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S&P/BMV IPC Forecasting Using Quantum Long Short-Term Memory

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Abstract. This paper presents results on time series forecasting using a Quantum Long Short-Term Memory, known by its acronym QLSTM. We present experimental results about the forecasting on the Mexican financial closing price dataset (S&P/BMV IPC), In addition to also making a regression on the time series related to the closing value of companies relevant to this financial index, where their probability distributions and their correlation dimensions are analyzed to validate that they show behaviors that are not linear and persistent, which describe their complex behavior and reinforce the idea of using more appropriate tools such as quantum neural networks. To evaluate the performance of the QLSTM, we make a continuous comparison of the results obtained by its classical counterpart, the LSTM neural network. It is noteworthy that this work represents the first publication in non-linear time series prediction applied to a Mexican stock index using quantum computing. See the code: `https://github.com/JordiFGonzalezC/QLSTM_Close`

Keywords. QLSTM, quantum finance, quantum time series forecasting, non-linear systems.

1 Introduction

1.1 Background

Our study builds on the QLSTM [7], proposed by Samuel Yen-Chi Chen, Shinjae Yoo, and Yao-Lung L. Fang in 2020 in their article "Quantum Long Short-Term Memory", where they show that this architecture can make regression on soft functions such as sines and cosines, but leaves open the question of whether it is capable of forecasting on time series with greater complexity. They introduced the architecture for the QLSTM which implements Variational Quantum Circuits (VQC) instead of using the classical LSTM architecture to reinterpret it into a quantum recurrent neural network.

As outlined in [7], the proposed architecture enables the encoding of real numbers into quantum states, essentially mapping these values to an angular value within a qubit. However, part of the objective of exploring quantum computing is to show and validate the hypothesis that it is possible to find the quantum version that behaves equal or better than the already known classic algorithms. In this case, experiment with this quantum recurrent neural network for estimating the closing values in the stock market for S&P/BMV IPC index using quantum simulators, we expect to have better performance and faster decrease in the loss functions than its classical counterpart.

S&P/BMV IPC, also known as the Mexican Stock Exchange (Bolsa Mexicana de Valores or BMV), is one of the major stock exchanges in Mexico. IPC (Índice de Precios y Cotizaciones) is the main stock market index in Mexico. For this case, four important companies that contribute to the calculation of the S&P/BMV IPC were analyzed. The chossing rule was to use data that have complete historical records from January 2020 to July 2024 in Investing.com website, so we can have a fairer comparison:

- Grupo Televisa SAB Unit (TLEVISACPO),
- Coca-Cola Femsa SAB de CV (KOFUBL),

- Grupo Carso, S.A.B. De C.V. (GCARSOA1),
- Grupo Financiero Inbursa, SAB De CV (GFINBURO).

Estimating these future values could help to reduce risks, because accurate predictions can help investors make informed decisions, anticipate market movements, and manage their portfolios more effectively.

The paper is structured as follows: Introduction, this section reviews the QLSTM architecture and describes its main components. Here, we also characterize the data that will be used and concludes on why these data are considered as non-linear, and according to [4], consider that presents persistent behaviors. In the Experimental Development section, we delve into the methodology employed to address the forecasting problem and the challenges encountered by using the QLSTM. Finally, in the Results section, we analyze and compare the prediction outcomes of the QLSTM against the LSTM and we highlight the results achieved by employing a quantum algorithm for time series forecasting.

1.2 Quantum Long Short-Term Memory Architecture

The QLSTM architecture is an innovative variant of the classical Long Short-Term Memory (LSTM) recurrent neural network that incorporates the principles of the Variational Quantum Circuits. In QLSTM, it's not possible to call this architecture totally quantum, since as can be seen in Fig. 1, there are still parts that are totally classical, such as additions, multiplications, sigma functions and hyperbolic tangents are the usual ones, so there are dependencies even with conventional computing, but this may vary between authors.

Like in the classic version LSTM [15], its quantum version is designed to process sequential data, making it well-suited for tasks like natural language processing and time series analysis, as reported in recent applied work on signal processing [22], which shows the value that quantum computing has added when used in certain processes to increase computing capacity due to its quantum properties.s, potentially offering advantages in tasks that involve complex temporal dependencies or large-scale data. Basically the QLSTM architecture is kind of same structure than its classical counterpart, contrasting the quantum version that uses the variational quantum circuits as is shown in the architecture in the Fig.1 as VQC boxes.

This type of recurrent neural network is commonly used in prediction tasks and natural language processing. In this context, x_t represents the input vector containing the time series data to be analyzed—specifically, the closing values for each company and the S&P/BMV IPC close value, which serve as the input to the network. The output (or also called the hidden state), h_t, is a vector of predicted values, while C_t, according to the LSTM architecture, refers to the cell memory. This memory acts as a collector that stores and discards information, facilitating the generation of future values, as usual in recurrent neural networks, the C_{t-1} and h_{t-1} vector, are the feedback values, due to the architecture operating within a range of 0 to 1, dictated by the activation functions, it is crucial to normalize the input data as a prerequisite [25].

The QLSTM shares the same gates as its classical counterpart, with each gate serving a distinct function.

