. Each particle in PSO is a potential solution that travels around the search space based on its own and its neighbors' experiences. [44-50].

In this instance, the process is carried out to forecast the Bitcoin time series [51]. A fuzzy system is proposed as a model for time series prediction this Sugeno and Mamdani Generalized Type-2 fuzzy system is proposed for the first time, it is made up of 1 output and 1 output called predicted with 32 possible fuzzy rules, for the prediction of time series, even a type-2 Generalized fuzzy system was implemented to compare the results and demonstrate that Type-2 Generalized Fuzzy Systems obtain are good results for time series prediction [52-54].

The contribution of this study lies in the development of the neural ensemble and its optimization using GA and PSO, along with the design of Interval Type-2, General Type-2, and Type-1 Fuzzy Inference Systems to the prediction process of Ensemble Neural Network (ENN).

This approach is tested using Bitcoin time series forecasting. A fuzzy system is proposed as a model to combine the ENN outputs and improve prediction. The fuzzy system consists of 1 output and 5 inputs, with 32 fuzzy rules for time series prediction. Furthermore, a comparison was performed, demonstrating that Generalized Type-2 systems offer favorable results for time series forecasting.

The rest of the Research work is structured as follows: Part 2 introduces the theoretical details of Generalized Type-2, Part 3 summarizes the procedure and describes the methodology. Part 4 simulation results, tables, and comparison. Finally, Part 5 summarizes the conclusion.

2 Generalized Type-2 Fuzzy System

The structure of the *l*th generic GT2 Zadeh rule for a GT2 fuzzy system is (*l* = 1, ..., *M*):

$$\tilde{\mathrm{R}}_{\mathrm{Z}}^{1} : \mathrm{IF}\ \mathrm{X}_1 \tilde{\mathrm{F}}_1^{\mathrm{l}}\ \text{and} \ldots\ \text{and}\ \mathrm{x_p}\ \text{is}\ \mathrm{x_p}\ \text{is}\ \tilde{\mathrm{F}}_{\mathrm{p,}}^{\mathrm{l}}\ \ \mathrm{THEN}\ \ \mathrm{y}\ \text{is}\ \tilde{G}^{1}, \tag{1}$$

whereas the structure of the *l*th generic GT2 Takagi, Sugeno, and Kang (TSK) rule for a GT2 fuzzy system is (*l* = 1, ..., *M*):

$$\begin{gathered}\tilde{\mathrm{R}}_{\mathrm{TSK}}^{1} : \mathrm{IF}\ \mathrm{X}_1 \tilde{\mathrm{F}}_1^{\mathrm{l}}\ \text{and} \ldots\ \text{and}\ \mathrm{x_p}\ \text{is}\ \mathrm{x_p}\ \text{is}\ \tilde{\mathrm{F}}_{\mathrm{p,}}^{\mathrm{l}} \\ \mathrm{THEN}\ \ \mathrm{y}\ \text{is}\ \ g^{1}(x_1 \ldots, \mathrm{x_p}),\end{gathered} \tag{2}$$

in both (1, 2), $\tilde{\mathrm{F}}_1^{\mathrm{l}} \in T_{X_1}, \ldots, and\ \ \tilde{\mathrm{F}}_{\mathrm{p}}^{\mathrm{l}}\ \in\ T_{X_p}$.

In (1), because $\tilde{G}^1 \in\ T_Y$ is a GT2 FS, it is described by its MF $\mu_{\tilde{G}}$ (y**,** *u*). In (2), although *y* does not seem to be a fuzzy set, it can be modeled as a type-2 fuzzy singleton **G̃**ᵢ, so it is made to resemble a GT2 Zadeh rule, where:

$$\mu_{\tilde{G}\,(y)} \equiv \begin{cases} \frac{1}{1} & when\ y\ g^1(x) \\ \frac{1}{0} & otherwise \end{cases}, \tag{3}$$

$$\begin{gathered}R^K : IF\ x_1\ is\ \tilde{F}_1^k\ \text{ and...and } x_n \text{ is } \tilde{F}_n^k \text{ THEN} \\ y_1\ \tilde{G}_1^k \text{ and...and } y_m \text{ is } \tilde{G}_m^k,\end{gathered} \tag{4}$$

$$H^K :\ x_1\ is\ \ \tilde{X}_1 \text{ and...and } x_n \text{ is } \tilde{X}_n, \tag{5}$$

$$y_1\ is\ \ \tilde{B}_1 \text{ and...and } y_m \text{ is } \tilde{B}_m, \tag{6}$$

