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Fuzzy Graphoidal Covering of Fuzzy Graphs and its Application in Cancer Classification under Artificial Neural Network Context

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Abstract. The classification of cancers concerning their nature and identification of the variant are very crucial things in health sciences as decision-making. The chance and nature of cancer in the human body are efficiently tackled by the signaling pathways. Also, these concepts are very vital for the up and down criteria of several types of oncogenes. There are uncertainties in the data of oncogene parameters and many ambiguities in the pathways concept. We have tied these two issues in one frame to overcome the vague sense of classification in this work. For fuzzy graphs, fuzzy graphoidal covers are helping for the evaluation of the best choice of execution of the pathways associated with different types of cancer for detecting their nature. Some specific conditions on the appearance of the edges and their adjacent vertices in a particular path are imposed on the path-cover concept to define this new parameter for a fuzzy graph incorporating the role of membership degree to edges and vertices. Additionally, a parameter corresponding to this cover set is defined to measure the impact of the fuzzy graphoidal cover in a fuzzy network. An efficient algorithm is designed with complexity analysis for finding fuzzy graphoidal covers. We have spotted the light on an artificial neural network perspective. Through a theoretical perspective of artificial neural network, this is very beneficial in the application part related to health-care i.e., cancer prediction and classification.

Keywords. Graphoidal cover, fuzzy graphs, fuzzy graphoidal covering number, covering problems, cancer classification.

1 Introduction

Among many important fundamental problems of classical graph theory, determination of shortest path betwixt any two arbitrary vertices is very crucial. For a graph G in crisp sense, a chain of vertices and edges which appears alternatively, opening and terminating by vertices is called a path, where no repeatation of vertices or edges in the sequence. The shortest path problem for this situation is for finding a path which has a least length between a particular set of two vertices.

Sum of weights corresponding to al edges in a path has been treated as the path's length. But, in real-life situation, each edge weight of a system may not be a certain real number. For real-life impreciseness, any imprecise number like interval numbers, trapezoidal fuzzy numbers etc. are used for assigning weights for the edges. Triangular fuzzy numbers (TFNs) has been applied here for representing vertex, edge membership values as well as weights of the edges for a fuzzy network.

For fuzzy network, sum of the fuzzy arc lengths (fuzzy numbers) is considered as fuzzy length of a fuzzy path. In this chapter, our seek is to develop an algorithm which will be performed for fuzzy network to obtain a fuzzy graphoidal cover with justified analysis and time complexity. Also, fuzzy graphoidal covering number is to be calculated for best possible cover.

Our proposed algorithm for this work is developed with the help of ranking procedure of fuzzy numbers and necessary definitions related to the topic. A concept of fuzzy number's ranking by possibility theory is introduced by Dubois and Prade [18]. Delgado et al. [16] have defuzzified the fuzzy numbers with the help of fuzzy relations. Packing concept by using covering idea is developed by Harary and Schwenk [21].

For crisp graphs, graphoidal covering number is defined in the study of Pakkiam and Arumugam [28]. The discussion on path graphs are reflected in the work of Broersma and Hoede [13]. Klein [23] has determined the shortest fuzzy paths for the first time. An extension on fuzzy shortest path problem is leaded by Lin and Chen [24]. Arumugam et al. [2] have given a detailed review on fuzzy graphs with graphoidal covers. An algorithm is designed by Okada and Soper [27] without any guidance for finding a non-dominated family of shortest paths for a specified pleasure level.

Shortest path problems are solved in an algorithmic way by Nayeem and Pal [26]. Fuzzy planar graphs has been proposed by Samanta and Pal [29]. Different variations in product bipolar fuzzy graphs has been shown by Ghorai and Pal [20]. Das et al. [15] have provided the centrality measures of fuzzy numbers. In 2020, graphoidal graphs and graphoidal digraphs are described by the research field of Arumugam and Bagga [3]. Also, Bhattacharya and Pal [4, 5, 6] have developed models with multi-objective optimization problems by combining covering problems. In a way of pairing the parametric relation, Dombi and Jonas [17] have ranked trapezoidal fuzzy numbers. Some modern trends in fuzzy graph theory are discussed in the book of Pal et al. [**?**]. Bhattacharya and Pal [7, 8, 9] have worked on various types of coverings with their real-life applications in different sectors of our society.

To include uncertainties and impreciseness related to real-life problems, we are interested to work on fuzzy graphs for finding fuzzy graphoidal covers. In existing literature, only edge weights are taken as fuzzy numbers on crisp graphs i.e., only distances are fuzzy in the network. But, in our proposed method, we have considered vertex membership values, edge membership values, edge weights as fuzzy numbers. As a result, both the distance and strength of a path will be fuzzy together which is not done earlier in literature. These motivate to this work on fuzzy graphs with fuzzy arc lengths and determine fuzzy graphoidal cover. A basic comprehensive study on graphoidal covers in fuzzy graphs and combined study with different optimization tool like particle swarm optimization are described by Bhattacharya and Pal [10, 11]. Also, Bhattacharya et al. [12] have applied the covering idea of fuzzy graphs for analyzing the sustainable development goal 'climate action'.