Forget Gate: The forget gate controls which information from the previous cell state should be discarded. It takes the previous hidden state and the current input, applies a sigmoid function, and produces a value between 0 and 1 for each element in the previous cell state C_{t-1}. This value determines the degree to which the previous information should be forgotten.

Input Gate: The input gate decides which values from the input will be used to update the cell state. This gate is composed of two parts: a sigmoid layer that determines which values will be updated, and a $\tanh$ layer that generates a vector of new candidate values that could potentially be added to the cell state.

Lastly, the **Output Gate** determines the output of the current cell. It combines the previous hidden state and the current input, applies a sigmoid function, and then multiplies this result by the $\tanh$ of the updated cell state. This process ultimately decides what the next hidden state h_t should be.

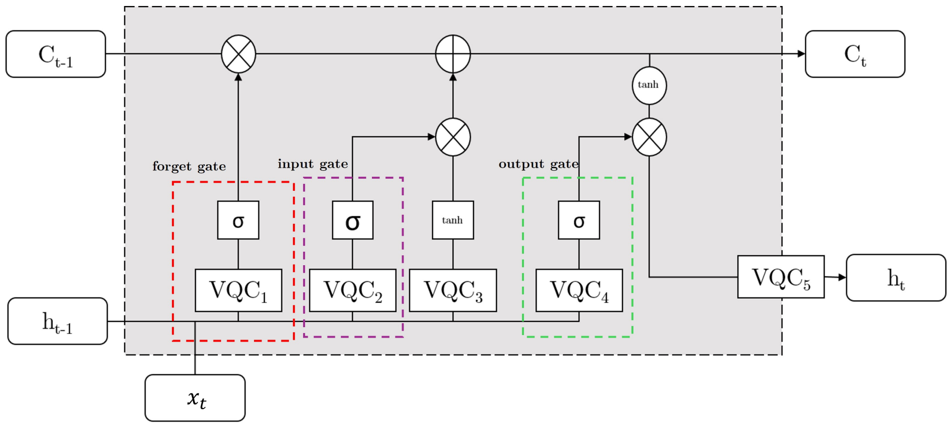

Fig. 1. QLSTM architecture [7]

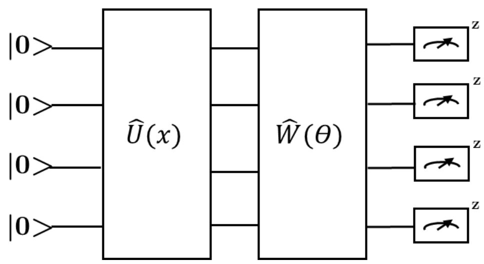

Fig. 2. Quantum circuit to encode data

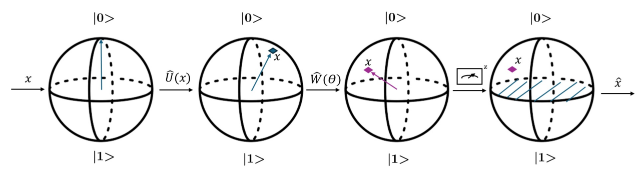

Fig. 3. Quantum states changing through the operators

1.3 Variational Quantum Circuit (VQC)

The quantum improvements comes from the using of VQC boxes that do the work of encoding the information into quantum states, and then seek an adequate distribution of the information entered within the qubit using a proposal function called in the literature "ansatz" that works experimentally (by trial and error) for this numerical data, here the best angular values are searched using some optimization method such as gradient descent [27], and in the end the expected value is obtained with a measurement. This process can be seen as the following general quantum circuit:

From Fig. 2, the quantum operator $\hat{U}(x)$ is the function that processes the time series data x. It is noteworthy that this can be seen as a transformation of data from the real numbers $\mathbb{R}$ to the Hilbert space $\mathcal{H}$, which, in the case of a qubit, is the Bloch sphere [3]. Next, the operator $\hat{W}(\theta)$ is a function that depends on parameters, in this case, angular ones, which are tuned using classical machine learning techniques [27]. The main objective is to entangle the qubits and reposition the values x on the Bloch sphere conveniently according to a parameter θ.

Analytically the transformation of the quantum state can be seen as the following expression:

$$
\begin{aligned}
|0\rangle^{\otimes n} &\rightarrow^{\hat{U}(x)} |\psi(x)\rangle = \hat{U}(x)|0\rangle^{\otimes n} \\
&\rightarrow^{\hat{W}(\theta)} \hat{W}(\theta)|\psi(x)\rangle = |\psi(x,\theta)\rangle.
\end{aligned} \tag{1}
$$

For the measurement an operator $\hat{M}$ is defined, then the complete circuit can be seen as the following function of the expected value of the quantum state $|\psi(x,\theta)\rangle$:

$$
f(x,\theta) = \langle\psi(x,\theta)|\hat{M}|\psi(x,\theta)\rangle = \hat{x}. \tag{2}
$$

These changes in the quantum state within the Bloch sphere can be better seen in Fig. 3. This description may be useful for understanding how data can be encoded within a qubit. Since VQC are parametric quantum circuits, it implies that a classical computer is necessarily required for their proper functioning and training. The forms of the quantum operations $\hat{U}(x)$ and $\hat{W}(\theta)$ are arbitrarily selected and do not follow any strict methodology for their creation, so their design is based on trial and error experimentally [20].