$$\begin{gathered}\tilde{\mu}(\mathrm{x,u}) = \mathrm{gaussmgauss2type2}(\mathrm{x,u},[\sigma, \mathrm{m}_1, \mathrm{m}_2, \\ \lambda, \rho]),\end{gathered} \tag{7}$$

$$\begin{gathered}\tilde{\mu}(x) = [\mu(\mathrm{x}), \bar{\mu}(\mathrm{x})] = \mathrm{igaussmtype2} \\ (\mathrm{x},[\sigma, \mathrm{m}_1, \mathrm{m}_2]),\end{gathered} \tag{8}$$

$$\mu_2 = \tilde{\mu}(x) = \exp \lfloor -\tfrac{1}{2} \ \left(\tfrac{\mathrm{x}-\mathrm{m}_1}{\sigma} \right)^2 \rfloor, \tag{9}$$

$$\mu_2 = \exp \lfloor -\tfrac{1}{2} \ \left(\tfrac{\mathrm{x}-\mathrm{m}_2}{\sigma} \right)^2 \rfloor, \tag{10}$$

$$\bar{\mu}(\mathrm{x}) = \begin{cases} \mu_1(x) & x < m_1 \\ 1 & m_1 \leq \ m_2 \\ \mu_2(x) & x < m_2 \end{cases}, \tag{11}$$

$$\underline{\mu}(x) = \begin{cases} \mu_2(x) & x \leq \ \frac{m_{1+}\ m_2}{2} \\ \mu_1(x) & x > \ \frac{m_{1+}\ m_2}{2} \end{cases}, \tag{12}$$

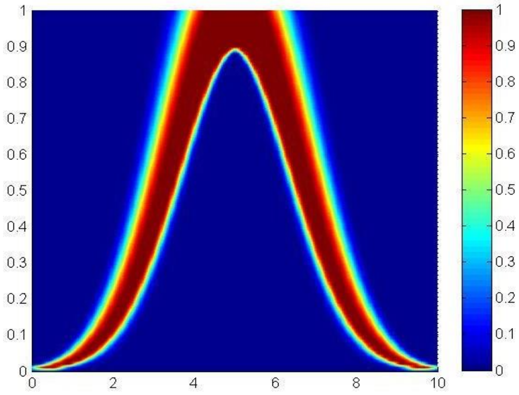

Fig. 1. Gaussian Membership function

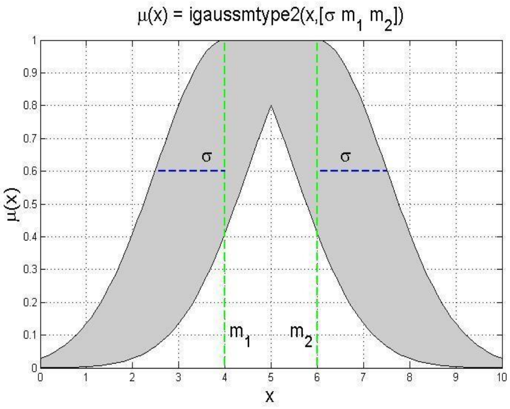

Fig. 2. Igaussmtype2

$$p_x=\exp\left[-\frac{1}{2}\left(\frac{x-m_1}{\sigma}\right)^2\right] \text{ where } m=\frac{m_1+m_2}{2}, \tag{13}$$

$$\delta=\bar{\mu}(x)-\underline{\mu}(x), \tag{14}$$

$$\sigma_u=\frac{\delta}{2\sqrt{3}}+\varepsilon, \tag{15}$$

$$c_1=p_x(1-\lambda), \tag{16}$$

$$\sigma_1=\sigma_u(1-\rho), \tag{17}$$

$$c_2=p_x(1-\lambda), \tag{18}$$

$$\sigma_2=\sigma_u(1-\rho), \tag{19}$$

where λ and are the uncertainty fraction of the nucleus and the support of the secondary membership function, respectively:

$$\tilde{\mu}(x,u)=\begin{cases}\exp\left[-\frac{1}{2}\left(\frac{u-c_1}{\sigma_1}\right)^2\right] & u\le c_1\\ 1 & c_1<u<c_2\\ \exp\left[-\frac{1}{2}\left(\frac{u-c_2}{\sigma_2}\right)^2\right] & u\le c_1\end{cases} \tag{20}$$