1.1 Focus of the Study

In this work, a fuzzy graphoidal covering problem and try to construct an efficient algorithm to solve it. Firstly, we take a fuzzy network in which all vertices and edges are uncertain. Triangular fuzzy numbers (TFNs) are used for assigning arc lengths. So, here fuzzy network is with fuzzy arc lengths. Then we have determined all possible fuzzy shortest paths betwixt any couple of vertices in a network. Now, we have implemented graphoidal covers for crisp graphs into a new way for defining fuzzy graphoidal cover. Most clearly, it can be said that it is a group of vertex-disjoint fuzzy shortest paths that will cover all the vertices in a fuzzy network to optimize different parameters involved in this fuzzy network. An algorithm is designed later part of this paper with proper analysis of time complexity of it.

Presently, here an introducing part for defining new parameters fuzzy graphoidal cover and related number for that cover has been described for a fuzzy graph. We have given a new comparison technique between two TFNs. An algorithm is designed to determine fuzzy graphoidal cover with an appropriate example. Also, the time complexity is determined. Lastly, analyze the cancer possibility and classification by the use of this concept of fuzzy graphoidal covers and made guidelines to identify the possibility of tumor in specific human organs. Finally, a comparison study is made to show efficiency of our developed algorithm in the application section.

2 Conceptual Foundations

This part sets forth the core definitions underpinning the subsequent discussions.

2.1 Essential Definitions

Definition 2.1. *[25] A non-null set V of vertices involving two associated functions, one for edges and another for vertices are such that $\mu : V \times V \to [0,1]$ and $\sigma : V \to [0,1]$ respectively maintaining the relation $\mu(x,y) \leq min\{\sigma(x), \sigma(y)\}$ for all $x, y \in V$ is entirely known as a fuzzy graph $G = (V, \sigma, \mu)$.*

Definition 2.2. *[1]*

For a graph $G = (V, E)$, a graphoidal cover is a collection $\zeta(G)$ of paths in G, that are not required to be open, that satisfies the subsequent conditions:

(i) Each path in $\zeta(G)$ contains no fewer than two vertices.

(ii) Any internal vertex of G belongs to appearing in not more than one path of $\zeta(G)$.

(iii) All edges in G is contained in at least exactly one path of $\zeta(G)$.

Definition 2.3. *[1] Let, ψ representing the collection ranging across all graphoidal covers of G. Then:*

$$\gamma(G) = min_{\zeta(G)\in\psi}|\zeta(G)|,$$

is known as the graphoidal covering number of that graph.

Definition 2.4. *[25] In a fuzzy graph $G = (V, \sigma, \mu)$ for a vertex u, the degree is defined as:*

$$deg(u) = \sum_{(u,v)\in E} \mu(u, v).$$

2.2 Distinction of two TFNs

Let $\tilde{A} = (a, b, c)$ be a TFN, where $a \leq b \leq c$ and the membership value of this number at b maximum and it is 1.

Definition 2.5. *The crisp value of a TFN $\tilde{A} = (a, b, c)$ is evaluated by the value of area of triangle formed by points $(a, 0)$, $(b, 1)$ and $c, 0$. That is:*

$$\Delta_{\tilde{A}} = \frac{1}{2}(c - a).$$

Definition 2.6. *Let two TFNs be $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$. A two dimensional similarity measure between $\tilde{A}_1$ and $\tilde{A}_2$ is known as credibility index which is a function such that $\Omega : \tilde{A}_1 \times \tilde{A}_2 \to [0,1]$ and given by the following expression:*

$$\Omega(\tilde{A}_1, \tilde{A}_2) = \frac{\mathcal{A}(\tilde{A}_{1\mathcal{R}} \cup \tilde{A}_{2\mathcal{L}})}{|\mathcal{A}_\mathcal{R}(\tilde{A}_1)| + |\mathcal{A}_\mathcal{L}(\tilde{A}_2)| - |\mathcal{A}(\tilde{A}_1 \cap \tilde{A}_2)|},$$

For clear description of the terms of Definition 8, let us consider the Figure 1. The area $\mathcal{A}(\tilde{A}_{1\mathcal{R}} \cup \tilde{A}_{2\mathcal{L}})$ is the area of the rectangle BEHG, $\mathcal{A}_\mathcal{R}(\tilde{A}_1)$ is the area of the region BDG, $\mathcal{A}_\mathcal{L}(\tilde{A}_2)$ is the area of the region CBH:

For TFNs $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$,
$\mathcal{A}(\tilde{A}_{1\mathcal{R}} \cup \tilde{A}_{2\mathcal{L}}) = (b_2 - b_1)$.