Once this part is understood, the VQC proposal given by [7] can be comprehended, where the part marked in red represents $\hat{U}(x)$ and in green $\hat{W}(\theta)$. According to their results, an adequate performance for encoding real information in the quantum neural network is reported.

It is necessary to question: Why does this ansatz work? Is it possible to propose a different trigonometric function for encoding? What happens if we add another layer of arbitrary quantum operators? For the moment, this study aims to use this validated proposal to forecast the S&P/BMV IPC index.

We can divide this quantum circuit into three layers, the first layer, known as the encoding layer, is where we map the real numbers into the qubits. This is achieved using Hadamard gates (H) and rotational operators (R_y, R_z) along the y, z axes. In the Fig. 4, it is shown that each element in the time series is mapped to each qubit using the $arctan$ function. The subsequent layer is the variational part, where qubits are entangled and rotated by certain angles (α, β, γ) around each x, y, z axis, respectively.

The experimentation considers only four qubits to execute the circuit. However, the code [12] allows to extend the experimentation to n qubits.

1.4 Data Categorization Using Grassberger-Procaccia Algorithm

The data for the S&P/BMV IPC and the closing values for Gurpo Carso, Coca-Cola, Inbursa and Televisa were obtained from the Investing.com site [31], the information obtained was from January 1, 2020, to July 22, 2024, it is known that the financial markets exhibit complex and often non-linear behavior [13], making it crucial for analysts and traders to employ sophisticated tools to grasp the inherent unpredictability. The use of the Grassberger-Procaccia algorithm to estimate the correlation for the analysis of time series and dynamical systems to quantify complexity and fractal structure is a common tool used in the literature [23].

The correlation dimension of the time series will be discussed as the result obtained after applying the Grassberger-Procaccia algorithm as D_2.

The correlation dimension, originally developed in the field of nonlinear dynamics and chaos theory [23], have found a unique and increasingly important role in the realm of finance. These mathematical constructs offer a quantitative framework for assessing the sensitivity of financial systems to initial conditions, shedding light on the intricate interplay of factors that drive market fluctuations. By examining the correlation dimension of financial time series data, analysts can gain valuable insights into the underlying dynamics of asset prices, market volatility, and risk.

To find the correlation dimension, we aplied the calculations for the Grassberger-Procaccia algorithm, from a Github code implementation [18].

This algorithm is a technique used in time series analysis to estimate the correlation dimension, bassically a measure of the complexity and fractal structure of a system for detecting the presence of non-linear behavior in a time series, the steps to estimate the correlation dimension value is as follows.

1.4.1 1. Phase Space Reconstruction

First, the phase space is reconstructed from the one-dimensional time series using the method of time delay. Vectors of embedding dimension m and a time delay τ are created. For a time series $x(t)$, the vectors in the phase space are:

$$\mathbf{X}(t) = (x(t), x(t+\tau), x(t+2\tau), \ldots, x(t+(m-1)\tau))\,.$$

1.4.2 2. Calculation of Distances in Phase Space

Next, the Euclidean distance between all pairs of points in the reconstructed phase space is calculated:

$$d_{ij} = \|\mathbf{X}(i) - \mathbf{X}(j)\|.$$

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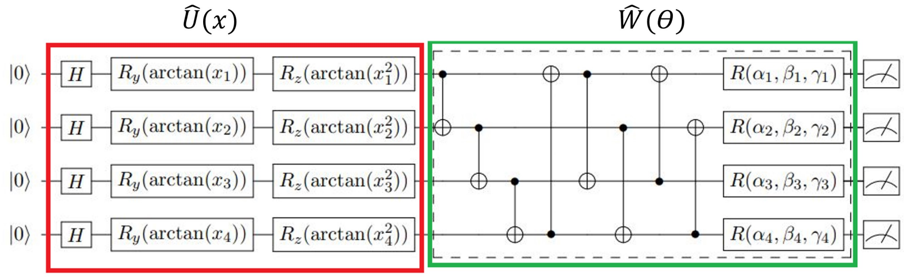


Fig. 4. Variational Quantum Circuit for QLSTM [7]

1.4.3 3. Correlation Function

The correlation function $C(r)$ is then defined, which is the fraction of pairs of points whose distance is less than a threshold r:

$$C(r) = \frac{2}{N(N-1)} \sum_{i<j} H(r - d_{ij}).$$

where H is the Heaviside step function, N is the total number of points in the series, and d_{ij} is the distance between points i and j.

1.4.4 4. Estimation of the Correlation Dimension

The correlation dimension D_2 is estimated by observing how $C(r)$ changes as r varies:

$$D_2 = \lim_{r \to 0} \frac{\log C(r)}{\log r}.$$

In practice, this is done by performing a linear regression of $\log C(r)$ against $\log r$ over a range of r where this relationship is approximately linear.

1.4.5 Estimation of Correlation Dimension for S&P/BMV IPC and Close Values

It is feasible to estimate certain metrics that indicate its non-linear nature. A comprehensive analysis was conducted to characterize the non-linearity of the S&P/BMV IPC. This involved examining the trends in behavior and memory of the prices of the shares comprising the S&P/BMV IPC index in the Mexican stock market, as well as the evolution of returns over time. The findings from these analyses led to the conclusion that the index exhibits a persistent behavior [4] instead of chaotic.