3 Model Description

In this section, the proposed model is formulated to optimize the Ensemble Recurrent Neural Network (ERNN) using PSO and GA. The objective is to determine the optimal number of modules within the ensemble, which are subsequently integrated using Generalized Type-2 and Interval Type-2 Fuzzy Systems. Figure 3 depicts the proposed model, which initiates with the acquisition of historical data [3]. optimization algorithms are immediately used to define the structural parameters of ERNN, including the number of modules (between to 5), the number of neurons per layer (between 1 to 30), and the number of layers (between 1 to 3). The outputs generated by the ensemble modules are subsequently integrated through Interval Type-2, Generalized Type-2, and Type-1. These Sugeno and Mamdani fuzzy systems, both of which output linguistic variables of the fuzzy systems, are represented by Gaussian membership functions. The efficiency of fuzzy integrators is systematically evaluated under varying lag and scale values to establish the optimal configuration of the membership functions that govern the fuzzy rules. Each fuzzy integrator employs two Gaussian membership functions, low prediction and high prediction, for both input and output variables. The results, summarized in the table headers, reveal the parameter combinations that yield the most accurate time series predictions. If the ensemble consists of five modules, the corresponding fuzzy system will include 5 input Fuzzy rules are then defined for the input variables and their respective membership functions. Given that each of the five inputs is represented by two membership functions, a total of 32 possible fuzzy rule combinations are generated.

3.1. Bitcoin Time Series Prediction

Bitcoin is a decentralized cryptocurrency that is based on the need for a central authority, meaning that no entity, such as a government or bank, controls its issuance or regulation. Its security is guaranteed through advanced cryptography, and its operation is based on blockchain technology.

Additionally, Bitcoin transactions are public and verifiable, although users can maintain a degree of anonymity as addresses are not directly linked to personal identities. Its use has expanded as a means of investment, transfer of value without intermediaries, and a financial alternative in unstable economies, although it faces challenges such as price regulatory uncertainty, volatility, regulatory uncertainty and the high energy consumption of its mining process.

The time series under study has 1200 data points corresponding to the period from January 1, 2020, to April 14, 2023. On the Y axis, we have the Bitcoin closing price, and on the X axis, we have the days as shown in Figure 4. 30% of this point was used for testing, and 70% for ERNN [55].

3.2 Overview Type-2 and Generalized Type-2 Fuzzy Systems

The Generalized Type-2 fuzzy system consists of five input variables and a single output variable, each associated with two membership functions. The fuzzy rules are obtained from all possible combinations of the input variables and their corresponding membership functions. Given five inputs with two membership functions each, a total of 32 possible rule combinations are obtained, as illustrated in Figure 5. Since the fuzzy system includes five input variables, each with two associated membership functions, a total of 32 possible fuzzy rules can be generated, as illustrated in Table 1. These rules are applied within the Generalized Type-2. Table 2 summarizes the main characteristics of the membership functions used in fuzzy systems integrated with recurrent neural networks for Bitcoin time series prediction. The membership functions used are Gaussian and are defined for both the output and input variables.