$\mathcal{A}_\mathcal{R}(\tilde{A}_1) = \frac{1}{2}(c_1 - b_1)$, *region formed by right spread of TFN $\tilde{A}_1$.*

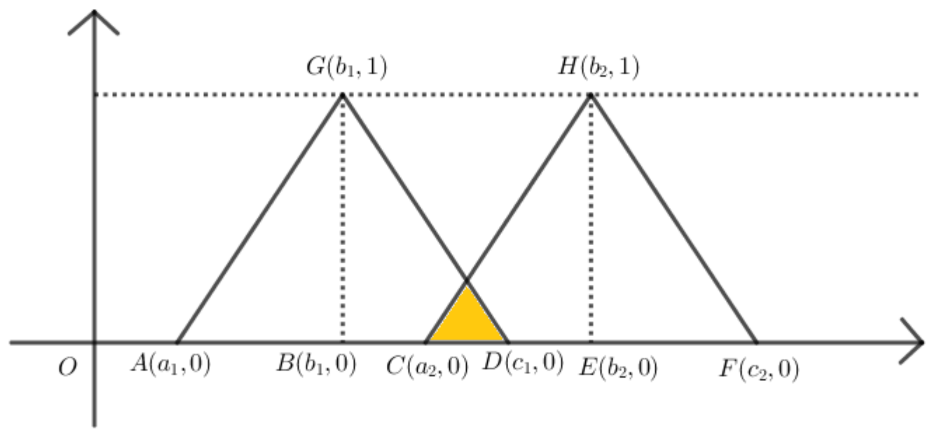

Fig. 1. Credibility index between two TFNs

$\mathcal{A}_{\mathcal{L}}(\tilde{A}_2) = \frac{1}{2}(b_2 - a_2)$, *region formed by left spread of TFN* $\tilde{A}_2$.

$\mathcal{A}(\tilde{A}_1 \cap \tilde{A}_2) = \frac{1}{2} \times \frac{(c_1 - a_2)(a_1 - a_2)}{(b_2 - a_2) - (b_1 - a_1)}$, *area of the common two dimensional region with right spread of* $\tilde{A}_1$ *and left spread of* $\tilde{A}_2$.

The parameter $\Omega(\tilde{A}_1, \tilde{A}_2)$ may be interpreted as a measure of how much the TFN $\tilde{A}_1$ precedes the TFN $\tilde{A}_2$.

Remark: From the definition of credibility index Ω, the following properties are immediately obtained.

(a) $0 \leq \Omega(\tilde{A}, \tilde{B}) \leq 1$ holds for any TFNs $\tilde{A} = (a_1, b_1, c_1)$ and, $\tilde{B} = (a_2, b_2, c_2)$.

(b) $\Omega(\tilde{A}, \tilde{B})$ is a continuous function of the variables a_i, b_i, c_i for $i = 1, 2$.

(c) If the TFN $\tilde{A}$ entirely precedes the TFN $\tilde{B}$, that is, $a_2 > c_1$, then $\Omega(\tilde{A}, \tilde{B}) = 1$.

(d) If $b_1 = b_2$, then $\Omega(\tilde{A}, \tilde{B}) = 0$.

(e) If $\Omega(\tilde{A}, \tilde{B}) > 0.5$, then $\tilde{A}$ is said to be totally governing over $\tilde{B}$ which is denoted by $\tilde{A} \leq_t \tilde{B}$.

(f) If $\Omega(\tilde{A}, \tilde{B}) < 0.5$, then $\tilde{A}$ is said to be fractionally governing over $\tilde{B}$ which is denoted by $\tilde{A} <_f \tilde{B}$.

(g) If $\Omega(\tilde{A}, \tilde{B}) = 0.5$, then $\tilde{A}$ is said to be not totally governing over $\tilde{B}$ which is denoted by $\tilde{A} \nleq_t \tilde{B}$.

Definition 2.7. *In a given fuzzy graph* $G = (V, \sigma, \mu)$, *let* P *be a fuzzy path. Then,* $\Pi(P)$ *is representing the distance:*

$$\Pi(P) = \sum_{u,v \in P} \frac{\mathfrak{L}_{uv}}{1 + \sigma(u)\sigma(v)},$$

where, for the shortest $u - v$ *path,* $\mathfrak{L}_{uv}$ *is the defuzzified length.*

Definition 2.8. *For a fuzzy graph* $G = (V, \sigma, \mu)$, *fuzzy graphoidal cover of* G *is a collection* $\mathfrak{P}(G)$ *of fuzzy shortest paths in* G *that meets the following criteria:*

(i) Each fuzzy shortest path in $\mathfrak{P}(G)$ *contains at least two vertices* u *and* v *such that* $\sigma(u) \neq 0$ *and* $\sigma(v) \neq 0$.

(ii) For any vertex $u \in V$ *with* $\deg(u) > 0$, *the vertex* u *appears occurring as an internal vertex in no more than one fuzzy shortest path in* $\mathfrak{P}(G)$.

(iii) For every edge $(u, v) \in E(G)$, *if* $\mu(u, v) > 0$, *then* (u, v) *belongs to at least any individual fuzzy shortest path in* $\mathfrak{P}(G)$.

Definition 2.9. *For a fuzzy graph* $G = (V, \sigma, \mu)$, *let* $\mathfrak{P}(G)$ *be a fuzzy graphoidal cover. Accordingly, the fuzzy graphoidal covering number is given by* N_{gc}, $N_{gc} = min\{\Pi(P) : P \in \mathfrak{P}(G)\}$.

The main principle to find (fuzzy) shortest path is labeling of vertices in two stages. In first stage, a provisional label is assigned to all vertices and based on a specific rule defined by the decision maker, a particular vertex get permanent label called eternal label.

3 Proposed Algorithm

The proposed algorithm with analysis of time complexity is discussed. The following steps are followed to design the algorithm:

(i) Set provisional labels to all non-eternal labeled vertices.

(ii) Select the least value out of the provisional labels, and then the vertex gets the eternal label.