Using correlation function, the obtained value for D_2 for S&P/BMV IPC is shown in the graph in Fig. 5 for different candidate embedding dimensions, this measure provides information about the complexity and structure of the underlying time series.

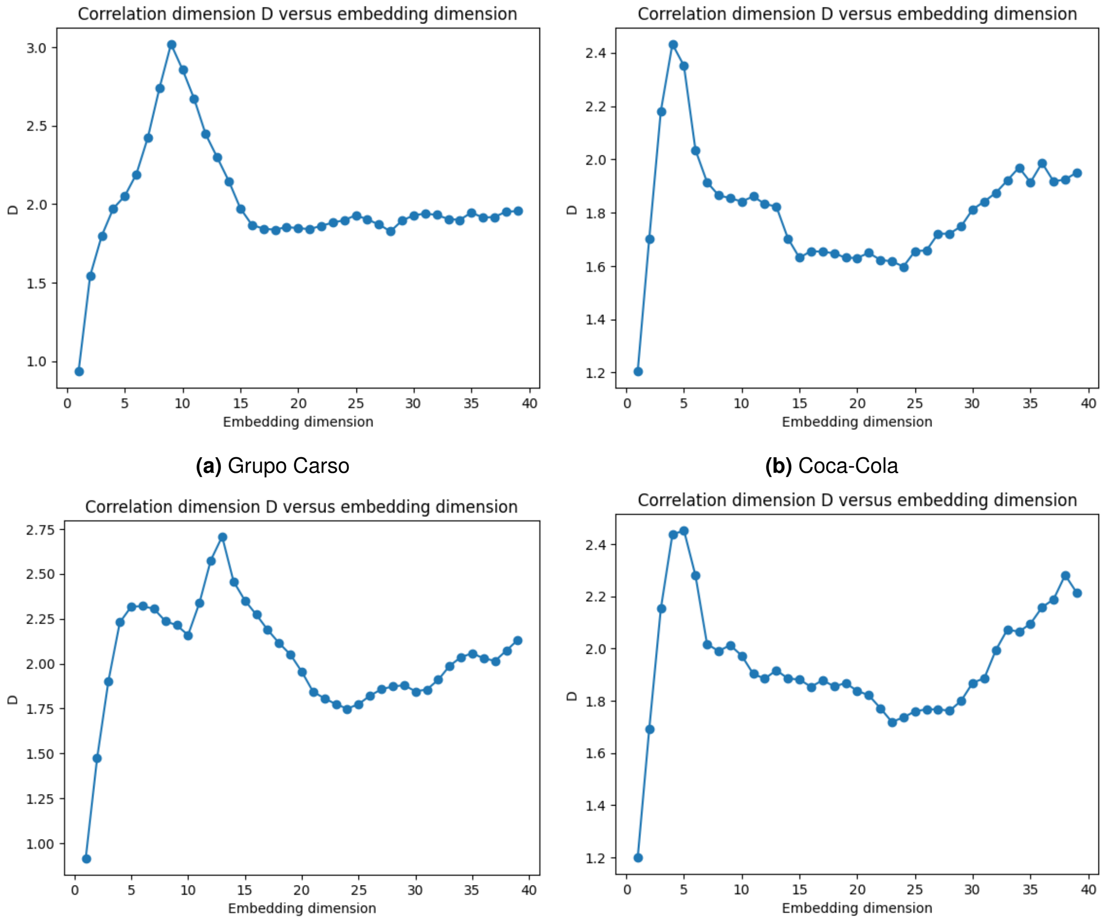
For the four graphs, it is noted that the trend as the time series is analyzed, the value of the correlation dimension becomes smaller, that is, it decreases, which suggests a more complex or non-linear structure in the dynamic system [30], but as it is inconclusive, the definition of persistent time series will be employed.

To reinforce the premise that the data do not present a linear behavior or rather, that they fit a probability distribution, the histograms of each set of data are obtained, as can be seen in the graphs, none of them can fit a normal distribution, so these time series can hardly be categorized as stochastic. As shown in Fig. 6d, the data distributions prevent us from using classical probability methods to make estimates on the S&P/BMV IPC, so the use of this type of quantum computing tools is justified.

2 Experimental Development

2.1 Methodology

To conduct the experiments, the data was gathered from the closing values of the S&P/BMV IPC index and individual companies, using the public website [31]. The data was then filtered to include



(a) Grupo Carso **(b)** Coca-Cola

(c) Inbursa **(d)** Televisa

Fig. 5. Correlation dimension for the closing values for a) Grupo Carso, b) Coca-Cola, c) Inbursa, and d) Televisa

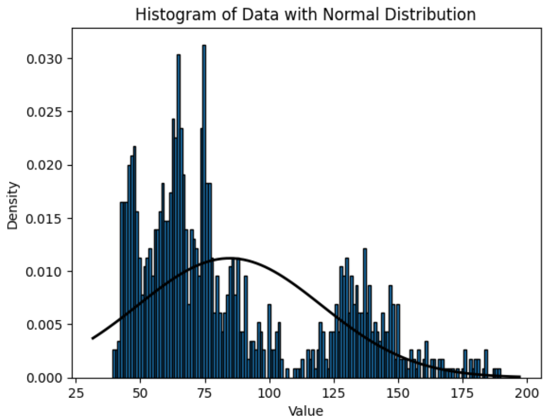
only historical information from January 1, 2020, to July 22, 2024.