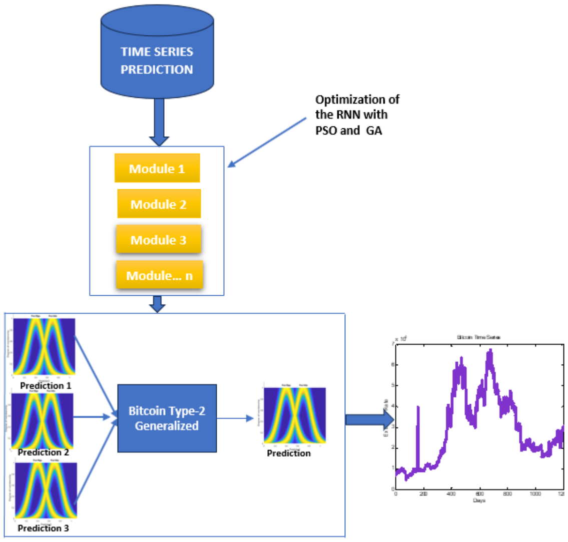

Fig. 3. Proposed Model

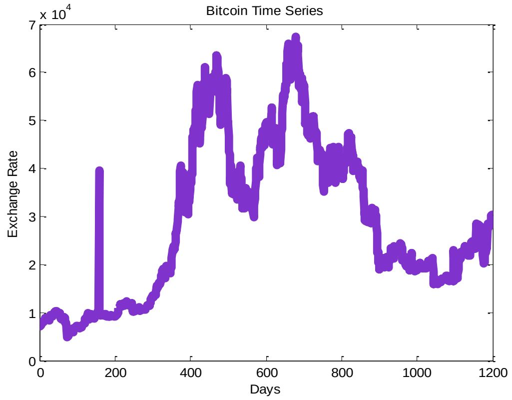

Fig. 4. Bitcoin Time Series

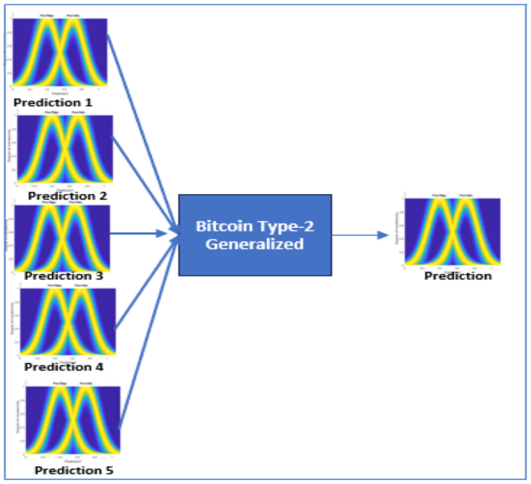

Fig. 5. Type-2 Generalized Fuzzy System Bitcoin for Time Series

Table 1. Rules of Generalized type-2

Rule	Perd 1	Pred2	Pred 3	Pred 4	Pred 5	Pred
1	PDLW	PDLW	PDLW	PDLW	PDLW	PDLW
2	PHH	PHH	PHH	PHH	PHH	PHH
3	PDLW	PDLW	PDLW	PDLW	PHH	PDLW
4	PHH	PHH	PHH	PHH	PDLW	PHH
5	PDLW	PDLW	PDLW	PHH	PHH	PDLW
6	PHH	PHH	PHH	PDLW	PDLW	PHH
7	PDLW	PDLW	PHH	PHH	PHH	PHH
8	NPHH	PHH	PDLW	PDLW	PDLW	PDLW
9	PDLW	PHH	PHH	PHH	PHH	PHH
10	PHH	PDLW	PDLW	PDLW	PHH	PDLW
11	PDLW	PHH	PDLW	PHH	PDLW	PDLW
12	PHH	PDLW	PHH	PDLW	PHH	PHH
13	PDLW	PHH	PDLW	PDLW	PDLW	PDLW
14	PHH	PDLW	PHH	PHH	PHH	PHH
15	PDLW	PDLW	PHH	PDLW	PDLW	PDLW
16	PHH	PHH	PDLW	PHH	PHH	PHH
17	PDLW	PDLW	PDLW	PHH	PDLW	PDLW
18	PHH	PHH	PHH	PDLW	PHH	PHH
19	PDLW	PDLW	PHH	PHH	PDLW	PDLW
20	PHH	PHH	PDLW	PDLW	PHH	PHH
21	PDLW	PHH	PHH	PDLW	PDLW	PDLW
22	PHH	PDLW	PDLW	PHH	PHH	PHH
23	PDLW	PDLW	PHH	PHH	PDLW	PDLW
24	PHH	PHH	PDLW	PDLW	PHH	PHH
25	PDLW	PHH	PHH	PDLW	PHH	PHH
26	PHH	PDLW	PDLW	PHH	PHH	PDLW
27	PDLW	PHH	PDLW	PHH	PDLW	PDLW
28	PHH	PDLW	PHH	PDLW	PHH	PHH
29	PDLW	PHH	PHH	PHH	PHH	PDLW
30	PHH	PDLW	PDLW	PHH	PHH	PDLW
31	PDLW	PHH	PDLW	PHH	PHH	PHH
32	PHH	PDLW	PHH	PDLW	PDLW	PDLW

Table 2. Parameters of generalized type-2 membership functions

Variable	Membership function type	Number of functins	Labels	Fuzzy Type
Input 1 to Input 5	Gaussiana	2	“low prediction”, “high prediction”	Mamdani and SugenoType-1, Type-2 and Generalized Type-2.
Output	Gaussiana	2	“low prediction”, “high prediction”	Sugeno and Mamdani Type-2, Generalized Type-2 and Type-1.