Repeat these two steps until all vertices get eternal labels.

(iii) Find all the possible fuzzy shortest paths in fuzzy graph G.

(iv) Find a minimal set of shortest paths of G n which every edge is contained in one or more paths. This collection will be known as fuzzy graphoidal cover of G.

Let us choose a vertex s as source vertex and the collection of all eternally labeled is denoted by V^*. At first stage, assume that δ-th provisional label of $v \in V - \{s\}$ is $L^P(v)$ with $L^P_\delta(s) = 0$. Then, we construct a collection of fuzzy shortest paths which are vertex disjoint and the union of all vertices in those fuzzy shortest paths will give the whole vertex set of G. So, M is collection of all fuzzy shortest paths in G and set of all vertices is V belonging to at least one path $P \in M$ but, not simultaneously. Then, we denote fuzzy graphoidal cover as,

$\mathfrak{P}(G) = \{M : P_i, P_j \in M$ such that $V(P_i) \cap V(P_j) = \phi$ and, $V = V^*\}$.

FUNCTION LEAST $(\tilde{Q}, \tilde{R})$

Input: Two TFNs $\tilde{Q}$ and $\tilde{R}$. They are neither totally or fractionally governing over other.

Output: Least between two TFNs $\tilde{Q}$ and $\tilde{R}$.

if $((\tilde{Q} \nleq_t \tilde{R})$ or $(\tilde{Q} <_f \tilde{R}))$
$least = \tilde{Q}$;
else
$least = \tilde{R}$;
endif;
return (least);
END LEAST.

3.1 Algorithm FUZZY-GRAPHOIDAL-COVER

Algorithm which is proposed in this work, is described here.

Input: The adjacency matrix D for G and source vertex s.

Output: Fuzzy graphoidal cover consists of fuzzy shortest paths of G with respect to fuzzy distance and fuzzy strength.

Step-1: Initiate eternal label of the source node with 0 and provisional labels for other remaining vertices as ∞. $//L^E_q(j), L^P_\delta$, DV denote respectively q-th eternal label of j-th vertex, δ-th provisional label of j-th node, whereas destination node get label of eternal type.

Step-2: Set $\mathfrak{P}(G) = \phi$.

If $\{V^* \neq V\}$ do // when all vertices of V are not getting eternal labels //

Step-2.1: Now, for any $j \in V - V^*$ do

$$L^E(j) = min\Big\{\frac{\mu(ij)}{\sigma(j)} \otimes \Delta(\alpha_{ij}) \forall i\Big\};$$

if $L^E(j)$ and $L^P(j)$ are of non-governing type

then $L^P_{\delta+1}(j) = L^E(j)$;
end if;
end for;

Step-2.2: If $L^P_q(j)$ is totally governing or, fractionally governing to other remaining provisional labels of members in the set $V - V^*$ then,

DV$\leftarrow j, V^* \leftarrow V^* \cup \{j\}$ and, $q \leftarrow q + 1$.

Step-3: Yet, in the case when there are many number of provisional labels (let, r labels are of type non-governing provisional) for the node j then, the process have to divide into r number of branches and repeat the step 2.2 for each branch and do the same.

Step-4: Reset $\mathfrak{P}(G) = \mathfrak{P}(G) \cup \{P_j\}$ for any j.

if $V(P_i) \cap V(P_j) = \phi$ for $P_i, P_j \notin \mathfrak{P}(G)$, where i, j are arbitrary indices.

$\mathfrak{P}(G) = \mathfrak{P}(G)$

else,

eliminate vertices from $V(P_i) \cap V(P_j)$

end if.

Step-4: END FUZZY-GRAPHOIDAL-COVER.

4 Exploration of the Algorithm

On the basis on the problem's branching factor and the computational complexity of the proposed algorithm can be sincerely analyzed. Consider the vertex s as source vertex of the fuzzy network. The whole procedure may be branched into many intermediate steps. Also, find all the fuzzy shortest paths which are vertex disjoint and the union of all vertices will form the whole vertex set of given fuzzy graph. We collect all such fuzzy shortest paths and called them fuzzy graphoidal cover for a fuzzy graph.

Initially, set, $s = \chi_0$, say, the eternal label. Without loosing triviality, we may assume that χ_1 be the next vertex to get the eternal label. Yet, in the case when a vertex has two provisional labels with equal value; but distinct edge-weights, then two branches occur and, both non-governing provisional labels will be set as eternal label in two parts. Also, for the next vertex, the same situation may occur to get eternal label, χ_2, say. In a similar manner, the most generalized situation is considered for the branches arising at every eternal label, then this system can be presented by a fuzzy tree F_T given in Figure 2.

In Figure 2 let, the fuzzy tree has m leaf and p number of fuzzy paths. Every path of these possibilities gives the fuzzy the minimal-length paths connecting s, source node to all other vertices of the fuzzy graph.