The closing value for each time series where mapped into $[0, 1]$ values, after that, the data was characterized by calculating correlation dimensions and generating histograms to define its nature. Subsequently, the code for the QLSTM model was developed.

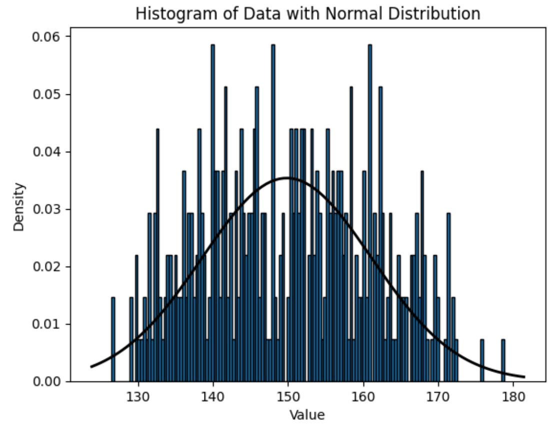
The training of the QLSTM was the most time consuming-time process in this work, the quantum parameters training could take 10 time more than a classic LSTM using the same data and computer, this could represent a disadvantage for our proposal. For the mean squared error (a loss metric function) is used to determine whether the training is improving or deteriorating. This is crucial for deciding when to stop the training and avoid overfitting, as continuing the process can lead to gradient explosion and an increase in error instead of a decrease.

3 Results

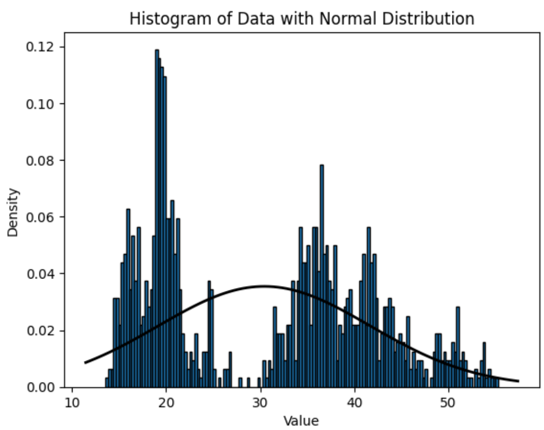
The prediction graphs for the closing values of Grupo Carso, Coca-Cola, Inbursa, and Televisa show similar behavior when using both LSTM and QLSTM. In fact, the graphs overlap and are difficult to distinguish from one another. This indicates



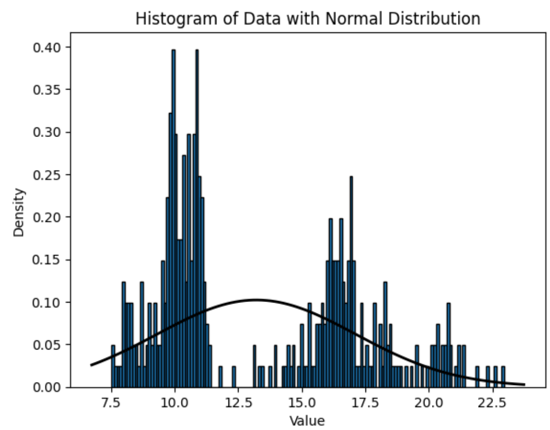
(a) Grupo Carso



(b) Coca-Cola



(c) Inbursa



(d) Televisa

Fig. 6. Histograms for the closing values for a) Grupo Carso, b) Coca- Cola, c) Inbursa, and d) Televisa

that for these particular companies, there is no significant impact on the behavior of the data, validating the premise that such financial time series follow a persistent trend.

The results in Fig. 8 compare LSTM and QLSTM architectures for forecasting the closing values of Grupo Carso, Coca-Cola, Inbursa, and Televisa. Both models generally track the actual stock prices closely, with the vertical dashed line indicating the test set start.

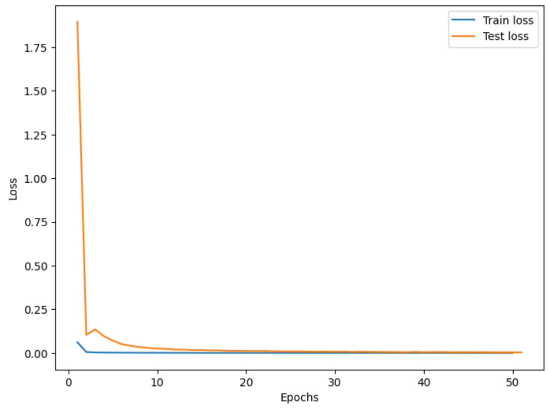
For Grupo Carso Fig. 8 a), both models perform similarly, accurately capturing the overall downward trend.

In case of Coca-Cola Fig. 8 b), both models also handle the general decline well, though LSTM shows a slight edge in predicting short-term fluctuations.