Table 3. Outcomes of the PSO applied to the ERNN

No.	No. Modules	No. Layers	No. Neurons	Duration	Prediction Error
1	4	2	24,10 23,18 12,18 13,16	02:04:20	0.057909
2	3	2	12,28 14,20 13,4	02:06:22	0.057528
3	4	2	19,21 14,22 21,11 24,18	02:03:10	0.05765
4	3	1	15 5 22	02:08:10	0.057992
5	4	1	14 8 20 19	02:06:13	0.057847
6	3	3	15,6,19 3,25,28 19,26,22	02:05:08	0.05765
7	4	1	4 13 15 4	02:06:01	0.057439
8	3	3	22,18,25 15,21,20 20,11,23	02:07:04	0.057601
9	3	2	24,14 2,23 17,13	02:08:22	0.057328
10	3	2	18,26 27,17 17,8	02:09:23	0.057434

Table 4. Results integration Mamdani type-1 fuzzy system for Bitcoin

No.	**Integration Mamdani type-1 fuzzy system**
1	0.6700
2	0.6700
3	0.6700
4	0.6700
5	0.6710
6	0.6700
7	0.6700
8	0.6700
9	0.6700
10	0.6700

Table 5. Integration Sugeno Type-1 fuzzy system for Bitcoin.

No.	Integration Sugeno Type-1 Fuzzy System
1	0.800
2	0.799
3	0.800
4	0.799
5	0.800
6	0.799
7	0.799
8	0.799

Table 3 presents outcomes of the PSO, based on the total of 30 experiments. For clarity, only the top 10 results are displayed.

Here the best prediction error was 0.0089454 in row 7Table 4 displays the integration outcomes of the Type-1 fuzzy integration performed with the PSO for the ERNN model.

4 Analysis and Simulation Outcomes

This section describes the outcomes of the tests performed, as shown in the tables and comparisons of the suggested fuzzy systems.

We consider Generalized type-2, Interval type-2, and Type-1.

Table 4 displays the integration outcomes of the Type-1 fuzzy integration performed with the PSO for the ERNN model

Table 5 presents the outcomes with the particle swarm algorithm (PSO) of the recurrent neural network ensemble with type-1.

Table 6 presents the integration results of the Mamdani Type 2 fuzzy system; different scale (Scl) and lag (Llag) values were tested.

Table 7 presents the integration results of the Sugeno Type 2 fuzzy system; different scale and lag values were tested. These results allow us to understand the influence of the Scale (Scl) and Lag (Llag) parameters on the predictive performance of the system.

Table 8 presents the results of integrating the ERNN with the generalized type-2 system with different scale (Scl) and lag (Llag) values.

Table 9 presents the Bitcoin time series results for the Sugeno Generalized Type-2. Different scale (Scl) and lag (Llag) values were set for each ERNN.

Table 10 summarizes the results of the Particle Swarm Optimization (PSO) algorithm across 30 experiments, highlighting only the top 10 outcomes for clarity.

The optimal performance was achieved in row 9, with a prediction error of 0.0079875, indicating the highest accuracy obtained by the algorithm in this series of tests.

Table 6. Results integration Mamdani type-2

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.7 Llag 0.3	Scl 0.9 Llag 0.1
0.3289	0.3281	0.3286	0.3329	0.3273	0.3298
0.3271	0.3266	0.3270	0.3295	0.3259	0.3250
0.3289	0.3281	0.3284	0.3326	0.3273	0.3298
0.3274	0.3256	0.3258	0.3259	0.3251	0.3249
0.3284	0.3278	0.3283	0.3317	0.3271	0.3298
0.3266	0.3263	0.3267	0.3285	0.3257	0.3250
0.3285	0.3278	0.3284	0.3318	0.3271	0.3298
0.3269	0.3265	0.3269	0.3292	0.3259	0.3250
0.3272	0.3267	0.3271	0.3297	0.3260	0.3251
0.3254	0.3256	0.3258	0.3259	0.3251	0.3249

Table 7. Results Integration type-2 fuzzy system

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.7 Llag 0.3	Scl 0.9 Llag 0.1
0.1781	0.1780	0.1780	0.1780	0.1781	0.1785
0.1490	0.1262	0.1429	0.1671	0.0838	0.0487
0.1781	0.1780	0.1780	0.1780	0.1781	0.1785
0.0880	0.0768	0.1045	0.1433	0.0468	0.0485
0.1781	0.1780	0.1780	0.1780	0.1781	0.1785
0.1461	0.1235	0.1414	0.1659	0.0804	0.0485
0.1781	0.1780	0.1780	0.1780	0.1781	0.1785
0.1470	0.1243	0.1416	0.1662	0.0816	0.0486
0.1302	0.1047	0.1274	0.1623	0.06198	0.0484
0.1350	0.1161	0.1378	0.1617	0.0619	0.0479