From source vertex $\chi_0 = s$, the possible paths to χ_1 are $(\chi_0)^0 \to (\chi_1)^0$ and, $(\chi_0)^0 \to (\chi_1)^1$, if $(\chi_1)^1$ exists, where $(\chi_1)^0$ and $(\chi_1)^1$ are two eternal labels of the vertex χ_1 with equal provisional label value but distinct edge-weights. Similarly, the paths from χ_0 to χ_2 are,

$(\chi_0)^0 \to (\chi_1)^0 \to (\chi_2)^0$, $(\chi_0)^0 \to (\chi_1)^0 \to (\chi_2)^1$, $(\chi_0)^0 \to (\chi_1)^0 \to (\chi_2)^2$, $(\chi_0)^0 \to (\chi_1)^1 \to (\chi_2)^0$, $(\chi_0)^0 \to (\chi_1)^1 \to (\chi_2)^1$ and, $(\chi_0)^0 \to (\chi_1)^1 \to (\chi_2)^2$.

All these paths exist if $\sigma((\chi_2)^2) > 0$, $\sigma((\chi_2)^1) > 0$, and $\sigma((\chi_1)^1) > 0$. Also, we have to find all the fuzzy lengths for these possible paths. Then, take the path with least value for fuzzy length. Here, all these paths have equal provisional label values but distinct fuzzy lengths i.e., these paths are non-governing.

When m branches exist for the fuzzy tree F_T, in the case of most generalized possibility, $O(mn)$ is the cardinality of the vertex set, since any branch of the fuzzy tree F_T has n number of existed vertices. For comparing the provisional labels $O(n)$ time is needed. On the other hand, for assigning eternal assigning labels to each vertex requires comparing provisional labels for not exceeding n nodes.

Therefore, this process requires $O(n)$ times. So, in total $O(n) \times O(mn) = O(mn^2)$ has been needed for generating all possible m number of fuzzy shortest paths. Also, to put m number of fuzzy shortest paths in a set, anyone require $O(m)$ times. So, to find fuzzy graphoidal cover with fuzzy shortest paths, $O(m) \times O(mn^2) = O(m^2n^2)$ time is required for a fuzzy graph.

Remark: In the best possible case, i.e., only a single branch of the tree F_T and assigning a permanent label to each vertex; $O(n^2)$ is the resulting time complexity. Hence, the time complexity in this case for determining fuzzy graphoidal cover is, $O(n^2)$.

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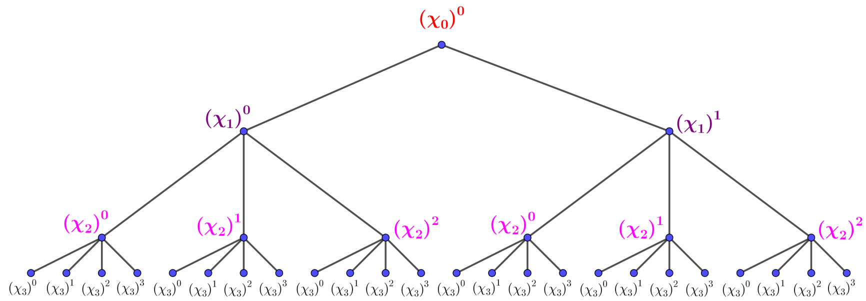

Fig. 2. Tree representation of branching possibilities

4.1 Modeling an Artificial Neural Network through Fuzzy Graph Representation

Throughout this work, the results were systematically developed on the basis of the fuzzy graphoidal covering number. This newly introduced parameter serves as a quantitative assessment of the covering set, namely the fuzzy graphoidal cover of a fuzzy graph. At the same time, prediction tasks in modern networks are efficiently performed using artificial neural networks (ANNs).

To leverage the advantages pertaining to our study of fuzzy graphoidal covering numbers, it becomes necessary to represent an ANN in the form of a fuzzy graph. The process begins with identifying the corresponding vertices and edges.

In real-world networks, data often involve uncertainty and imprecision, which may be expressed through artificial neural networks (ANNs). To more effectively capture these characteristics, fuzzy graphs provide a suitable framework compared to traditional crisp graphs. An ANN typically consists of three main layers: the layers for input, processing (hidden), and output. Raw data are fed into the input layer, computational processes and training occur within the hidden layer, and the final outcomes are produced in the output layer. Each of these layers contains multiple nodes that function in the course of execution, these nodes are treated as vertices of the fuzzy graph. Connections between layers facilitate the flow of information to subsequent stages; however, data transfer is restricted to interactions between specific nodes.

Whenever an edge connects the two nodes for the purpose of data transfer, an edge is formed; which is in the fuzzy graph then represented as an edge.

4.2 Demonstrating an Example with the Fuzzy Graphoidal Covering Number on an Artificial Neural Network

Subsequent to the conversion procedure described above, a fuzzy graph is generated representing an artificial neural network (ANN). Consider a three-layer ANN consisting of five input and eight hidden nodes, and four nodes in the output layer. In this representation, fuzzy weights may be linked to the graph's edges depending on the case study under consideration. The notation $w_{i,j}$ denotes the weight between vertices k_i and k_j. The fuzzy graph captures the overall data flow of the ANN; however, the directionality of edges is not taken into account, as it does not affect the proposed results and definitions. The resulting fuzzy graph is illustrated in Figure 3.

In Figure 3, the ANN structure as a fuzzy graph is with 15 vertices and 17 edges. It is clear that there arises five shortest paths as following.