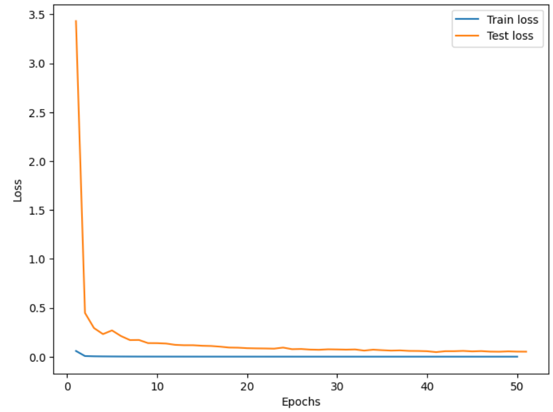
For Inbursa Fig. 8 c), the models adapt well to sharp price increases, with QLSTM offering slightly better alignment with real data during the test period.

Lastly, in Televisa Fig. 8 d), both models effectively capture significant price movements, showing strong predictive performance overall.

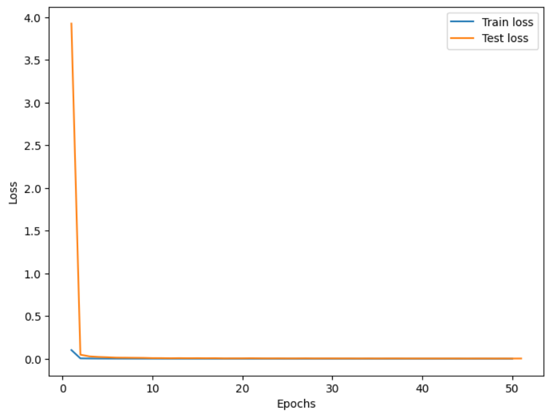
In Table 1, last five days of results are shown, the obtained error (difference between the closure value and the predicted value) is still something that could be improved. However, the index values are around the order of 1×10^4.



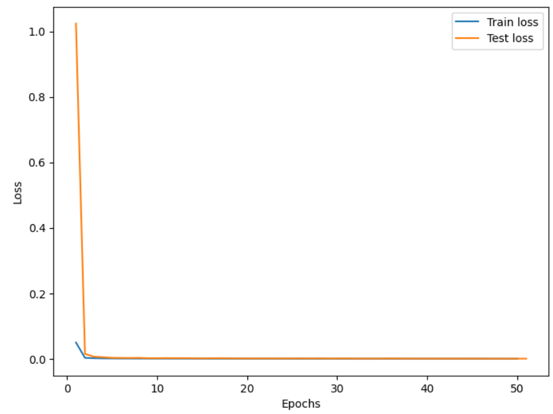
(a) Grupo Carso



(b) Coca-Cola



(c) Inbursa



(d) Televisa

Fig. 7. Loss function for training and testing data sets in the QLSTM architecture for forecasting the closing values of four companies: a) Grupo Carso, b) Coca-Cola, c) Inbursa, and d) Televisa

Table 1. Last five errors obtained in S&P/BMV IPC forecasting using the QLSTM

Date	Close	Model forecast	Error
22.07.2024	44437.23	44395.95	41.28
19.07.2024	44624.85	44693.60	-68.76
18.07.2024	44495.30	44484.23	11.07
17.07.2024	44157.81	44152.37	5.44
16.07.2024	44470.91	44507.41	-36.50

This outcome reinforces the notion that the financial market is complex but it not impossible to use the current technology to forecast its behavior. It underscores the importance of continuing to research these behaviors and raises questions about how these architectures can be improved to be more robust against such uncontrollable situations.

In Fig. 9 presents the forecasting results for the S&P/BMV IPC index using LSTM and QLSTM architectures. Both models appear to closely follow the actual trend of the index, particularly during the test period, capturing key fluctuations and price movements. The SP/BMV IPC shows considerable volatility, with a significant dip early in the dataset followed by a gradual upward trend. Both models perform comparably well in predicting these movements, with minimal divergence from the real data throughout the test set.

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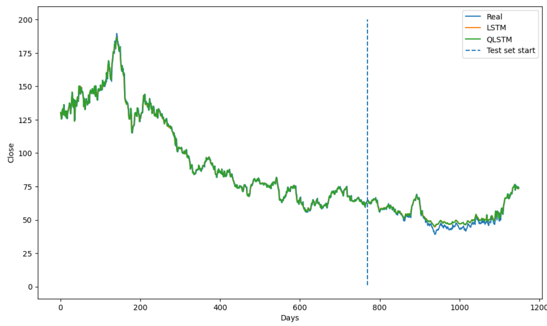
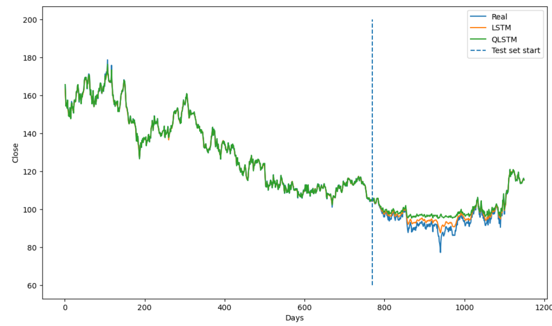
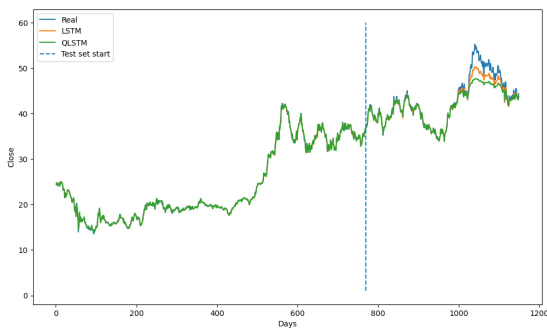