Table 8. Results integration Mamdani Type-2 Generalized fuzzy system for the Bitcoin

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.7 Llag 0.3	Scl 0.9 Llag 0.1
0.3230	0.3230	0.3230	0.3230	0.3228	0.3230
0.3236	0.3235	0.3235	0.3238	0.3235	0.3233
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3228	0.3230
0.3286	0.3274	0.3274	0.3299	0.3275	0.3253
0.3230	0.3230	0.3230	0.3230	0.3228	0.3230
0.3231	0.3231	0.3231	0.3232	0.3231	0.3231
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3395	0.3354	0.3352	0.3432	0.3259	0.3281

Table 9. Results integration Sugeno Type-2 Generalized fuzzy system for the Bitcoin

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.7 Llag 0.3	Scl 0.9 Llag 0.1
0.2573	0.2098	0.2495	0.2670	0.2468	0.2180
0.2882	0.2848	0.2850	0.2918	0.2840	0.2686
0.2557	0.2098	0.2495	0.2670	0.2468	0.2180
0.2479	0.2461	0.2468	0.2513	0.2439	0.2315
0.2883	0.2848	0.2851	0.2918	0.2841	0.2687
0.2562	0.2117	0.2486	0.2571	0.2658	0.2176
0.2894	0.2860	0.2862	0.2929	0.2852	0.2697
0.2599	0.2109	0.2443	0.2655	0.2457	0.2173
0.2870	0.2836	0.2838	0.2906	0.2828	0.2674
0.2929	0.2895	0.2898	0.2961	0.2888	0.2730
0.2650	0.2618	0.2622	0.2688	0.2605	0.2464
0.2933	0.2898	0.2901	0.2966	0.2890	0.2730

Table 10. This table presents the results of ERNN with the GA

No	Pm	Pc	Num Modules	Num Layers	Num Neuron	Duration	Prediction Error
1	0.05	0.5	2	2	20,5 20,5	01:14:25	0.05616
2	0.08	1	2	3	2,19,22 2,19,22	01:15:11	0.05618
3	0.03	0.04	2	1	5 17	01:14:33	0.056025
4	0.06	0.3	2	1	12 12	01:14:25	0.05619
5	0.07	0.5	2	1	30 27	01:15:11	0.056144
6	0.8	1	2	1	5 16	01:14:33	0.56134
7	0.8	0.9	2	1	7 11	01:14:29	0.056054
8	0.002	0.1	2	1	9 16	01:18:23	0.056095
9	0.05	0.7	2	1	21 21	01:11:21	0.056167
10	0.01	1	2	1	30 30	01:17:22	0.056002

Table 11. Results integration Mamdani type-1 fuzzy system for Bitcoin

No.	Integration Mamdani
1	0.0740
2	0.0740
3	0.0740
4	0.0740
5	0.0740
6	0.0740
7	0.0740
8	0.0740
9	0.0740
10	0.0740

Table 12. Results integration Sugeno type-1 fuzzy system for the Bitcoin

No.	Integration Sugeno
1	0.01680
2	0.01680
3	0.01681
4	0.01680
5	0.01680
6	0.01680
7	0.01680
8	0.01680
9	0.01681
10	0.01681

Table 13. Results integration Mamdani Type-2 fuzzy system for Bitcoin

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.5 Llag 0.5	Scl 0.9 Llag 0.1
0.3270	0.3266	0.3270	0.32922	0.3251	0.32504
0.3253	0.3254	0.3257	0.3256	0.3259	0.3248
0.3263	0.3261	0.3265	0.3279	0.3259	0.3249
0.3269	0.3265	0.3269	0.3293	0.3251	0.3250
0.3269	0.3265	0.3269	0.3293	0.3256	0.3250
0.3270	0.3266	0.3270	0.3294	0.3259	0.3250
0.3270	0.3265	0.3270	0.3293	0.3259	0.3250
0.3270	0.3266	0.3270	0.3294	0.3259	0.3250
0.3270	0.3266	0.3270	0.3295	0.3259	0.3250
0.3272	0.3267	0.3271	0.3298	0.3256	0.3250

Table 11 shows the type-1, using a recurrent neural network ensemble (ERNN) with the Genetic Algorithm (GA).

Table 12. shows the type-1 fuzzy integration, using a recurrent neural network ensemble (ERNN) with the Genetic Algorithm (GA).