$\mathcal{P}_1 = \{\tau_1, \tau_6, \tau_{10}, \tau_{12}\}$, $\mathcal{P}_2 = \{\tau_2, \tau_7, \tau_{10}, \tau_{13}\}$, $\mathcal{P}_3 = \{\tau_3, \tau_8, \tau_{11}, \tau_{14}\}$, $\mathcal{P}_4 = \{\tau_4, \tau_8, \tau_{11}, \tau_{15}\}$, and $\mathcal{P}_5 = \{\tau_4, \tau_{11}, \tau_{15}\}$.

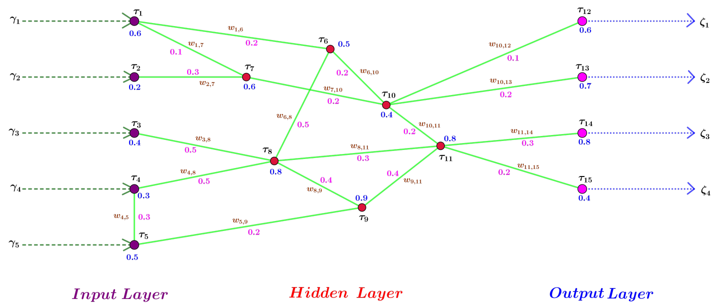

Fig. 3. ANN structure as a fuzzy graph

Loss in mean train-score	Scaled value	Loss in stdev train-score	Scaled value
0.005	0.05	0.002	0.02
0.009	0.09	0.003	0.03
0.043	0.43	0.011	0.11
0.004	0.04	0.001	0.01
0.012	0.12	0.006	0.06
0.055	0.55	0.016	0.16
0.003	0.33	0.001	0.01
0.110	1.1	0.089	0.89
0.021	0.21	0.068	0.68
0.062	0.62	0.085	0.85
0.020	0.2	0.046	0.46

Fig. 4. Data table for 2D-ANN

Accordingly, for this fuzzy graph, the set of all fuzzy shortest paths constitutes a fuzzy graphoidal cover, described as follows:

$$\mathfrak{P}(\mathcal{G}) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5\}.$$

For these constituents of the fuzzy graphoidal cover, we computed the corresponding distances and subsequently obtained the minimum:

$\Pi(\mathcal{P}_1) = \dfrac{0.2+0.2+0.1}{1+0.6\times 0.6} = 0.37,\ \Pi(\mathcal{P}_2) = \dfrac{0.3+0.2+0.2}{1+0.2\times 0.7} = 0.61,\ \Pi(\mathcal{P}_3) = \dfrac{0.5+0.3+0.3}{1+0.4\times 0.8} = 0.83,\ \Pi(\mathcal{P}_4) = \dfrac{0.5+0.3+0.3}{1+0.3\times 0.4} = 0.98,\ \Pi(\mathcal{P}_5) = \dfrac{0.2+0.4+0.2}{1+0.5\times 0.4} = 0.67.$

Hence, the fuzzy graphoidal covering number corresponding to the considered ANN (fuzzy graph) is expressed as $\mathcal{N}_{gc}(\mathfrak{G}) = min\{\Pi(\mathfrak{P}_i) : i = 1, 2, 3, 4, 5\} = min\{0.37, 0.61, 0.83, 0.98, 0.67\} = 0.37$.

4.3 2D-ANN Model

In a two-dimensional artificial neural network (2D-ANN), the kernel operates on matrix-type inputs. The model primarily consists of two dense layers of sizes 512 and 128, which are scaled as 5.12 and 1.28, respectively. For the present study, we assessed the drop in training-score values corresponding to the stride type $(2, 2)$, as shown in Figure 4. Membership levels of the graph's edges and vertices associated with the 2D-ANN model were assigned based on data extracted from the dataset table. In Figure 4, the green-coloured scaled values represent the vertices' fuzzy association values. Furthermore,

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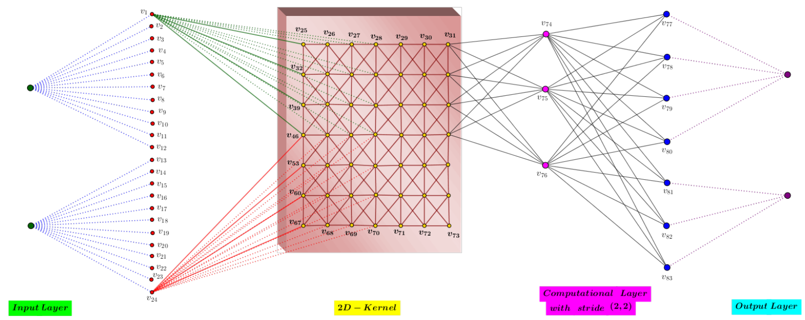

Fig. 5. Fuzzy graph G constructed for the 2D-ANN model

by analyzing the distances of fuzzy paths and evaluating the fuzzy graphoidal covering number, we aim to classify human organs as either tumor suppressors or oncogenes.

In this case, the highest and lowest values of the mean training score loss are 0.110 and 0.003, respectively, yielding an average of approximately 0.055. Accordingly, all mean training scores at least 0.055 are selected and are indicated in dark brown. Similarly, for the standard deviation of the training score, the highest and lowest values are 0.089 and 0.002, respectively, with an average of 0.0455. For this case, all values less than 0.045 have been considered in the construction. Based on these assumptions and considerations, the fuzzy graph presented in Figure 5 has been developed.