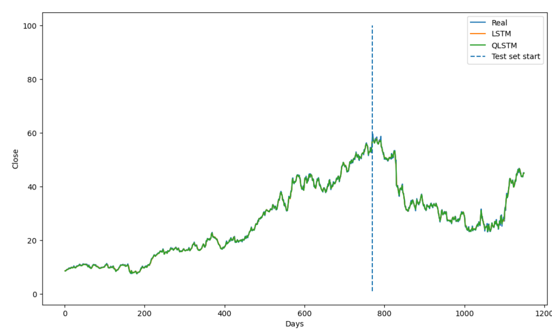
(a) Grupo Carso



(b) Coca-Cola



(c) Inbursa



(d) Televisa

Fig. 8. Results comparison for a) Grupo Carso, b) Coca- Cola, c) Inbursa, and d) Televisa time series forecasting

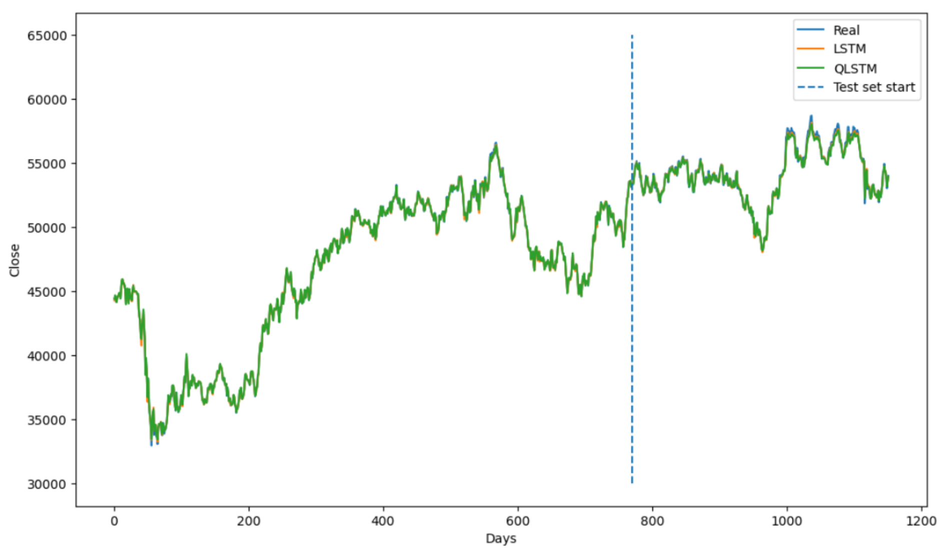


Fig. 9. Forecasting results for S&P/BMV IPC

In the heat map Fig.10, we can observe the correlation between the real time series vs. the forecasts, in case for Grupo Carso, the correlation between the LSTM prediction and the actual data

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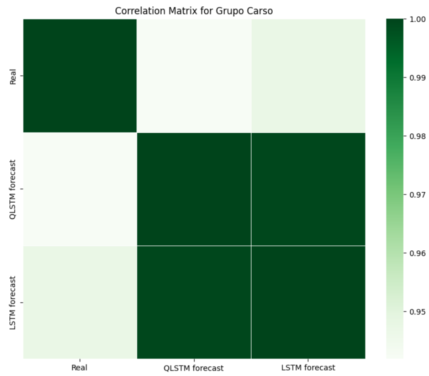
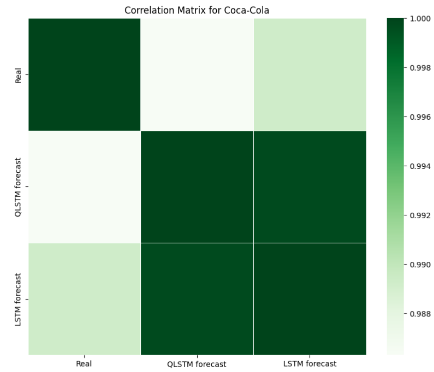
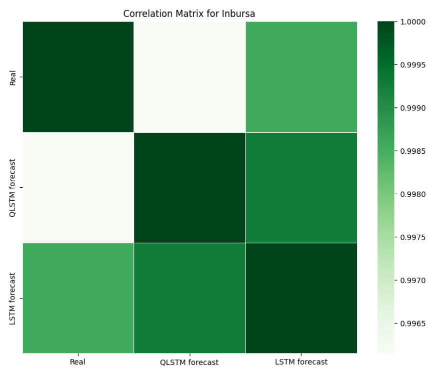


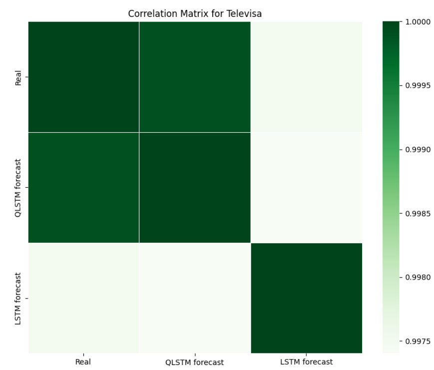
(a) Grupo Carso



(b) Coca-Cola



(c) Inbursa



(d) Televisa

Fig. 10. Heat map for a) Grupo Carso, b) Coca-Cola, c) Inbursa and d) Televisa time series correlation

is slightly higher than its quantum version.