Table 14. Results integration Sugeno type-2 fuzzy system for the Bitcoin

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.5 Llag 0.5	Scl 0.9 Llag 0.1
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755
0.1632	0.1880	0.1705	0.0732	0.1680	0.1755

Table 15. Results integration type-2 generalized fuzzy system for Bitcoin

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.5 Llag 0.5	Scl 0.9 Llag 0.1
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230
0.3230	0.3230	0.3230	0.3230	0.3230	0.3230

Table 13 presents the integration results of the Mamdani Type 2 fuzzy system; different scale and lag values were tested.

These results allow to understand the influence of the Scale (Scl) and Lag (Llag) parameters on the predictive performance of the system.

Table 14 presents the integration results of the Sugeno Type 2; different scale and lag values were tested.

These results allow to understand the influence of the Scale (Scl) and Lag (Llag) parameters on the predictive performance of the system.

Table 16. Results integration Sugeno Type-2 Generalized fuzzy system for the Bitcoin

Scl 0.6 Llag 0.4	Scl 0.4 Llag 0.3	Scl 0.3 Llag 0.3	Scl 0.5 Llag 0.5	Scl 0.5 Llag 0.5	Scl 0.9 Llag 0.1
0.2177	0.2114	0.2114	0.2242	0.2114	0.2046
0.2068	0.2055	0.2056	0.2077	0.2054	0.1966
0.2124	0.2090	0.2090	0.2188	0.2090	0.2033
0.2185	0.2120	0.2120	0.2251	0.2120	0.2049
0.2187	0.2122	0.2122	0.2254	0.2122	0.2050
0.2188	0.2123	0.2124	0.2255	0.2123	0.2050
0.2184	0.2119	0.2119	0.2250	0.2119	0.2048
0.2176	0.2114	0.2114	0.2242	0.2114	0.2046
0.2175	0.2113	0.2113	0.2241	0.2113	0.2046
0.2185	0.2120	0.2120	0.2252	0.2120	0.2049

Table 17. Mamdani type-1 t statistical test

Fuzzy Type System	Mean	Standard deviation	Mean Error	T Value	P Value
Gaussian PSO	0.670103	0.000310	0.000058	-734.29	0.000
Gaussian GA	0.740103	0.000409	0.000076		

Table 18. t-statistical test results for the Bitcoin time series using the Sugeno Type-1 fuzzy system.

Fuzzy Type System	Mean	Standard deviation	Mean Error	T Value	P Value
Gaussian PSO	0.0799345	0.0000484	0.0000090	6562.04	0.000
Gaussian GA	0.0167966	0.0000186	0.0000034		

Table 19. Mamdani Type-2 t statistical test

Fuzzy Type System	Mean	Standard deviation	Mean Error	T Value	P Value
Gaussian PSO	0.3119	0.0434	0.0079	-1.64	0.107
Gaussian GA	0.3249280	0.0000883	0.000016		

Table 15 presents the integration results of the Mamdani Generalized Type-2 fuzzy system; different scale and lag values were tested.

These results allow us to understand the influence of the Scale (Scl) and Lag (Llag) parameters on the predictive performance of the system.

Table 16 presents the Bitcoin time series results for the Sugeno type-2 generalized fuzzy system. Different Scale (Scl) and Lag (Llag) values were set for each ERNN.

Table 20. Sugeno Type-2t statistical test

Fuzzy Type System	Mean	Standard deviation	Mean Error	T Value	P Value
Gaussian PSO	0.1022	0.0614	0.011	-0.99	0.328
Gaussian GA	0.141	0.204	0.38		

Table 21. Mamdani Generalized Type-2 t statistical test

Fuzzy Type System	Mean	Standard deviation	Mean Error	T Value	P Value
Gaussian PSO	0.3085	0.0443	0.0082	0.18	0.858
Gaussian GA	0.30703	0.00308	0.00057		

Table 22. Sugeno Generalized. Type-2 t-statistical test

Fuzzy Type System	Mean	Standard deviation	Mean Error	T Value	P Value
Gaussian PSO	0.2251	0.0577	0.011	2.01	0.050
Gaussian GA	0.20360	0.00264	0.00049		

4.1 Comparison of results

This section presents a comparative analysis between the particle swarm optimization algorithm and the genetic algorithm.

Statistical tests were performed on the results obtained from Generalized Type-2, Type-1 and Interval Type-2, Fuzzy Systems, there are Sugeno and Mamdani configurations, also considering different scale and lag values.