In Figure 5, the constructed fuzzy graph excludes the dotted pathways in the input and output layers. Each input-layer artificial nodes are processed using a $(4, 4)$ kernel. Membership levels for the kernel's artificial nodes are 8, 16, 32, and 64, respectively. In the case of the 2D-ANN, a stride of type $(2, 2)$ is considered, where the stride defines how many pixels are skipped by which the filter shifts across the input matrix. With a stride of 2, the filters are moved by two pixels simultaneously. The proposed algorithm, FUZZY-GRAPHOIDAL-COVERING, has been applied to this constructed fuzzy graph. In this setup, the computational layer is fully connected.

$\mathcal{P}_1 = \{v_1, v_{25}, v_{26}, v_{27}, v_{28}, v_{29}, v_{30}, v_{31}, v_{74}, v_{77}\}$ and, $\Pi(P_1) = 1.268$.

$\mathcal{P}_2 = \{v_4, v_{32}, v_{33}, v_{34}, v_{35}, v_{36}, v_{37}, v_{38}, v_{74}, v_{78}\}$ and, $\Pi(P_2) = 1.569$.

$\mathcal{P}_3 = \{v_8, v_{39}, v_{40}, v_{41}, v_{42}, v_{43}, v_{44}, v_{45}, v_{75}, v_{79}\}$ and, $\Pi(P_3) = 1.359$.

$\mathcal{P}_4 = \{v_{12}, v_{46}, v_{47}, v_{48}, v_{49}, v_{50}, v_{51}, v_{52}, v_{75}, v_{80}\}$ and, $\Pi(P_4) = 1.906$.

$\mathcal{P}_5 = \{v_{16}, v_{53}, v_{54}, v_{55}, v_{56}, v_{57}, v_{58}, v_{59}, v_{76}, v_{83}\}$ and, $\Pi(P_5) = 0.903$.

$\mathcal{P}_6 = \{v_{20}, v_{60}, v_{61}, v_{62}, v_{63}, v_{64}, v_{65}, v_{66}, v_{76}, v_{82}\}$ and, $\Pi(P_6) = 0.967$.

$\mathcal{P}_7 = \{v_{24}, v_{67}, v_{68}, v_{69}, v_{70}, v_{71}, v_{72}, v_{73}, v_{76}, v_{81}\}$ and, $\Pi(P_7) = 1.38$.

$$\therefore N_{gc}(G_2) = min\{\Pi(P_i) : i = 1, 2, ..., 7\} = 0.903.$$

Here, the lowest value is occurred for the path $\mathcal{P}_5$ in the constructed fuzzy graph associated with two dimensional kernel. We impose the main 7 types of pathways are RTK pathway, Cell cycle pathway, PI3K pathway, P53 pathway, Notch pathway, Wnt pathway, Myc pathway for any particular case study with the computed seven fuzzy paths in this illustration. Then, we can say that the minimum possibility for a human organ to be a tumor suppressor or oncogene is analyzed with respect to the fifth pathway of cancer cell signaling, i.e., Notch pathway.

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5 Conclusion

This chapter introduces a new parameter for fuzzy graph i.e., fuzzy graphoidal cover of fuzzy graph and fuzzy graphoidal covering number associated to this cover. An efficient algorithm to find fuzzy graphoidal cover is implemented in this paper with proper justification and analysis. We have calculated the time complexity for our developed algorithm which is $O(m^2n^2)$. An illustrative example to show the use of algorithm is described for better understanding. Also, a new technique to compare two TFNs are given in this chapter with the help of a function. All the defined parameters and designed algorithm are used to investigate cancer cell signaling process which is one of the most important topic in molecular biology. Finally, we have shown the fuzzy graphoidal covering number for fuzzy graph associated to 2D-ANN is 0.903 which is much closer to 1 and reflecting the efficiency of using two dimensional kernel in such theoretical computation. Also, we have deduced some strategies and make theoretical experimental results rather than laboratory base studies and this is very helpful fr real-life decision-maker.

In future research, we aim to explore different variations of covering numbers and investigate their efficiency and structural characteristics. Additionally, we plan to integrate advanced optimization techniques from operations research with the concept of coverings in fuzzy graphs, with the objective of developing improved methodologies for addressing complex real-world problems.

References

1. **Acharya, B. D., Sampathkumar, E. (1987).** Graphoidal covers and graphoidal covering number of a graph. Indian J. pure appl. Math, 18 (10): 882-890.

2. **Arumugam, S., Acharya, B. D., Sampathkumar, E. (1996).** Graphoidal covers of a graph: A Creative Review. Proceedings of the National Workshop on Graph theory and Its Applications, pp: 1-28.

3. **Arumugam, S., Bagga, J. S. (2020).** Graphoidal graphs and graphoidal digraphs: a generalization of line graphs. AKCE International Journal of Graphs and Combinatorics, 17:2, 660-664.

4. **Bhattacharya, A., Pal, M. (2022).** Fuzzy tree covering number for fuzzy graphs with its real-life application in electricity distribution system. Sadhana; 47:280 Indian Academy of Sciences. https://doi.org/10.1007/s12046-022-01998-w.

5. **Bhattacharya, A., Pal, M. (2021).** Optimization in business strategy as a part of sustainable economic growth using clique covering of fuzzy graphs, Soft Computing, doi: 10.1007/s00500-021-05670-z.