A similar behavior is seen for Coca-Cola and Inbursa, where the classical version slightly outperforms the quantum one.

However, for Televisa, there is a clear improvement in correlation with the quantum version.

In contrast, for the estimation and prediction of the S&P/BMV IPC shown in Fig. 11, the QLSTM architecture has better results than the classical version.

4 Conclusion

This work represents a continuation of the exploration into the application of Quantum Long Short-Term Memory (QLSTM). The proposed architecture utilizes Variational Quantum Circuits (VQC), aiming to predict non-linear time series, particularly closing values in the S&P/BMV IPC stock market index.

An achievement of this work is demonstrating the possibility of matching the performance of the most effective classical algorithms with quantum computing. In the metrics presented, it was observed how the quantum version showed no distinction from the classical one. Furthermore, the application of these methods for time series prediction demonstrates their effectiveness even in the case of complex time series, which, in theory, pose a significant challenge for analysis and prediction. This serves as a clear example of the capabilities quantum machine learning has in predicting systems that, in principle, cannot be forecasted from a deterministic or stochastic perspective.

As shown in the heat map results, the correlation of the output data presents similar or, in some

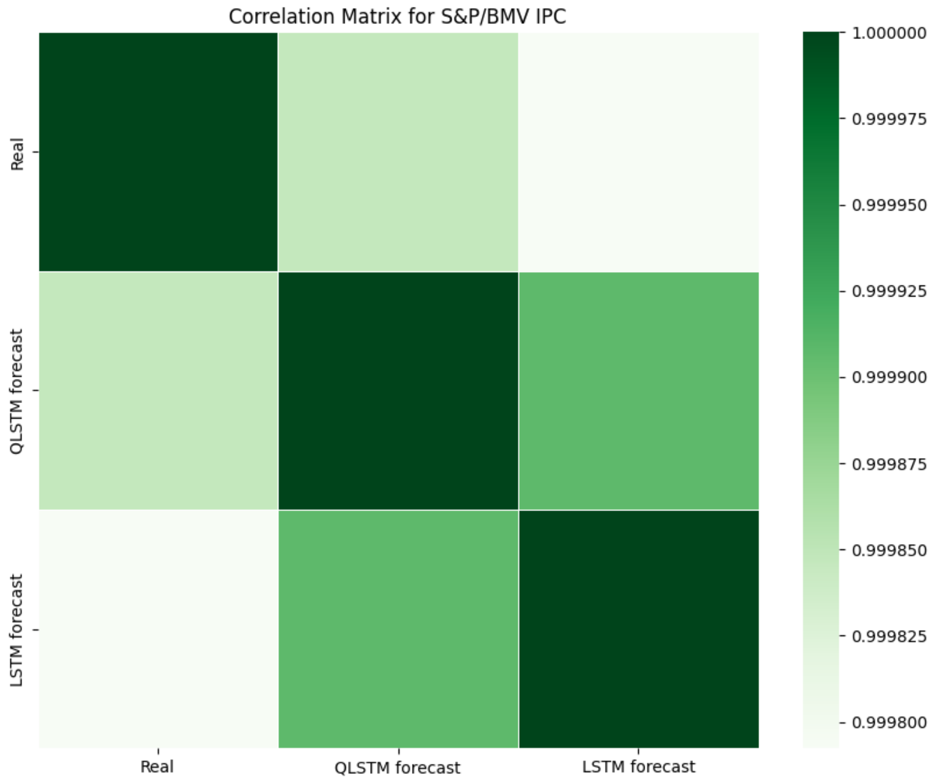


Fig. 11. Heat map for S&P/BMV IPC time series correlation

cases, a better performance for the quantum version, which opens a favorable panorama for the use of quantum computing technologies in the area of finance.

In conclusion, the results obtained from applying LSTM and QLSTM architectures to predict the S&P/BMV IPC and the closing values of Grupo Carso, Coca-Cola, Inbursa, and Televisa demonstrate the potential of both models. The S&P/BMV IPC and the closing values for individual companies show a persistent trend that both models can capture effectively, this reinforces the importance of continuing to research and develop more robust prediction models that can be applied in the financial market.

For future work, we aim to implement these models using a real quantum computer, alongside the design of a quantum circuit for quantum error correction (QEC). Exploring alternative quantum solutions is always intriguing, and although the current issues of quantum noise and high costs make real-world usage and operation challenging, the potential of quantum computing to become a reality in the near future could significantly advance the field of Quantum Finance. The continuous development and improvement of quantum technologies promise exciting opportunities for achieving breakthroughs in financial forecasting and beyond.

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