4.1.1 Mamdani Type-1 with PSO and GA

A comparison was made with Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) using Mamdani type-1 fuzzy systems. The statistical t-test was used to predict the Bitcoin time series, employing Gaussian membership functions, also using different scale and lag values.

A 95% confidence interval was used. The statistical test indicates no significant difference between the two methods; however, the GA method shows a smaller standard deviation, as shown in Table 19. The sample size is 29.

4.1.2 Sugeno Type-1 with PSO and GA

To compare the results of two Sugeno type-1 using the two optimization algorithms, PSO and GA for the prediction of For the Bitcoin time series, a statistical t-test was applied, employing Gaussian membership functions along with various scale and lag values a 95% confidence interval was used, and The statistical test performed tells us that there is significant difference in the two methods, Gaussian PSO, since the value of t is positive, indicating that the mean of Gaussian PSO is significantly greater than that of Gaussian GA are shown in the Table 18. The number of samples is 29.

4.1.3 Mamdani Type-2 with PSO and GA

A comparison was made between the GA and PSO using Mamdani type-2 fuzzy systems. The statistical t-test was used to predict the Bitcoin time series, employing Gaussian membership functions, also using different scale and lag values. A 95% confidence interval was used. The statistical test indicates no significant difference between the two methods; however, the GA method shows a smaller standard deviation, as shown in Table 19. The sample size is 29.

4.1.4 Sugeno Type-2 with PSO and GA

To compare the results of two Sugeno Type-2 fuzzy systems optimized using PSO and GA for Bitcoin time series prediction, the statistical t test was used, in which Gaussian membership functions were used also using the different scale and lag valuesPS a 95% confidence interval was used, and theº statistical test performed tells us indicating that no significant difference exists between the two methods, but the GA has a lower standard deviation value are shown in the Table 20. The number of samples is 29.

4.1.5 Mamdani Generalized Type-2 with PSO and GA

To evaluate and compare the effectiveness of two Mamdani Generalized Type-2 optimized using PSO and GA for Bitcoin time series prediction, a statistical t-test was conducted. Gaussian membership functions were applied, considering different scale and lag parameters. The analysis was carried out with a 95% confidence interval. The t-test results show no statistically significant difference between the two optimization techniques; however, the GA achieved a lower standard deviation, as shown in Table 21. The evaluation was based on a sample size of 29.

4.1.6 Sugeno Generalized Type-2 with PSO and GA

A comparison was made between the results of two Sugeno type-2 generalized fuzzy systems for Bitcoin time series prediction. For this test, Gaussian membership functions were used, incorporating different scale and delay values. For this, we used the statistical t-test and considered a 95% confidence interval for the analysis. Since the t-value is positive, it indicates that the average performance of the Gaussian PSO is significantly higher than that of the Gaussian GA, as shown in Table 22. The analysis was based on a sample size of 29.

5 Conclusions

In this study, Sugeno and Mamdani fuzzy systems, specifically Interval Type-2, Generalized Type-2, and Type-1, were used to aggregate the responses of ensemble recurrent neural networks (ERNN) for Bitcoin, aiming to achieve the lowest prediction error. In summary, this proposed approach is suitable for forecasting complex time series. The Type-2 Generalized Fuzzy Systems, both Mamdani and Sugeno, demonstrated superior or comparable performance to standard Type-1 and Type-2 systems regarding prediction accuracy, especially under different scale and lag parameter settings. This reflects their robustness to the inherent uncertainty in volatile time series such as cryptocurrency. The combination of recurrent neural networks with evolutionary algorithms (PSO and GA) enabled effective exploration of the solution space, optimizing architectures and configurations that, when integrated using fuzzy systems, significantly contributed to reducing prediction error.

The t-tests applied to compare the different fuzzy systems and optimization techniques revealed statistically significant differences in some cases, particularly between PSO and GA. In particular, the Generalized Sugeno Type-2 systems integrated with PSO yielded better means, while GA showed lower standard deviation in some scenarios, indicating greater stability. The implementation of optimized Neural Networks in combination with Generalized Type-2 represents a robust and effective alternative for the prediction of complex time series, highlighting their practical applicability and their ability to efficiently handle uncertainty and variability. Future work can be developed, optimizing Generalized Type-2 (Sugeno and Mamdani) aggregators with the particle swarm algorithm to find better prediction error. these could be optimized for membership function parameters, number of rules, and membership type, as in [56].

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