6. **Bhattacharya, A., Pal, M. (2021).** Fuzzy covering problem of fuzzy graphs and its application to investigate the Indian economy in new normal. Journal of Applied Mathematics and Computing, 68, 479–510.

7. **Bhattacharya, A., Pal, M. (2021).** Fifth Sustainable Development Goal, Gender equality in India: Analysis by Mathematics of Uncertainty and Covering of Fuzzy Graphs. Neural Computing and Applications, 33, 15027–15057.

8. **Bhattacharya, A., Pal, M. (2021).** Vertex covering problems of fuzzy graphs and their application in CCTV installation. Neural Computing and Applications; 33:5483–5506. doi:10.1007/s00521-020-05324-5.

9. **Bhattacharya, A., Pal, M. (2023).** A Fuzzy Graph Theory Approach to the Facility Location Problem: A Case Study in the Indian Banking System. Mathematics, 11, 2992. https://doi.org/10.3390/math11132992.

10. **Bhattacharya, A., Pal, M. (2024).** Prediction on nature of cancer by fuzzy graphoidal covering number using artificial neural network. Artificial Intelligence in Medicine; 148: 102783.

11. **Bhattacharya, A., Pal, M. (2024).** Covering of fuzzy graphs and its application in emergency aircraft landing by particle swarm optimization. Applied Soft Computing; 165: 112035.

12. **Bhattacharya, A., Muhiuddin, G., Al-subie, T., Al-Kadi, D., Pal. M. (2025).** Improvement strategy making in sustainable developmental goal for Climate Action by fuzzy vertex covering of fuzzy graphs. New Mathematics and Natural Computation; DOI: 10.1142/S1793005726500390.

13. **Broersma, H. J., Hoede, C. (1989).** Path graphs. J. Graph Theory, 13 (4); 427-444.

14. **Chowdhury, S., Sarkar, R. R. (2014).** Comparison of human cell signaling pathway databases-evaluation, drawbacks and challenges. Databases, 1-25; doi: 10.1093/database/bau126.

15. **Das, K., Samanta, S., Pal, M. (2018).** Study on centrality measures in social networks: a survey. Social Network Analysis and Mining, 8 (1), doi: 10.1007/s13278-018-0493-2.

16. **Delgado, M., Verdegay, J. L., Vila, M. A. (1988).** A procedure for Ranking Fuzzy Numbers using Fuzzy Relations. Fuzzy Sets and Systems, 26: 49-62.

17. **Dombi, J., Jonas, T. (2020).** Ranking trapezoidal fuzzy numbers using a parametric relation pair. Fuzzy Sets and Systems, 399: 20-43.

18. **Dubois, D., Prade, H. (1983).** Ranking Fuzzy Numbers in the Setting of Possibility Theory. Information Sciences, 30, 183-224.

19. **Ge, Z., Leighton, J., Wang, Y., Peng, X., Chen, H., Sun, Y., Yao, F., Li, J., Zhang, H., et al. (2018).** Integrated Genomic Analysis of the Ubiquitin Pathway across Cancer Types. Cell Rep 23. doi.org/10.1016/j.celrep.2018.03.047.

20. **Ghorai, G., Pal, M. (2017).** Certain types of product bipolar fuzzy graphs. International Journal of Applied and Computational Mathematics, 3 (2): 605-619, doi: 10.1007/s40819-015-0112-0.

21. **Harary, F., Schwenk, A. J. (1972).** Evolution of the path number of a graph; covering and packing in graphs-II. Graph Theory and Computing, Eds. R.C. Road, Academic Press, New York; 39-45.

22. **Hoadley, K. A., Yau, C., Hinoue, T., Wolf, M. D., Lazar, J. A., Drill, E., Shen, R., Taylor, M. A., Cherniack, D. A., Akbani, R., et al. (2018).** Cell-of-Origin Patterns Dominate the Molecular Classification of 10,000 Tumors from 33 Types of Cancer. Cell 173. https://doi.org/10.1016/j.cell.2018.03.022.

23. **Klein, C. M. (1991).** Fuzzy shortest paths. Fuzzy Sets and Systems, 39 (27-41).

24. **Lin, K., Chen, M. (1994).** The Fuzzy Shortest Path Problem and its Most Vital Arcs. Fuzzy Sets and Systems, 58: 343–353.

25. **Mordeson, J. N., Peng, C. S. (1994).** Operation on fuzzy graphs. Information Sciences, 79: 159–170.

26. **Nayeem, S. K., Pal, M. (2005).** Shortest Path Problem on a Network with Imprecise Edge Weight. Fuzzy Optimization and Decision Making, 4: 293–312.

27. **Okada, S., Soper, T. (2000).** A Shortest Path Problem on a Network with Fuzzy Arc Lengths. Fuzzy Sets and Systems, 109: 129-140.

28. **Pakkiam, C. Arumugam, S. (1989).** On the graphoidal covering number of a graph. Indian J. Pure Appl. Math., 20 (4): 330-333.

29. **Samanta, S., Pal, M. (2015).** Fuzzy Planar Graphs. IEEE Transactions on Fuzzy Systems, 23 (6): 1936-1942, doi: 10.1109/TFUZZ.2014.2387875.

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